The effect of server's breakdowns on the performance of finite-source retrial queueing systems

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OUTLOOK

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- **Applications**
- **Mathematical model**
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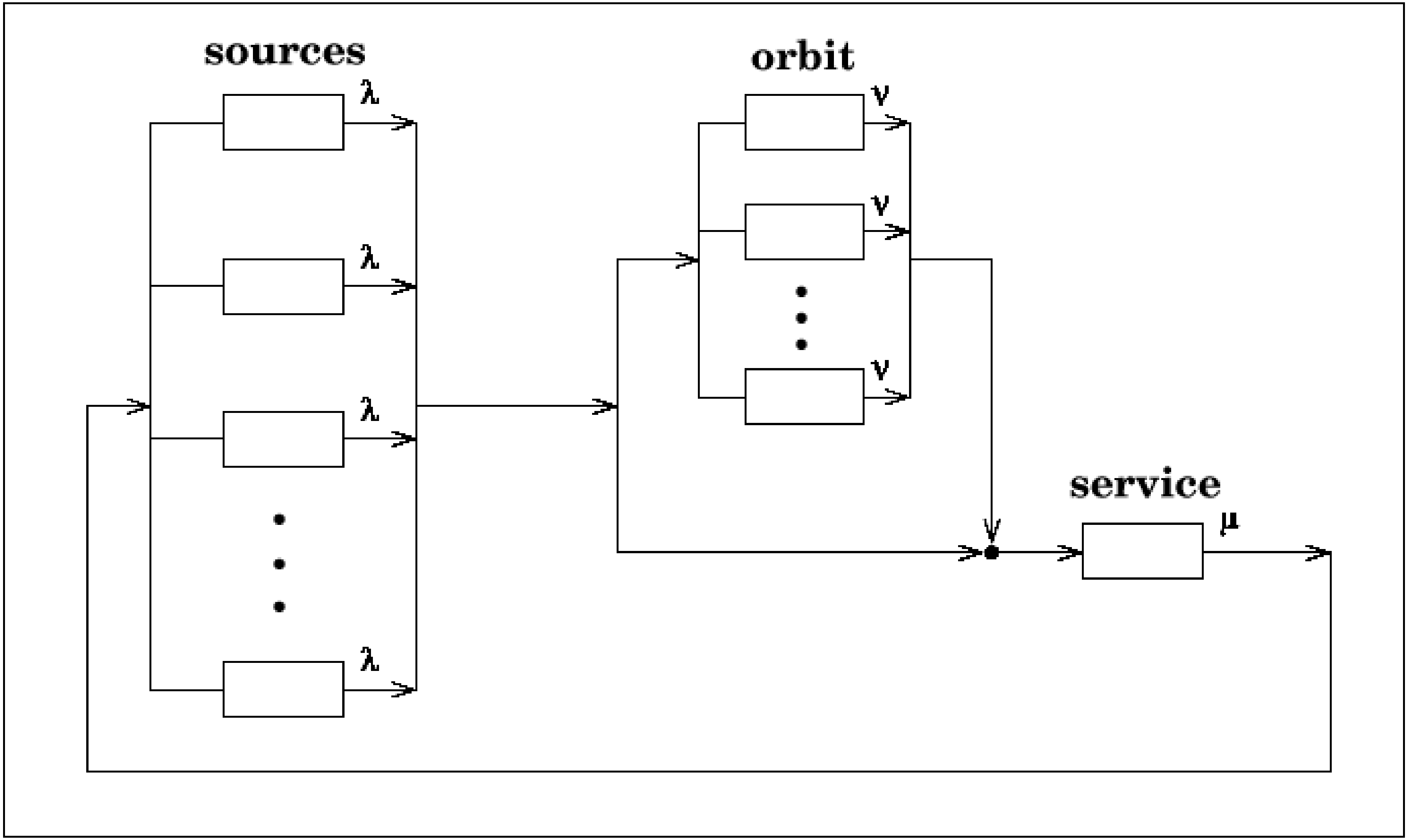
The queueing model



Applications

- **magnetic disk memory systems**
- **local area networks with CSMA/CD protocols**
- **collision avoidance local area network modeling**

Mathematical model

The system state at time t can be described with the process

$$X(t) = (Y(t); C(t); N(t))$$

where $Y(t) = 0$ if the server is up, $Y(t) = 1$ if the server is failed,

$C(t) = 0$ if the server is idle, $C(t) = 1$ if the server is busy,

$N(t)$ is the number of sources of repeated calls at time t.

