# HETEROGENEOUS FINITE-SOURCE RETRIAL QUEUES


**B. Almási, G. Bolch*, J. Sztrik**

Institute of Mathematics and Informatics, University of Debrecen

Debrecen, Hungary

*Department of Computer Science, University of Erlangen

Erlangen, Germany

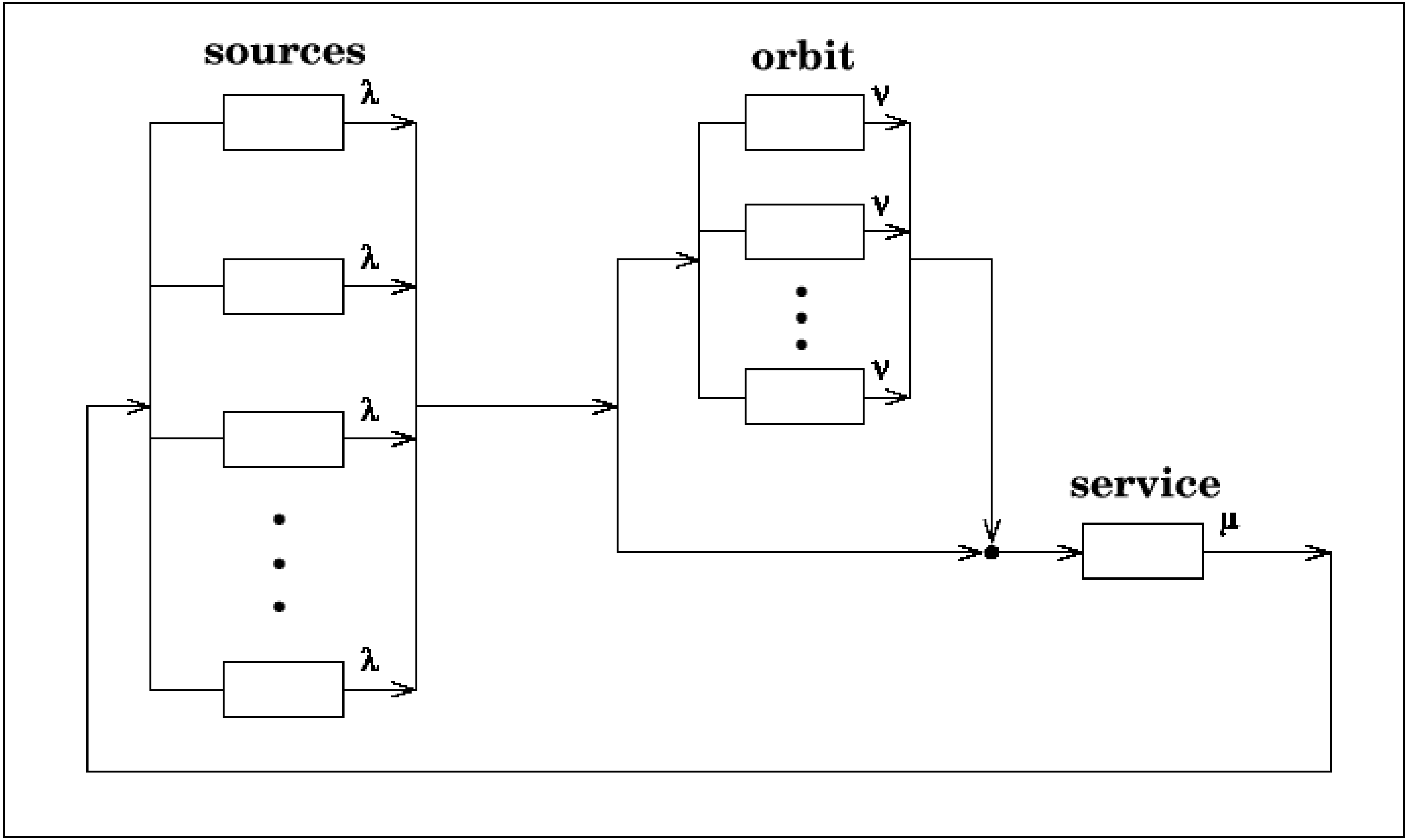
*e-mail: jsztrik@math.klte.hu*

www: http://it.math.klte.hu/user/jsztrik/

# OUTLOOK

- **The queueing model**
- **Applications**
- **Mathematical model**
- **Evaluation Tool MOSEL**
- **Case studies**
- **References**

# The queueing model



# Applications

- magnetic disk memory systems
- local area networks with CSMA/CD protocols
- collision avoidance local area network modeling

