HOMOGENEOUS FINITE-SOURCE RETRIAL QUEUES WITH SERVER SUBJECT TO BREAKDOWNS AND REPAIRS

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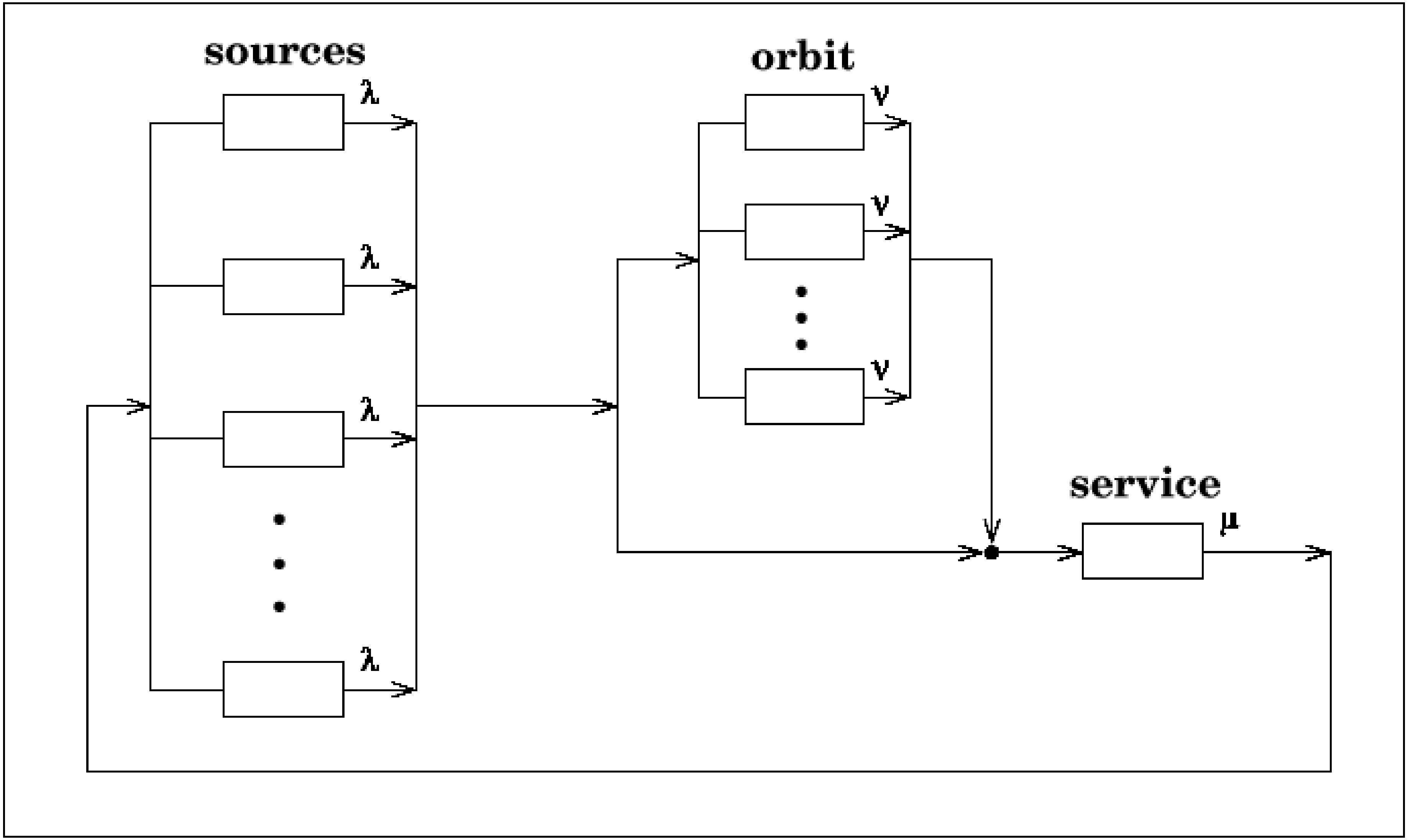
OUTLOOK

