UNIVERSITY OF DEBRECEN
FACULTY OF INFORMATICS


# Teaching Queueing Theory Based on Digital Technologies

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# Outline

- Origin of Queueing Theory
- Classifications of Queueing Systems
- Software Support
- QSA, Queueing Systems Assistance
- References

# Origin of Queueing Theory

Agner Krarup Erlang, 1878-1929

- "The Theory of Probabilities and Telephone Conversations", Nyt Tidsskrift for Matematik B, vol 20, 1909.
- "Solution of some Problems in the Theory of Probabilities of Significance in Automatic Telephone Exchanges", Elektrotkeknikeren, vol 13, 1917.
- "The life and works of A.K. Erlang", E. Brockmeyer, H.L. Halstrom and Arns Jensen, Copenhagen: The Copenhagen Telephone Company, 1948.

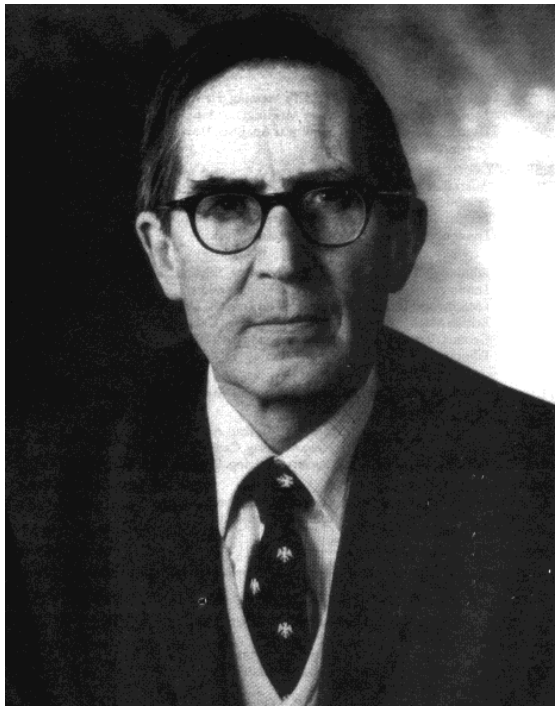
**Queueing Theory Homepage**

http://web2.uwindsor.ca/math/hlynka/queue.html

# Applications

- Telephony, Call Centers
- Manufacturing
- Inventories
- Dams
- Supermarkets
- Computer and Communication Systems
- Sensor Networks, IoT
- Infocommunication Networks, Clouds
- Hospitals
- Many others

# Kendall's Notation



David G. Kendall, 1918-2007

A/B/c/K/m/Z

# Performance Metrics

- Utilizations
- Mean Number of Customers in the System / Queue
- Mean Response / Waiting Time
- Mean Busy Period Length of the Server
- Distribution of Response / Waiting Time
- Distribution of the Busy Period
- Distribution of Number of Customers Served during a Busy Period
- Distribution of Number of Retrials until Service Completion

# Solution Methodologies

- Analytical
- Numerical
- Asymptotic
- Simulation
- Tool Supported Solutions

# Tool Supported Modeling

- University of Dortmund: *HIT*, *HiQPN*, *APNN*
  http://ls4-www.infromatik.uni-dortmund.de/tools.html/
- University of Illinois at Urbana-Champaign: *MÖBIUS*
  http://www.mobius.uiuc.edu/
- University of Erlangen: *PEPSY*, *MOSEL*
  http://www4.informatik.uni-erlangen.de/Projects/MOSEL/
- University of Oxford: *PRISM*
  http://www.prismmodelchecker.org/

# Software and Information

http://web2.uwindsor.ca/math/hlynka/qsoft.html

http://mason.gmu.edu/~jshortle/QtsPlus-4-0.zip

**QSA ( Queueing Systems Assistance)**

https://qsa.inf.unideb.hu

**Lecture Notes**

https://irh.inf.unideb.hu/~jsztrik/education/16/SOR_Main_Angol.pdf

https://irh.inf.unideb.hu/~jsztrik/education/16/Queueing_Problems_Solutions_2021_Sztrik.pdf

# Introduction of QSA and Case Studies

**Example 1**

Customers arrive to a 2 server system according to a Poisson process with rate 3. The service times are exponentially distributed with parameter 2.

**Find** the minimum capacity of the system for which the probability of blocking is less than 0.01 and the probability that the waiting time exceeds 1.8 minutes is less than 0.05.

# Case Studies

**Example 2**

We have a finite-source system with 50 sources, the request generation times are exponentially distributed with rate 0.5. The service times are exponentially distributed for all the 5 servers with intensity 2.

**Find** the minimum capacity of the system for which the probability of blocking is less than 0.01 and the probability that the waiting time exceeds 3.5 minutes is less than 0.05.

# Case Studies

**Example 3**

Let us see an M/M/1 system with arrival intensity 1 and the following costs, cost of service per server per unit time CS = 2, cost of waiting in the system per customer per unit time CW =2, cost of idleness per server per unit time CI=10, cost of service rate per server per unit time CSR=10, reward per customer per unit time R=5.

**Find** the optimal value for the service intensity which minimize the expected total cost per unit time with linear objective function.

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- Cai, L., Shen, X., Mark, J.W. *Multimedia Services in Wireless Internet, Modeling and Analysis*, John Wiley Sons, 2009
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- Misis, J., Misic, V.B. *Performance Modeling and Analysis of Bluetooth Networks: Polling, Scheduling and Traffic Control*, Auerbach Publications, 2006


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