# Mathematical model

$$P(0;0) = \lim_{t\to\infty} P(C(t) = 0, N(t) = 0)$$

$$P(j;0) = \lim_{t\to\infty} P(\alpha_1 = j, N(t) = 0), \quad j = 1, ..., K$$

$$P(0;i_1,...,i_k) = \lim_{t\to\infty} P(C(t) = 0, \beta_1 = i_1, ..., \beta_k = i_k), \quad k = 1, ..., K-1$$

$$P(j;i_1,...,i_k) = \lim_{t\to\infty} P(\alpha_1 = j, \beta_1 = i_1, ..., \beta_k = i_k), \quad k = 1, ..., K-1.$$

Once we have obtained these limiting probabilities the **main system performance measures** can be derived in the following way.

**1. The server utilization with respect to source $j$**

$$U_j = \text{P ( the server is busy with source } j \text{ )}$$

that is, we have to summarize all the probabilities where the first component is $j$. Formaly

$$U_j = \sum_{k=0}^{K-1} \sum_{i_1,\dots,i_k \neq j} P(j; i_1, \dots, i_k)$$

Hence the **server utilization**

$$U = E[C(t) = 1] = \sum_{j=1}^{K} U_j.$$

Let us denote by $P_W^{(i)}$ the steady state probability that request $i$ is waiting ( staying in the orbit ). It is easy to see that

$$P_W^{(i)} = \sum_{j=0, j \neq i}^{K} \sum_{k=1}^{K-1} \sum_{i \epsilon (i_1, ..., i_k)} P(j; i_1, ..., i_k).$$

Similarly, it can easily be seen, that the steady state probability $P^{(i)}$ that request $i$ is in the service facility (it is under service or waiting in the orbit) is given by

$$P^{(i)} = P_W^{(i)} + U_i.$$

## 2. Mean response time of source $i$

Let us denote by $E[T_i]$ the mean response time of customer $i$, and by $\gamma_i$ the **throughput** of request $i$, that is, the mean number of times that request $i$ is served per unit time. These are related by

$$\gamma_i = \frac{1}{E[T_i] + 1/\lambda_i} = \lambda_i(1 - P^{(i)}) = \mu_i U_i, \qquad i = 1, ..., K. \tag{1}$$

For $P^{(i)}$ we have

$$P^{(i)} = \frac{E[T_i]}{E[T_i] + 1/\lambda_i} = \gamma_i E[T_i] = 1 - \frac{\gamma_i}{\lambda_i} \qquad i = 1, ..., K. \tag{2}$$

which represents **Little's theorem** for request $i$ in the service facility. It is easy to see that as a consequence of (1) we have

$$P^{(i)} = 1 - \frac{\mu_i}{\lambda_i} U_i$$

and

$$P_W^{(i)} = P^{(i)} - U_i = 1 - \frac{\mu_i + \lambda_i}{\lambda_i} U_i.$$

Alternatively, by the help of (2) we can express the mean response time $E[T_i]$ for request $i$ in terms of $U_i$ as

$$E[T_i] = \frac{P^{(i)}}{\lambda_i(1 - P^{(i)})} = \frac{1 - \frac{\mu_i}{\lambda_i} U_i}{\mu_i U_i} = \frac{\lambda_i - \mu_i U_i}{\lambda_i \mu_i U_i}. \tag{3}$$

## 3. Mean waiting time of source $i$

The mean waiting time of request $i$ is given by

$$E[W_i] = E[T_i] - \frac{1}{\mu_i} = \frac{1}{\gamma_i} - \frac{1}{\lambda_i} - \frac{1}{\mu_i} = \frac{\lambda_i - (\mu_i + \lambda_i) U_i}{\lambda_i \mu_i U_i}. \tag{4}$$

At the same time we have another **Little's theorem** for request $i$ waiting for service. Namely

$$P_W^{(i)} = \frac{E[W_i]}{E[T_i] + 1/\lambda_i} = \gamma_i E[W_i] \qquad i = 1, ..., K.$$

**4. Mean number of calls staying in the orbit or in service**

$$M = E[C(t) + N(t)] = \sum_{i=1}^{K} P^{(i)} = \sum_{i=1}^{K} (1 - \frac{\mu_i}{\lambda_i} U_i) = K - \sum_{i=1}^{K} \frac{\mu_i}{\lambda_i} U_i.$$