- **The queueing model**
- **Applications**
- **Mathematical model**
- **Evaluation Tool MOSEL**
- **Case studies**
- **References**

The queueing model



Applications

- **magnetic disk memory systems**
- **local area networks with CSMA/CD protocols**
- **collision avoidance local area network modeling**

Mathematical model

The system state at time t can be described with the process

$$X(t) = (Y(t); C(t); N(t))$$

where $Y(t) = 0$ if the server is up, $Y(t) = 1$ if the server is failed,

$C(t) = 0$ if the server is idle, $C(t) = 1$ if the server is busy,

$N(t)$ is the number of sources of repeated calls at time t.

We define the stationary probabilities:

$$P(q;r;j) = \lim_{t\to\infty} P(Y(t) = q, C(t) = r, N(t) = j)$$

$$q = 0,1, \quad r = 0,1, \quad j = 0,...,K-1.$$

Once we have obtained these limiting probabilities the **main system performance measures** can be derived in the following way.

1. Utilization of the server

$$U_S = \sum_{j=0}^{K-1} P(0,1,j)$$

2. Utilization of the repairman

$$U_R = \sum_{q=0}^{1} \sum_{j=0}^{K-1} P(1,q,j)$$

3. Availability of the server

$$A_S = \sum_{q=0}^{1} \sum_{j=0}^{K-1} P(0,q,j) = 1 - U_R$$

4. The mean number of sources of repeated calls

$$N = E[N(t)] = \sum_{q=0}^{1} \sum_{r=0}^{1} \sum_{j=0}^{K-1} jP(q,r,j)$$

5. The mean number of calls staying in the orbit or in service

$$M = E[C(t) + N(t)] = \sum_{q=0}^{1} \sum_{r=0}^{1} \sum_{j=0}^{K-1} (r+j)P(q,r,j)$$

6. The mean rate of generation of primary calls

$$\overline{\lambda} = \begin{cases} \lambda E[K - C(t) - N(t); Y(t) = 0] & \text{for blocked case,} \\ \lambda E[K - C(t) - N(t)] & \text{for unblocked case.} \end{cases}$$

7. The mean response time

$$E[T] = M/\overline{\lambda}$$

8. The mean waiting time

$$E[W] = N/\overline{\lambda}$$

9. The blocking probability of a primary call

$$B = \begin{cases} \frac{\lambda E[K-C(t)-N(t);Y(t)=0;C(t)=1]}{\overline{\lambda}} & \text{for blocked case,} \\ \\ \frac{\lambda E[K-C(t)-N(t);C(t)=1]}{\overline{\lambda}} & \text{for unblocked case.} \end{cases}$$