We define the stationary probabilities:

$$P(q;r;j) = \lim_{t\to\infty} P(Y(t) = q, C(t) = r, N(t) = j)$$

$$q = 0, 1, \quad r = 0, 1, \quad j = 0, ..., K^*,$$

$$\text{where } K^* = \begin{cases} K - 1 & \text{for blocked case,} \\ K - r & \text{for unblocked case.} \end{cases}$$

Once we have obtained these limiting probabilities the **main system performance measures** can be derived in the following way.

1. Utilization of the server

$$U_S = \sum_{j=0}^{K-1} P(0, 1, j)$$

2. Utilization of the repairman

$$U_R = \sum_{q=0}^{1} \sum_{j=0}^{K^*} P(1, q, j)$$

3. Availability of the server

$$A_S = \sum_{q=0}^{1} \sum_{j=0}^{K^*} P(0, q, j) = 1 - U_R$$

4. The mean number of calls staying in the orbit or in service

$$M = E[N(t) + C(t)] = \sum_{q=0}^{1} \sum_{r=0}^{1} \sum_{j=0}^{K^*} j P(q, r, j) + \sum_{q=0}^{1} \sum_{j=0}^{K-1} P(q, 1, j).$$

5. Utilization of the sources

$$U_{SO} = \begin{cases} \frac{E[K - C(t) - N(t); Y(t)=0]}{K} & \text{for blocked case,} \\ \frac{K-M}{K} & \text{for unblocked case.} \end{cases}$$

6. Overall utilization

$$U_O = U_S + K U_{SO} + U_R.$$

7. The mean rate of generation of primary calls

$$\overline{\lambda} = \begin{cases} \lambda E[K - C(t) - N(t); Y(t) = 0] & \text{for blocked case,} \\ \lambda E[K - C(t) - N(t)] & \text{for unblocked case.} \end{cases}$$

8. The mean response time

$$E[T] = M/\overline{\lambda}$$

9. The blocking probability of a primary call

$$B = \begin{cases} \dfrac{\lambda E[K-C(t)-N(t);Y(t)=0;C(t)=1]}{\overline{\lambda}} & \text{for blocked case,} \\ \\ \dfrac{\lambda E[K-C(t)-N(t);C(t)=1]}{\overline{\lambda}} & \text{for unblocked case.} \end{cases}$$

