Queueing Theory and its Applications

A Personal View

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Outline

- Origin of Queueing Theory
- Classifications of Queueing Systems
- Applications
- Solution Methods
- Basic Formulas and Laws
- Recent Developments
- Hungarian Contributions
- References

Origin of Queueing Theory

Agner Krarup Erlang, 1878-1929

- "The Theory of Probabilities and Telephone Conversations", Nyt Tidsskrift for Matematik B, vol 20, 1909.
- "Solution of some Problems in the Theory of Probabilities of Significance in Automatic Telephone Exchanges", Elektrotkeknikeren, vol 13, 1917.
- "The life and works of A.K. Erlang", E. Brockmeyer, H.L. Halstrom and Arns Jensen, Copenhagen: The Copenhagen Telephone Company, 1948.

Queueing Theory Homepage

http://web2.uwindsor.ca/math/hlynka/queue.html

Murphy's Law of Queue

- If you change queues, the one you have left will start to move faster than the one you are in now.
- Your queue always goes the slowest
- Whatever queue you join, no matter how short it looks, will always take the longest for you to get served.

Google search for "Queueing Theory ": 188 000

Applications

- Telephony
- Manufacturing
- Inventories
- Dams
- Supermarkets
- Computer and Communication Systems
- Call Centers
- Infocommunication Networks
- Hospitals
- Many others

Kendall's Notation

David G. Kendall, 1918-2007

A/B/c/K/m/Z

Performance Metrics

- Utilizations
- Mean Number of Customers in the System / Queue
- Mean Response / Waiting Time
- Mean Busy Period Length of the Server
- Distribution of Response / Waiting Time
- Distribution of the Busy Period

Solution Methodologies

- Analytical
- Numerical
- Asymptotic
- Simulation
- Tools

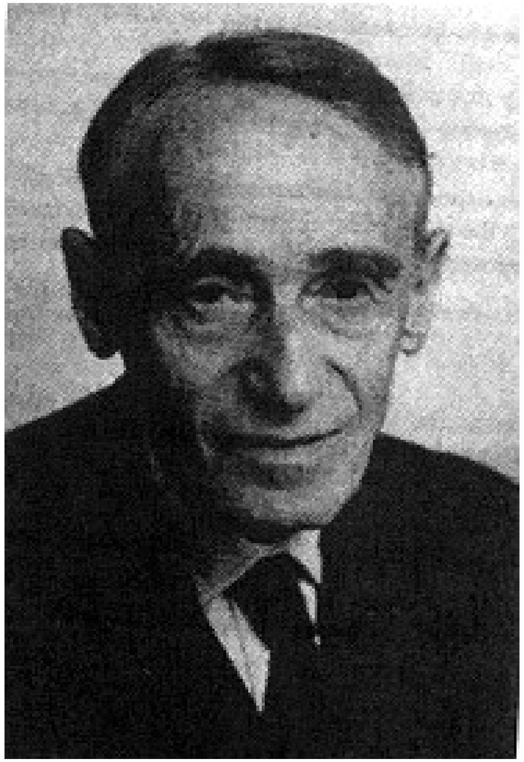
Erlang Loss Formulas, M/G/c/c Systems

$$B(c,\rho) = p_c = \frac{\rho^c/c!}{\sum_{n=0}^{c} \rho^n/n!} \qquad \rho = \lambda E(B)$$