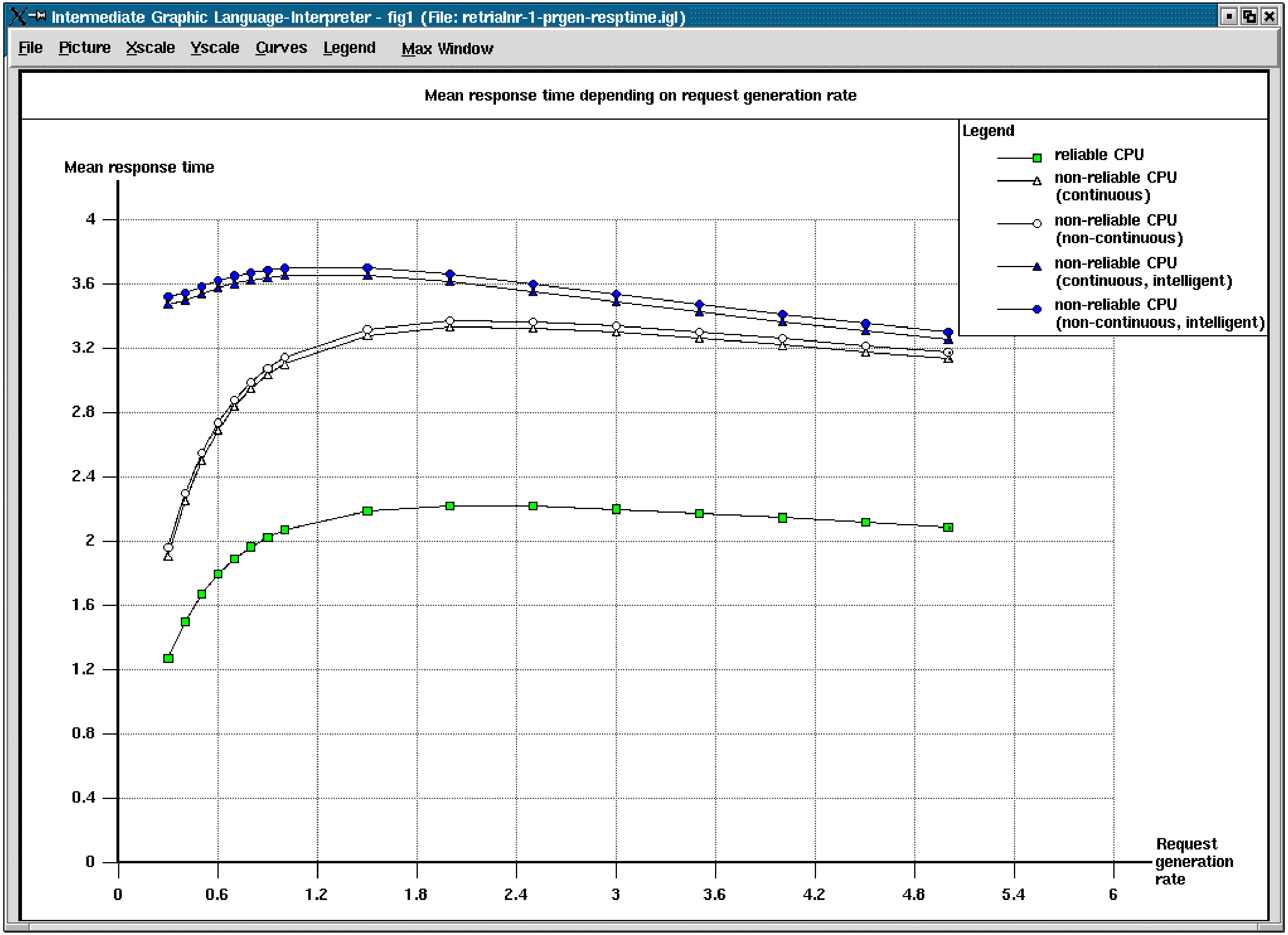
Evaluation Tool MOSEL

MOSEL (Modeling, Specification and Evaluation Language) developed at the University of Erlangen, Germany, is used to formulate and solved the problem.