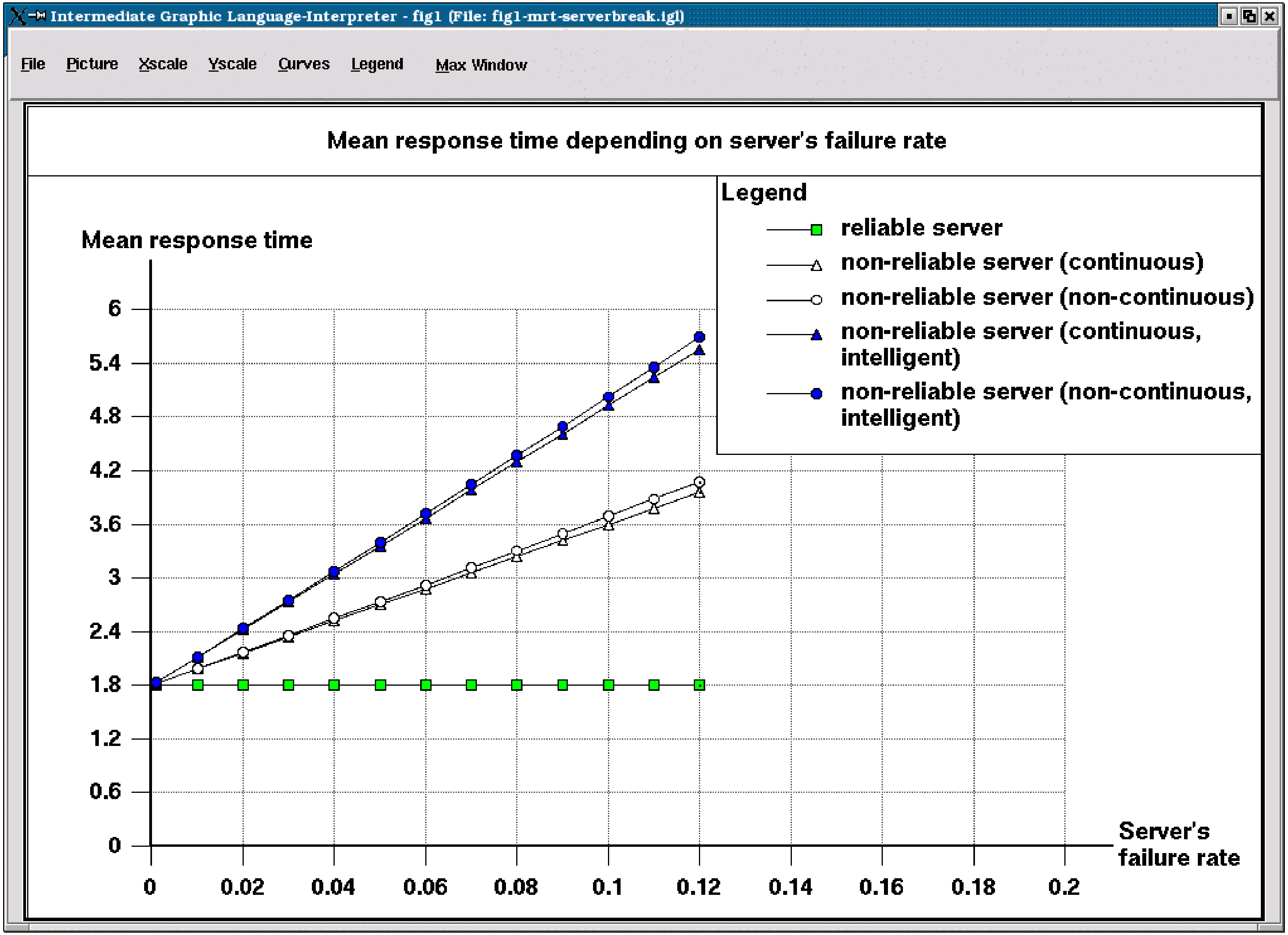
Evaluation Tool MOSEL

MOSEL (Modeling, Specification and Evaluation Language) developed at the University of Erlangen, Germany, is used to formulate and solved the problem.