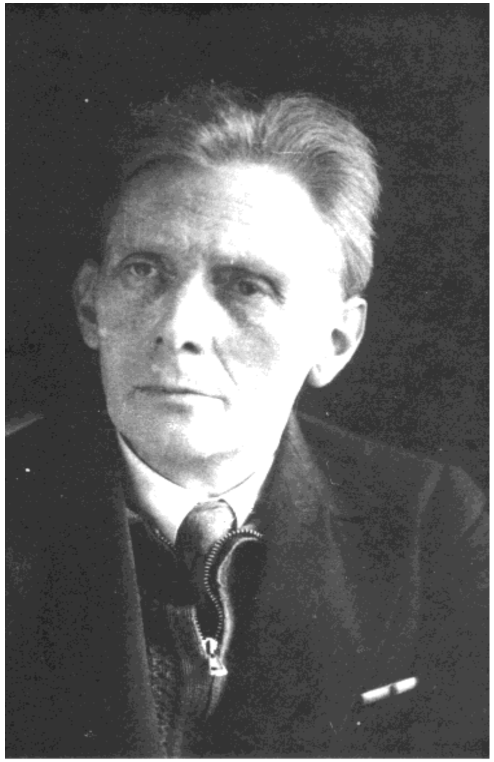
$$B(c,\rho) = \frac{\rho B(c-1,\rho)/c}{1+\rho B(c-1,\rho)/c} = \frac{\rho B(c-1,\rho)}{c+\rho B(c-1,\rho)}$$

$$B(0,\rho) = 1$$

Pollaczek-Khintchine Formulas, M/G/1 Systems



Felix Pollachek, 1892-1981



Alexander Y. Khintchine, 1894-1959

Mean Value Formulas

$$C_X^2 := \frac{\mathrm{Var}[X]}{(E[X])^2}$$

$$E[W] = E[R]\frac{\rho}{1-\rho} = \frac{E[B]}{2}\frac{\rho}{1-\rho}(1 + C_B^2)$$

$$E[T] = E[B]\left(1 + \frac{\rho(1 + C_B^2)}{2(1-\rho)}\right)$$

Transform Formulas

$$G_N(z) = L_B(\lambda(1-z)) \cdot \frac{(1-\rho)(1-z)}{L_B(\lambda(1-z)) - z}$$

$$L_T(s) = L_B(s)\frac{s(1-\rho)}{s - \lambda + \lambda L_B(s)}$$

Little's Law

$$E(L) = \lambda E(S)$$

Recent Developments

Boris Vladimirovich Gnedenko, 1912-1995

Recent Developments

Leonard Kleinrock, 1934 -

Hot Topics

22nd International Teletraffic Congress

September 7-9, 2010, Amsterdam, The Netherlands

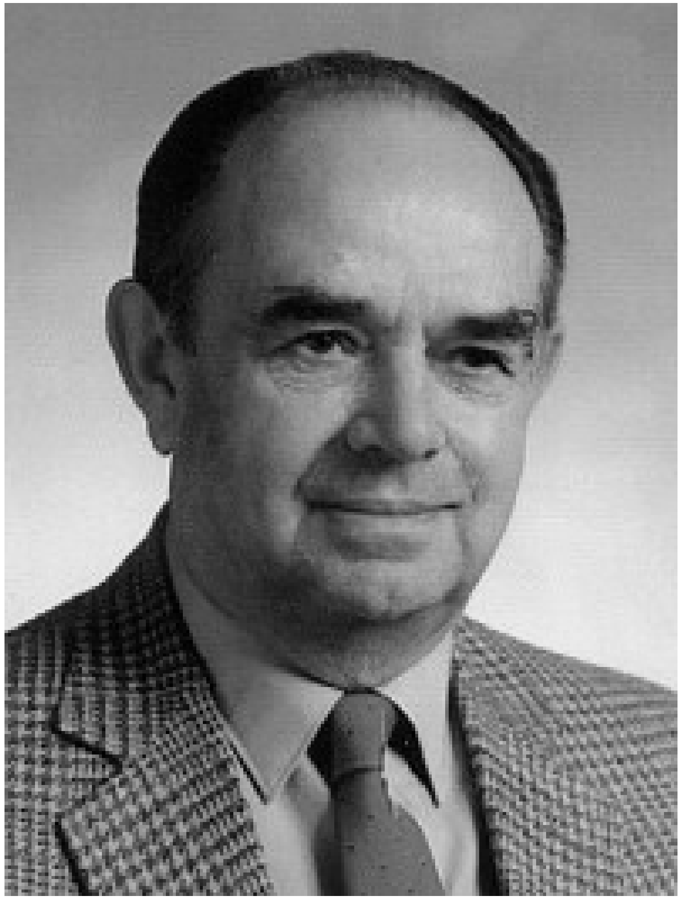
- Performance of wireless/wired networks
- Business models for QoS
- Performance and reliability tradeoffs
- Performance models for voice, video, data and P2P applications
- Scheduling algorithms
- Simulation methods and tools