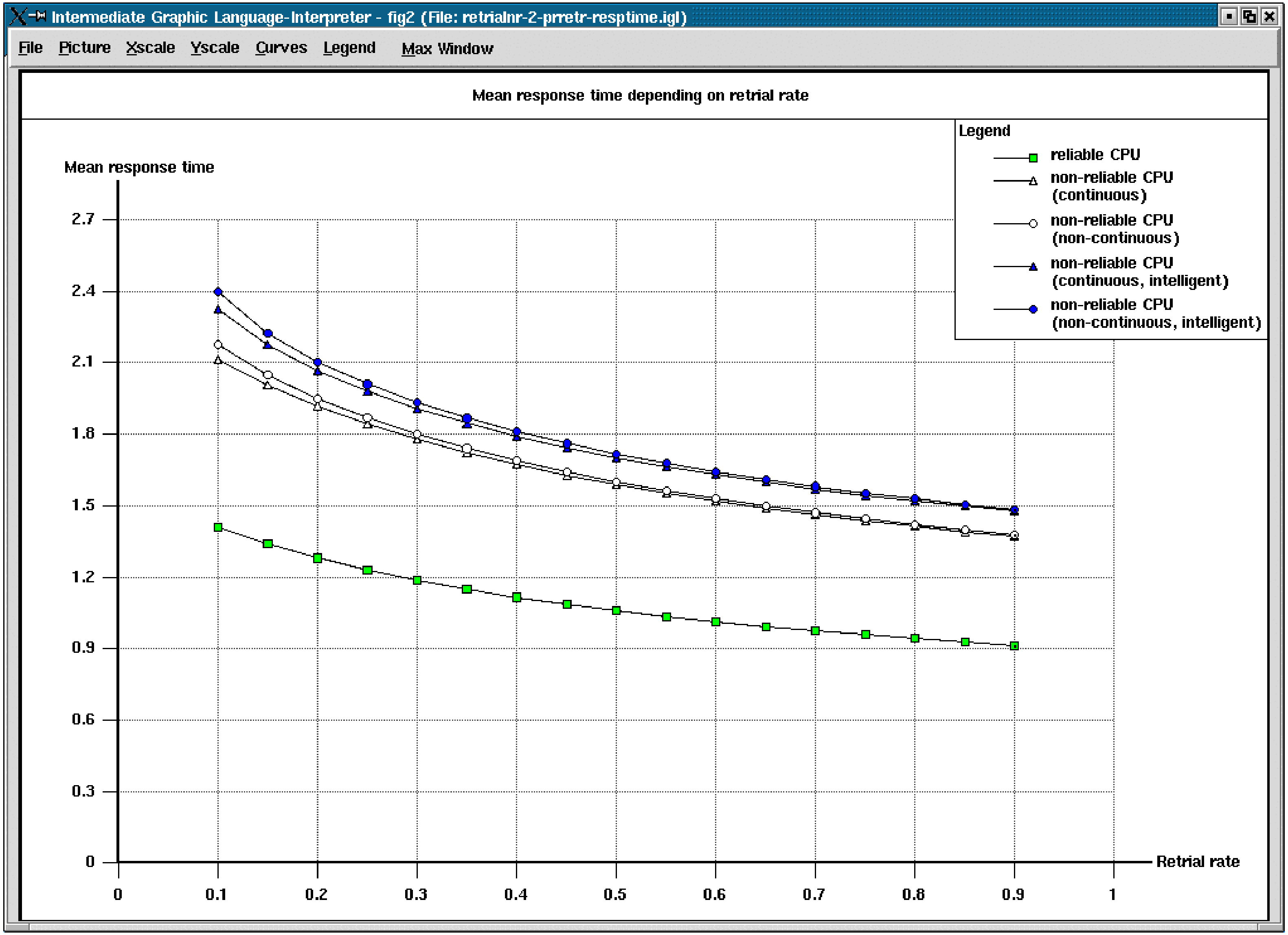
Case studies

Validation of results

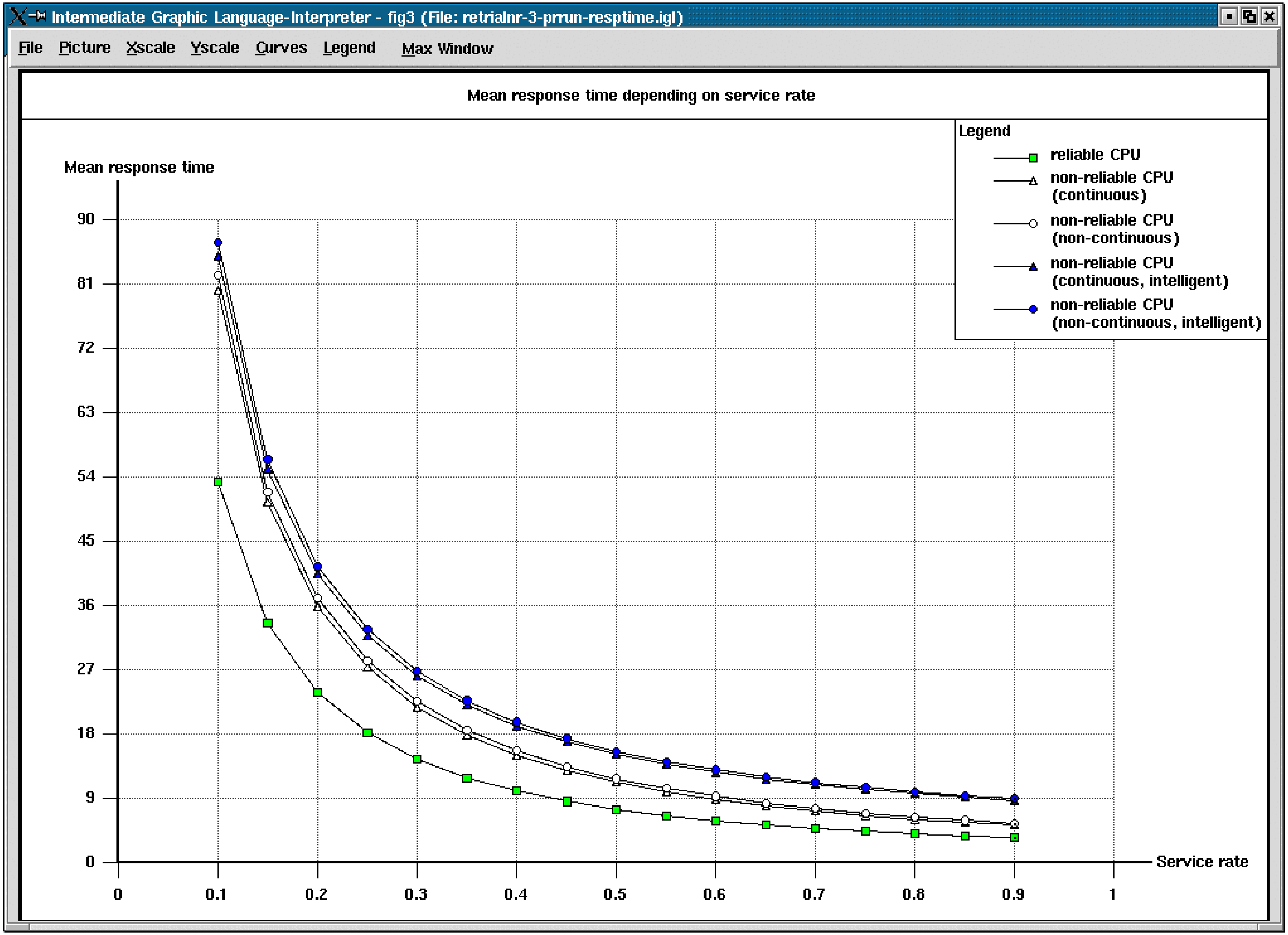
	non–rel. retrial(cont.)	non–rel. retrial(orbit)	non–rel. FIFO
Number of sources:	3	3	3
Request's generation rate:	0.1	0.1	0.1
Service rate:	1	1	1
Retrial rate:	1e+25	1e+25	-
Server's failure rate:	0.01	0.01	0.01
Server's repair rate:	0.05	0.05	0.05
Utilization of the server:	0.22327965614	0.22327965229	0.22327964521
Utilization of the sources:	0.74426549207	0.74426549375	0.74426549684
Mean response time:	1.43606563309	1.43606560277	1.43606554705