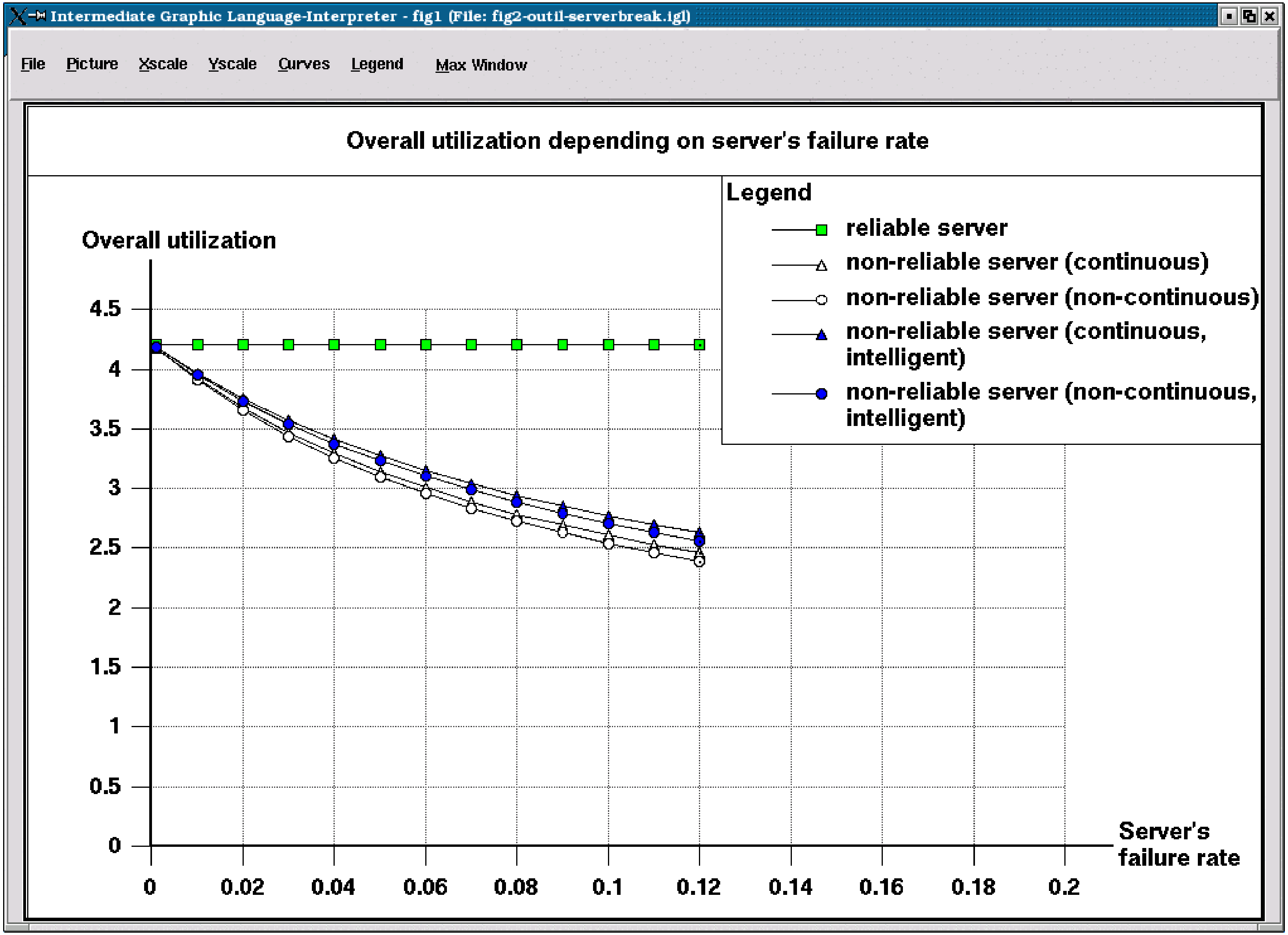
Case studies

	K	λ	μ	ν	δ, γ	τ
Figure 1	6	0.8	4	0.5	x axis	0.1
Figure 2	6	0.1	0.5	0.5	x axis	0.1
Figure 3	6	0.1	0.5	0.05	x axis	0.1
Figure 4	6	0.8	4	0.5	0.05	x axis
Figure 5	6	0.05	0.3	0.2	0.05	x axis
Figure 6	6	0.1	0.5	0.05	0.05	x axis