Madrid Conference, 2010

Third Madrid Conference on Queueing Theory

June 28 - July 1, 2010
to be held in Toledo

Hungarian Contributions



Lajos Takács, 1924 -

Hungarian Contributions

- Eötvös Loránd University
  (A. Benczúr, L. Lakatos, L. Szeidl)
- Budapest University of Technology and Economics
  (L. Györfi, M. Telek, S. Molnár)
- University of Debrecen
  (J. Tomkó, M. Arató, B. Almási, A. Kuki, J. Sztrik)

My Most Cited Paper, 22 citations

On the finite-source $\vec{G}/M/r$ Queue

European Journal of Operational Research 20 (1985) 261-268

Steady-State Probabilities

$$Q_{i_1,\ldots,i_k} = (n-k)!\left(r!r^{n-r-k}\mu^{n-k}\lambda_{i_1},\ldots,\lambda_{i_k}\right)^{-1}c_n,$$

$$(i_1,\ldots,i_k) \in C_k^n, \qquad k = 0,1,\ldots,n-r.$$

$$Q_{i_1,\ldots,i_k} = \left(\mu^{n-k}\lambda_{i_1},\ldots,\lambda_{i_k}\right)^{-1}c_n,$$

$$(i_1,\ldots,i_k) \in C_k^n, \qquad k = n-r,\ldots,n.$$

$$\hat{Q}_k = \sum_{(i_1,\ldots,i_k)\in C_k^n} Q_{i_1,\ldots,i_k},$$

$$\hat{Q}_k = \frac{n!}{r!k!r^{n-k-r}}\left(\frac{\lambda}{\mu}\right)^{n-k}\hat{Q}_n, \qquad ha \;\; 0 \le k \le n-r,$$

$$\hat{Q}_k = \binom{n}{k}\left(\frac{\lambda}{\mu}\right)^{n-k}\hat{Q}_n, \qquad ha \;\; n-r \le k \le n.$$

$$\hat{P}_k = \binom{n}{k} \left(\frac{\lambda}{\mu}\right)^k \hat{P}_0, \qquad ha \;\; 0 \leq k \leq r,$$

$$\hat{P}_k = \frac{n!}{r!(n-k)!r^{k-r}} \left(\frac{\lambda}{\mu}\right)^k \hat{P}_0, \qquad ha \;\; r \leq k \leq n.$$

Performance Metrics

Utilizations

$$Q^{(i)} = \sum_{k=1}^{n} \sum_{i \in (i_1,\ldots,i_k) \in C_k^n} Q_{i_1,\ldots,i_k}. \qquad U^{(i)} = Q^{(i)}$$

$$U_{CPU} = \frac{1}{r}\left(\sum_{k=1}^{r} k\hat{P}_k + r\sum_{k=r+1}^{n} \hat{P}_k\right) = \frac{\bar{r}}{r},$$

Mean Values

$$\bar{W}_i = \frac{1}{\lambda_i} \cdot \frac{1 - Q^{(i)}}{Q^{(i)}} - \frac{1}{\mu}, \qquad i = 1, \ldots, n.$$

$$\bar{T}_i = \bar{W}_i + 1/\mu = \left(1 - Q^{(i)}\right) \left(\lambda_i Q^{(i)}\right)^{-1}, \qquad i = 1, \ldots, n.$$

$$\sum_{i=1}^{n} \lambda_i \bar{T}_i Q^{(i)} = \bar{N}.$$

Java Applets and Information

- http://irh.inf.unideb.hu/user/jsztrik/education/09/index.html

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Bibliography

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- Khintchine, A.Y. *Mathematical Methods in the Theory of Queueing, Second Edition*, Hafner Publication Company, 1969
- Kleinrock, L. *Queueing Systems, Vol. I-II*, John Wiley Sons, 1976
- Takács, L. *Introduction to the Theory of Queues*, Oxfrod University Press, 1962

Thank You
for Your
Attention