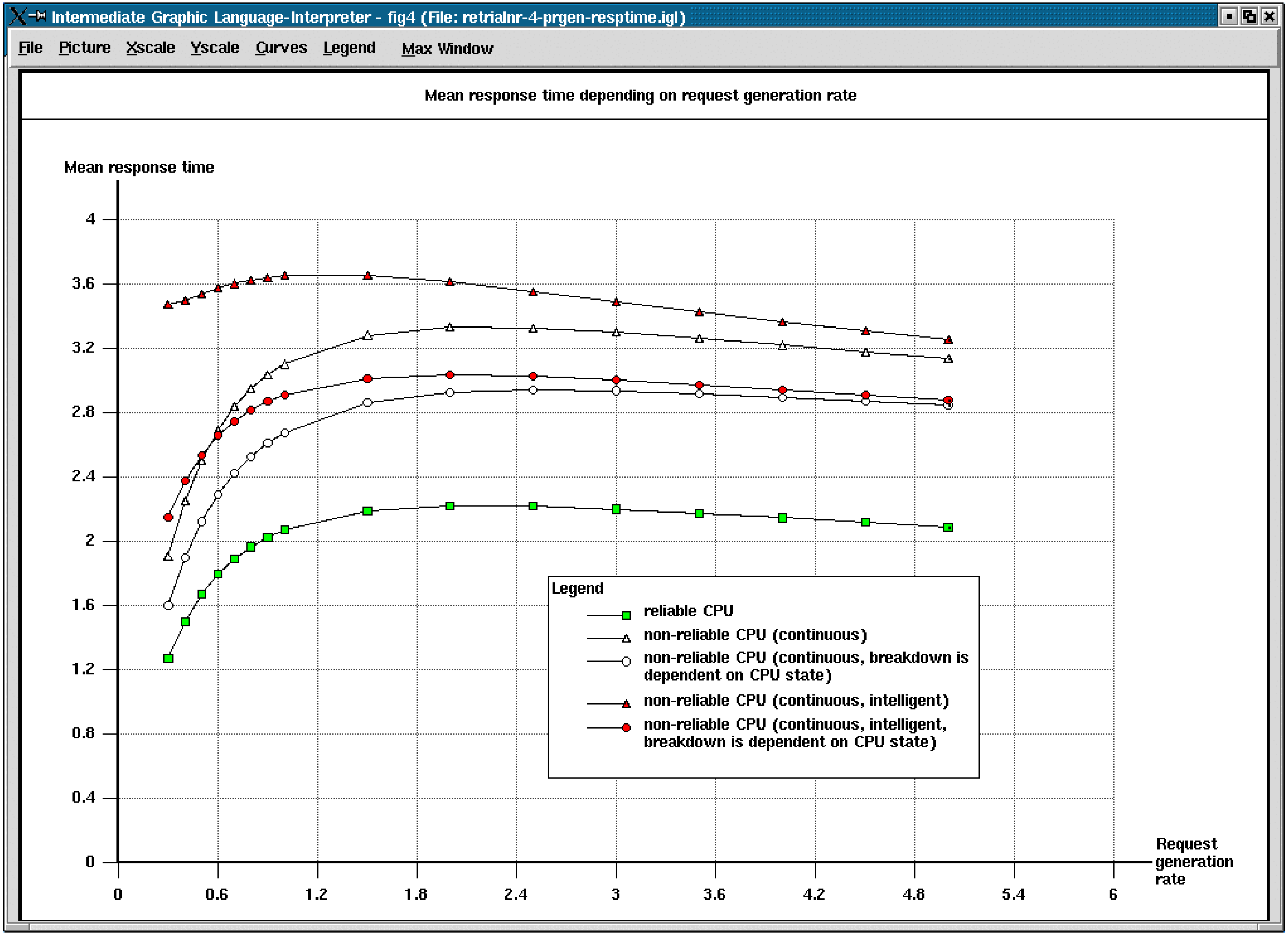
$E[T]$ versus primary request generation rate