**5. Mean number of sources of repeated calls**

$$N = E[N(t)] = \sum_{i=1}^{K} P_W^{(i)} = \sum_{i=1}^{K} (1 - \frac{\mu_i + \lambda_i}{\lambda_i} U_i) = K - \sum_{i=1}^{K} \frac{\mu_i + \lambda_i}{\lambda_i} U_i.$$

## 6. Mean rate of generation of primary calls

$$\overline{\lambda} = \sum_{i=1}^{K} \gamma_i = \sum_{i=1}^{K} \lambda_i (1 - P^{(i)}) = \sum_{i=1}^{K} \mu_i U_i.$$

## 7. Blocking probability of primary call $i$

$$B_i = \frac{\lambda_i \sum_{j=1, j \neq i}^{K} \sum_{k=0}^{K-1} \sum_{i \neq i_1, \ldots, i_k} P(j; i_1, \ldots, i_k)}{\overline{\lambda}}.$$

Hence **blocking probability of primary calls**

$$B = \sum_{i=1}^{K} B_i$$

In particular, in the case of **homogeneous calls**

$$U_i = E[C(t)]/K, \quad i = 1, ..., K, \quad N = K - \frac{(\lambda + \mu)U}{\lambda},$$

$$\overline{\lambda} = \lambda E[K - C(t) - N(t)] = \mu U,$$

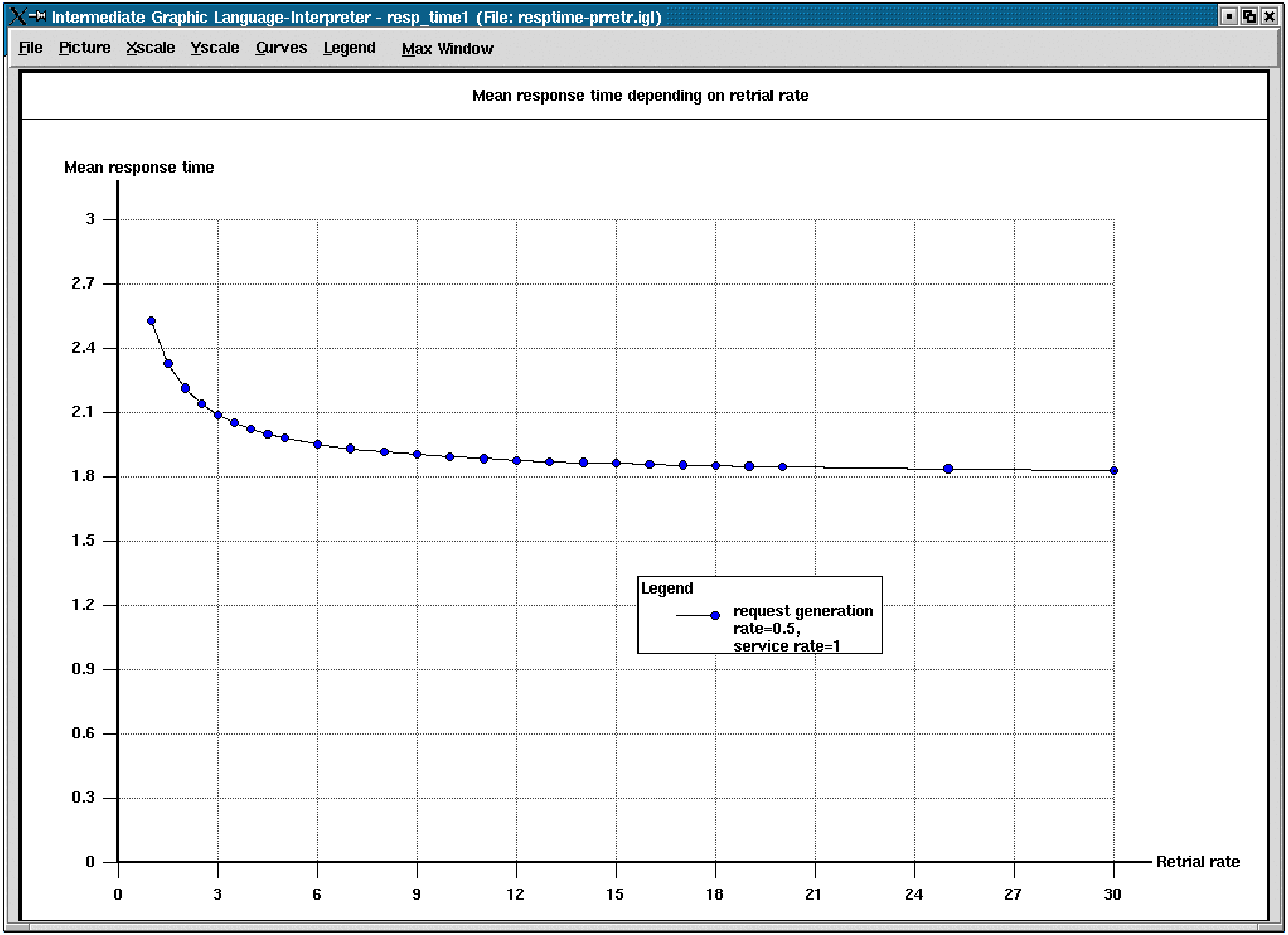
$$E[W] = \frac{N}{\overline{\lambda}} = K - \frac{1}{\mu U} - \frac{1}{\lambda} - \frac{1}{\mu},$$