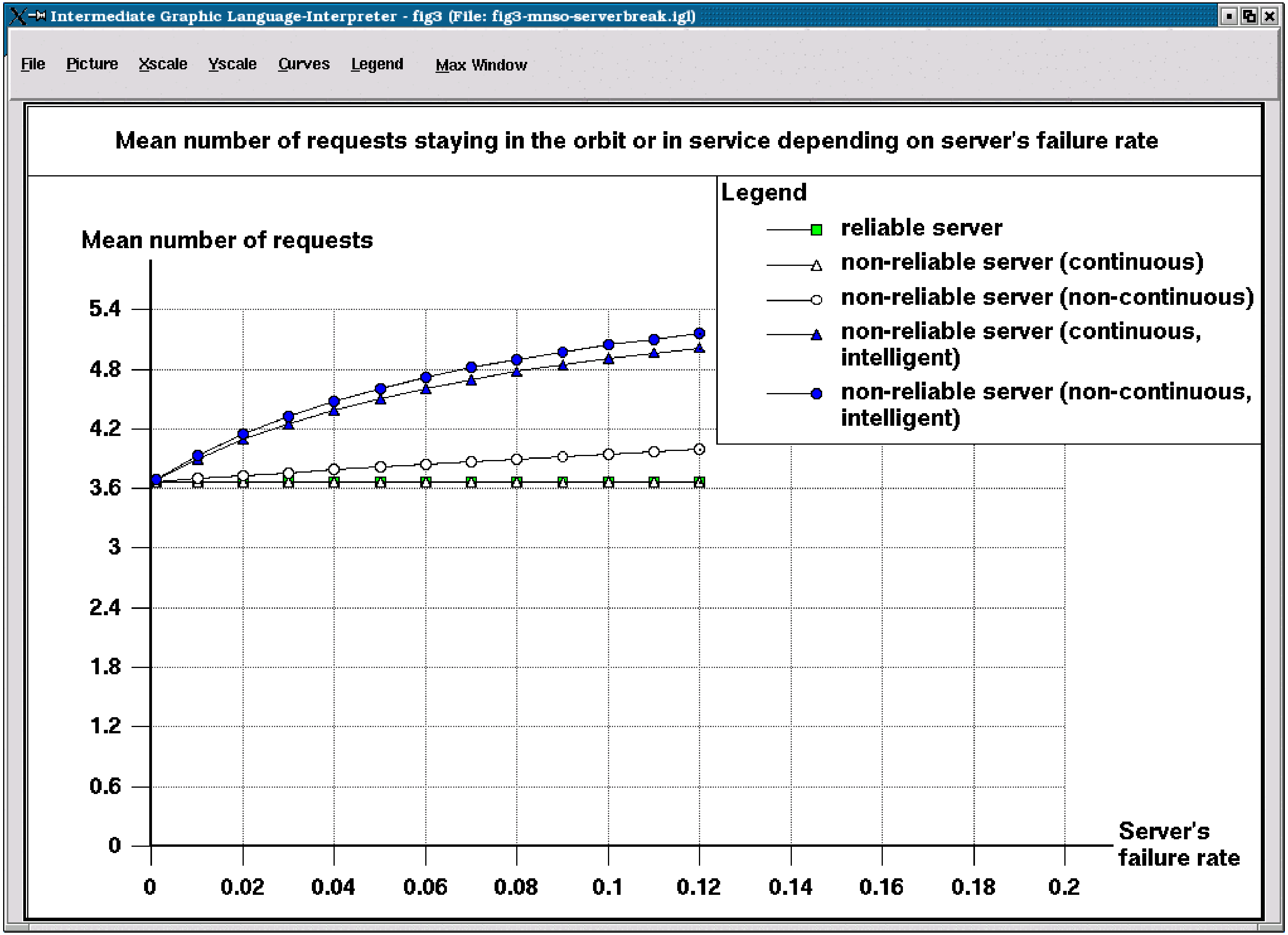
Input system parameters



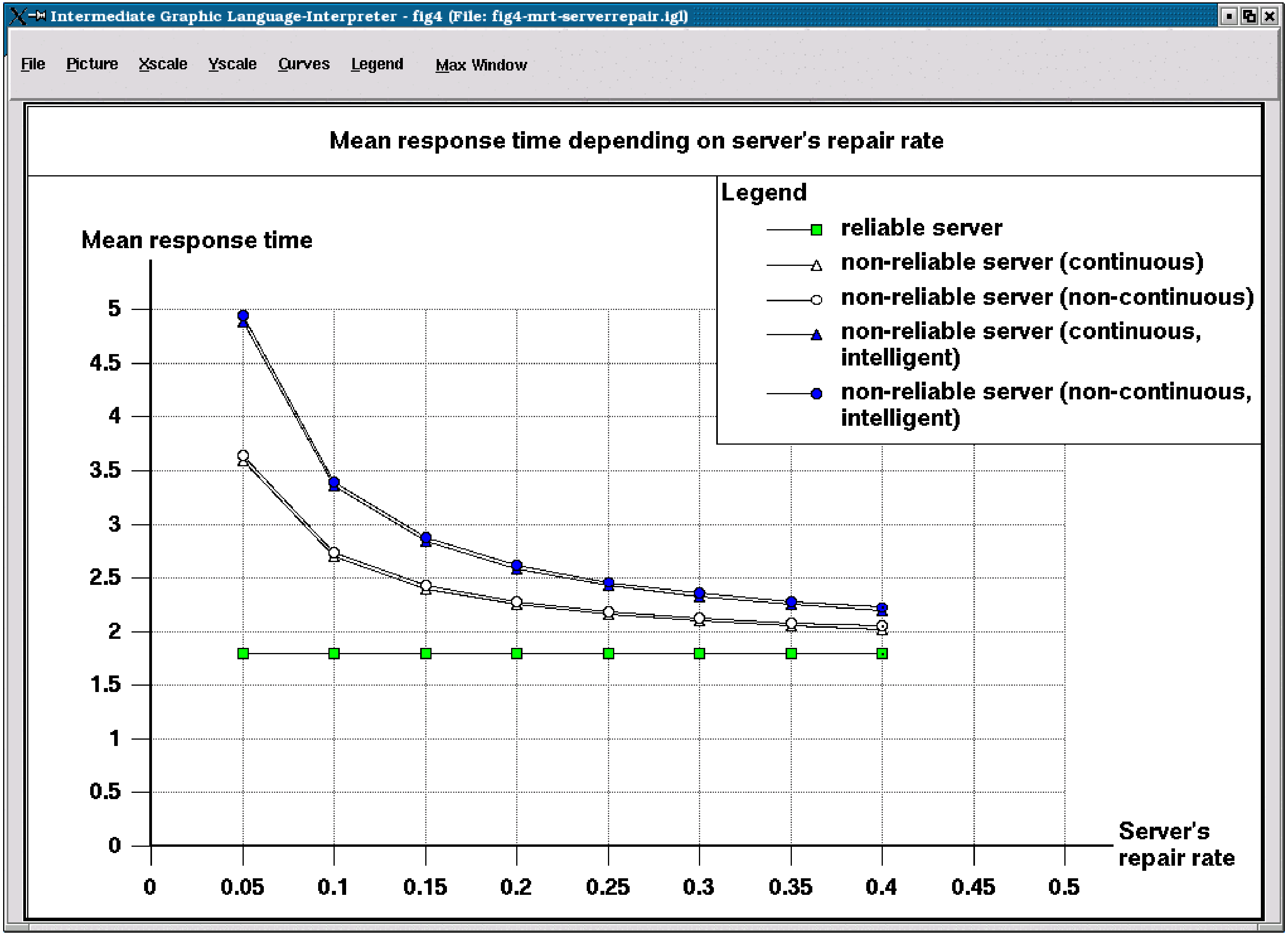
$E[T]$ versus server's failure rate



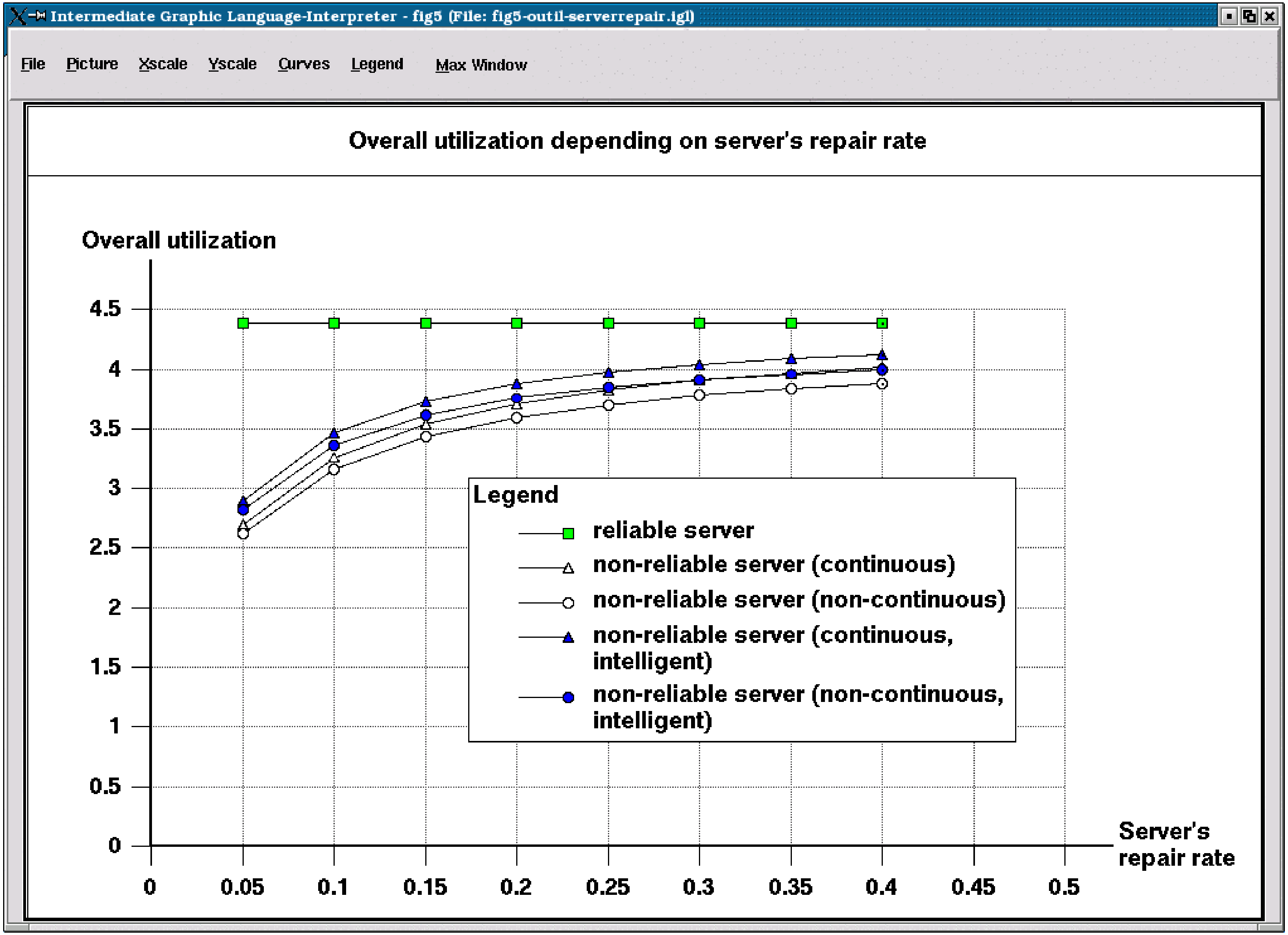
U_O versus server's failure rate



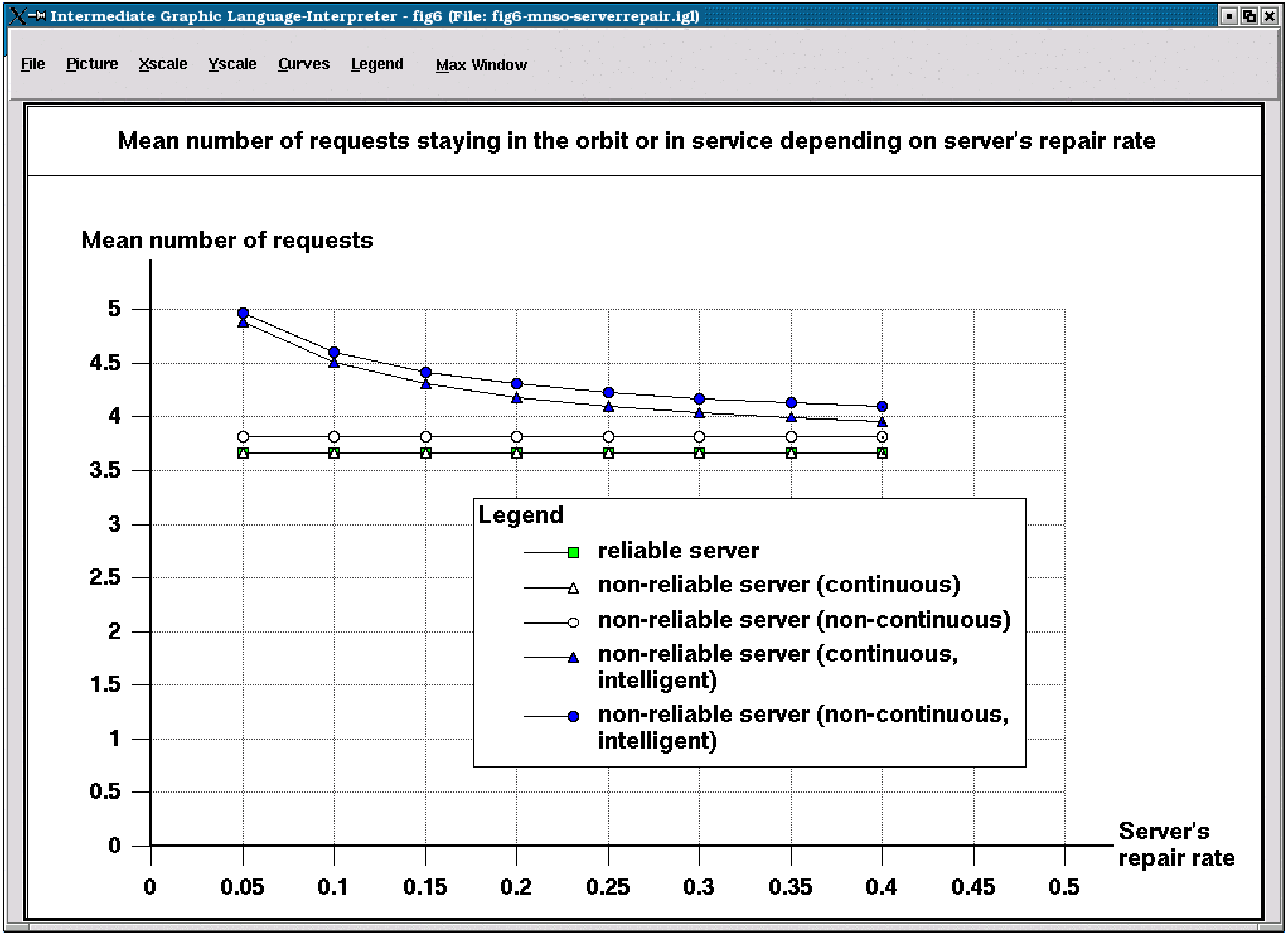
M versus server's failure rate



$E[T]$ versus server's repair rate



U_O versus server's repair rate



M versus server's repair rate

References

[1] **Almási B., Roszik J., and Sztrik J.** Homogeneous finite-source retrial queues with server subject to breakdowns and repairs, *Computers and Mathematics wit Applications* (submitted for publication).

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[3] **Falin G.I. and Templeton J.G.C.** *Retrial queues*, Chapman and Hall, London, 1997.

[4] **Falin G.I. and Artalejo J.R.** A finite source retrial queue, *European Journal of Operational Research* 108(1998) 409-424.

[5] **Wang Jinting, Cao Jinhua and Li Quanlin** Reliability analysis of the retrial queue with server breakdowns and repairs, *Queueing Systems Theory and Applications* 38(2001), 363–380.