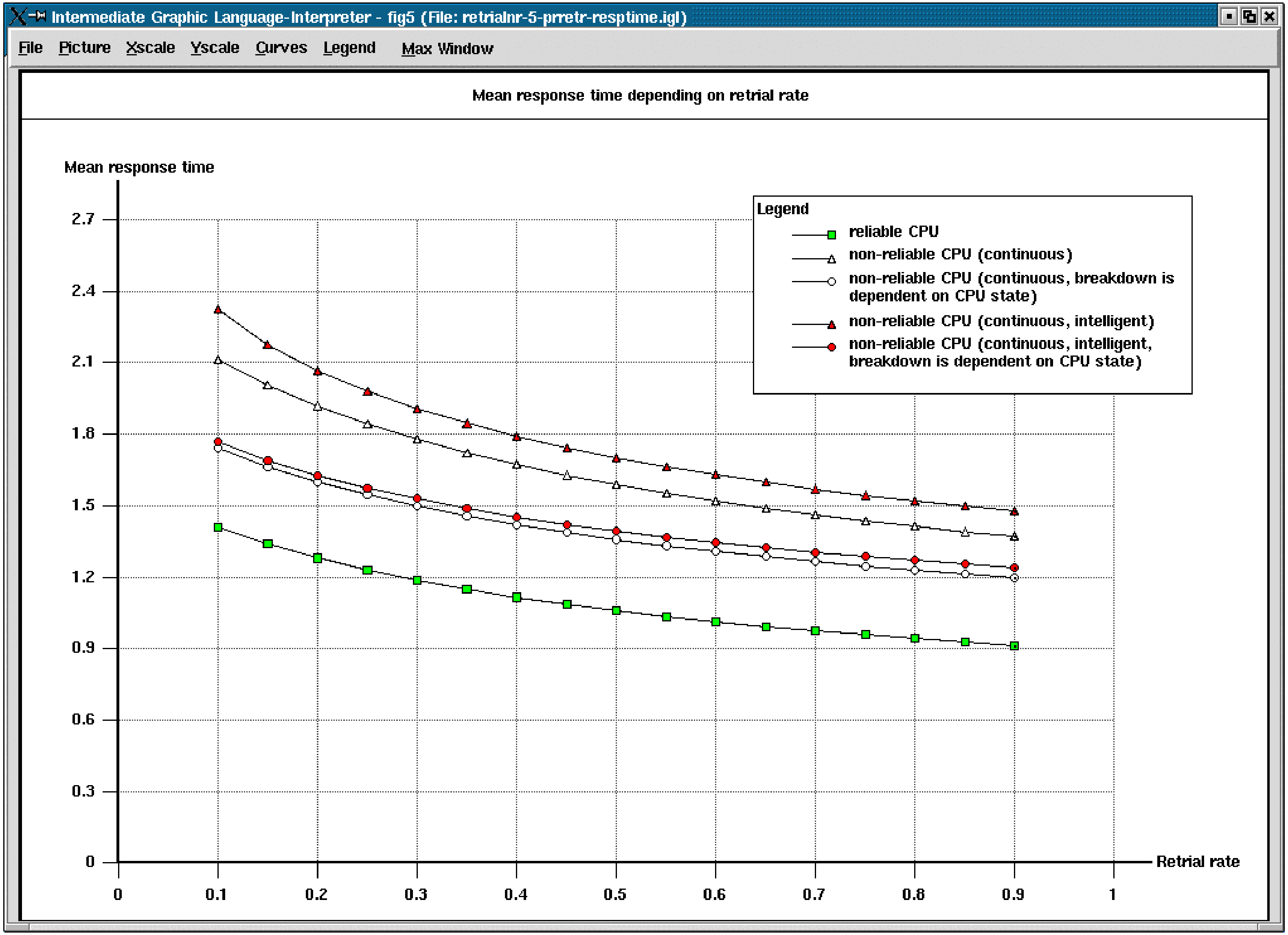
$E[T]$ versus retrial rate



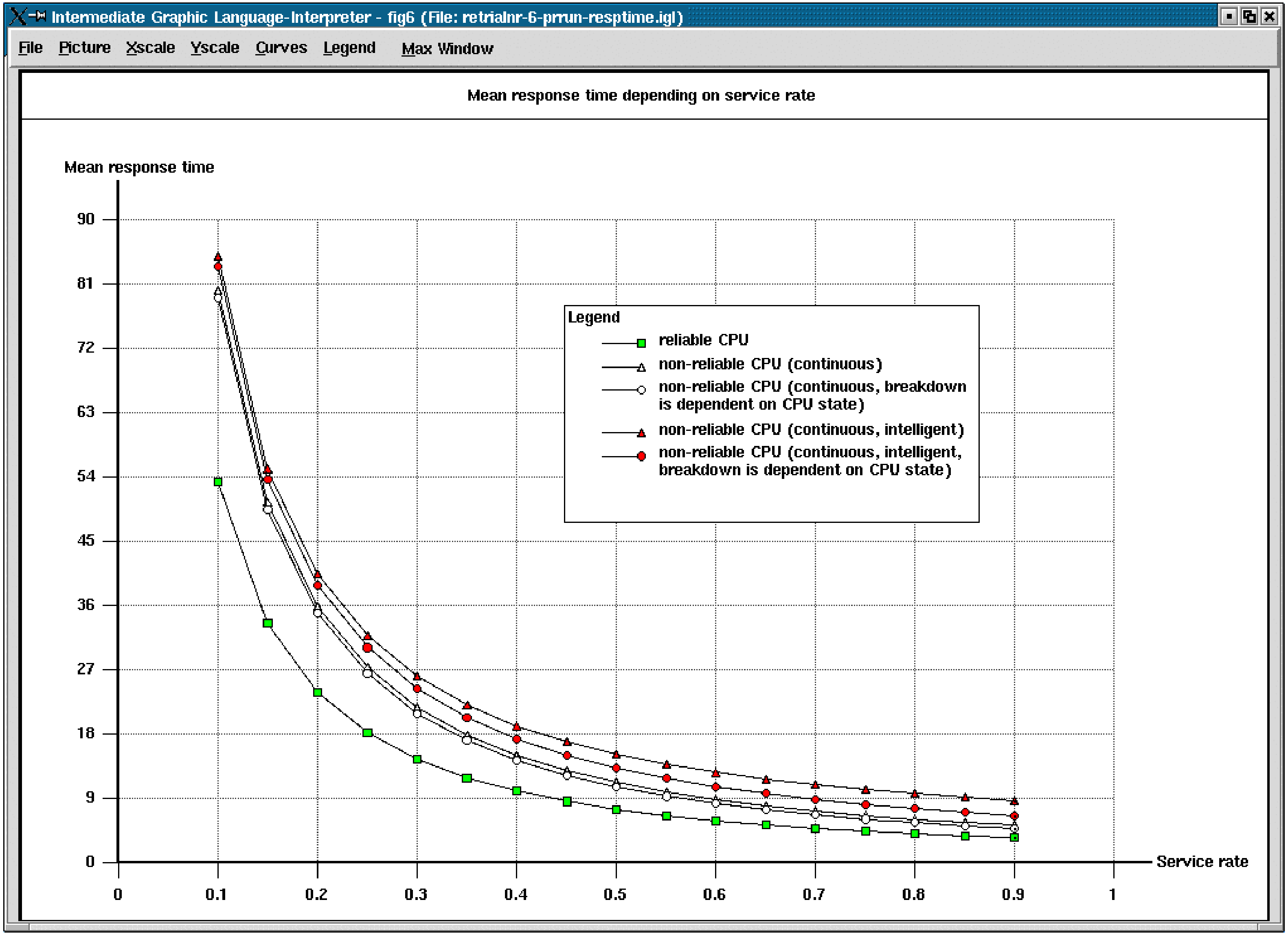
$E[T]$ versus service rate



$E[T]$ versus primary request generation rate



$E[T]$ versus retrial rate



$E[T]$ versus service rate

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[5] **Wang Jinting, Cao Jinhua and Li Quanlin** Reliability analysis of the retrial queue with server breakdowns and repairs, *Queueing Systems Theory and Applications* 38(2001), 363–380.