$$B = \frac{\lambda E[K - C(t) - N(t); C(t) = 1]}{\lambda E[K - C(t) - N(t)]}.$$

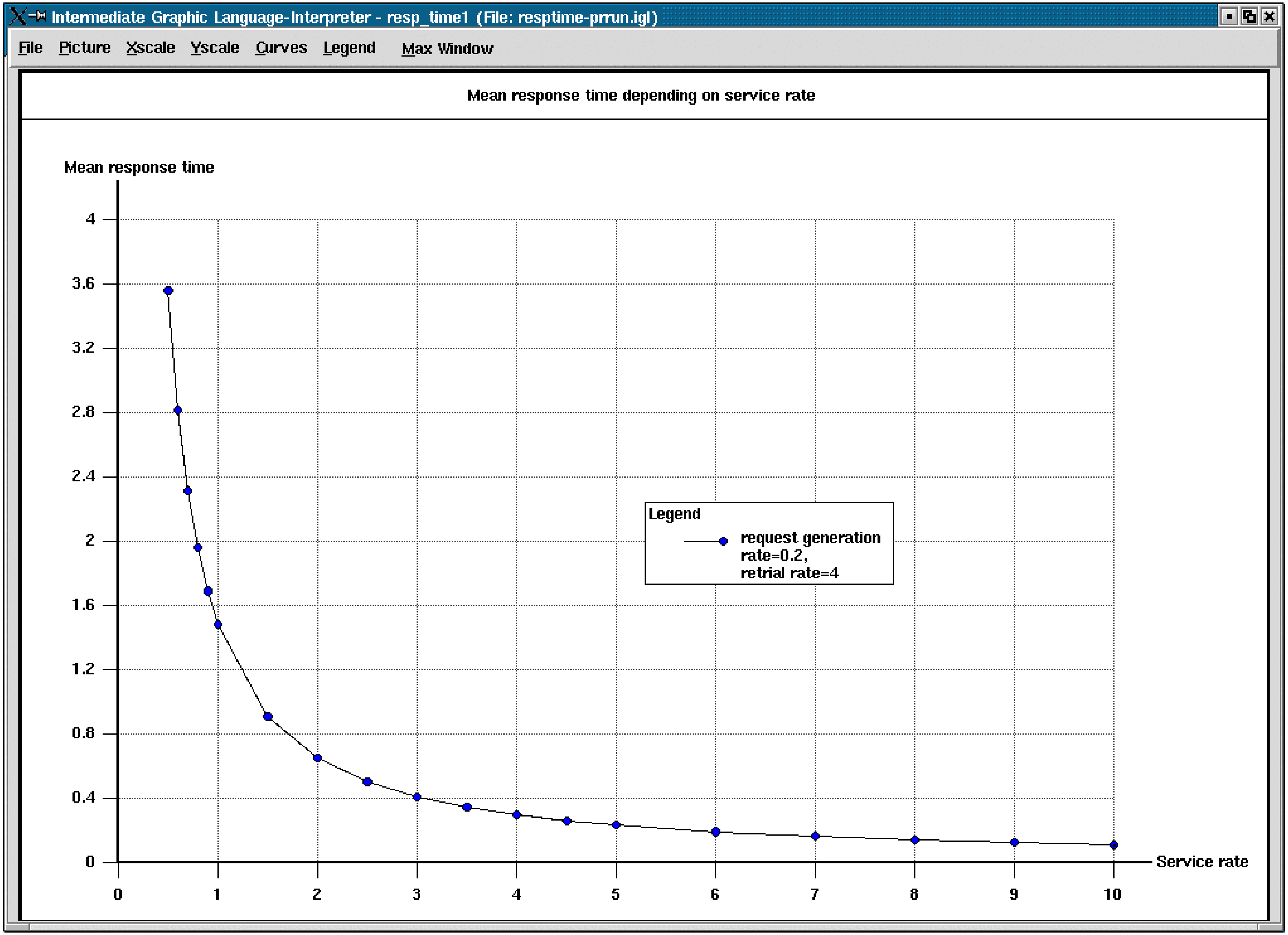
# Evaluation Tool MOSEL

**MOSEL ( Modeling, Specification and Evaluation Language )** developed at the University of Erlangen, Germany, is used to formulate and solved the problem.

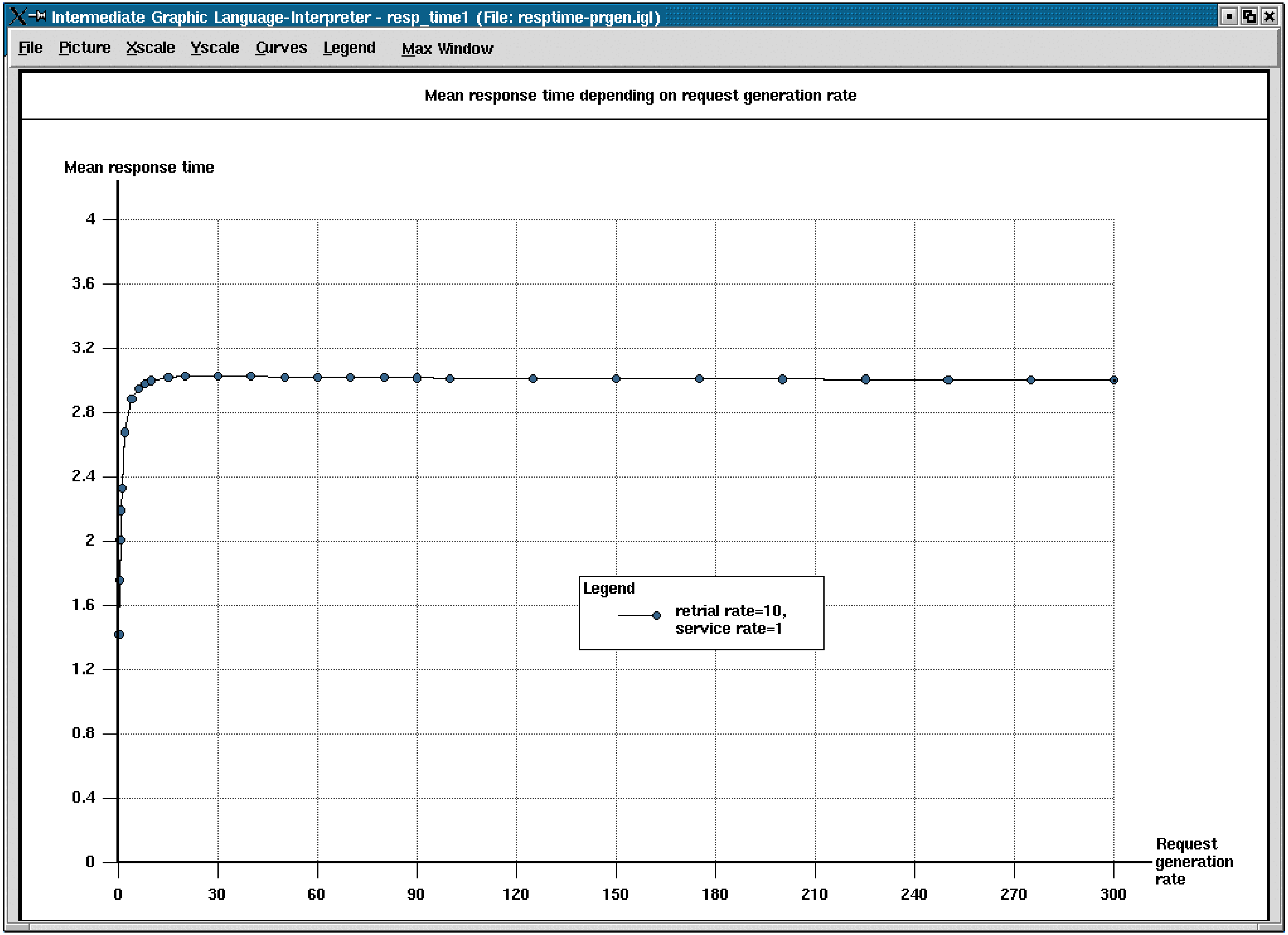
# Case studies



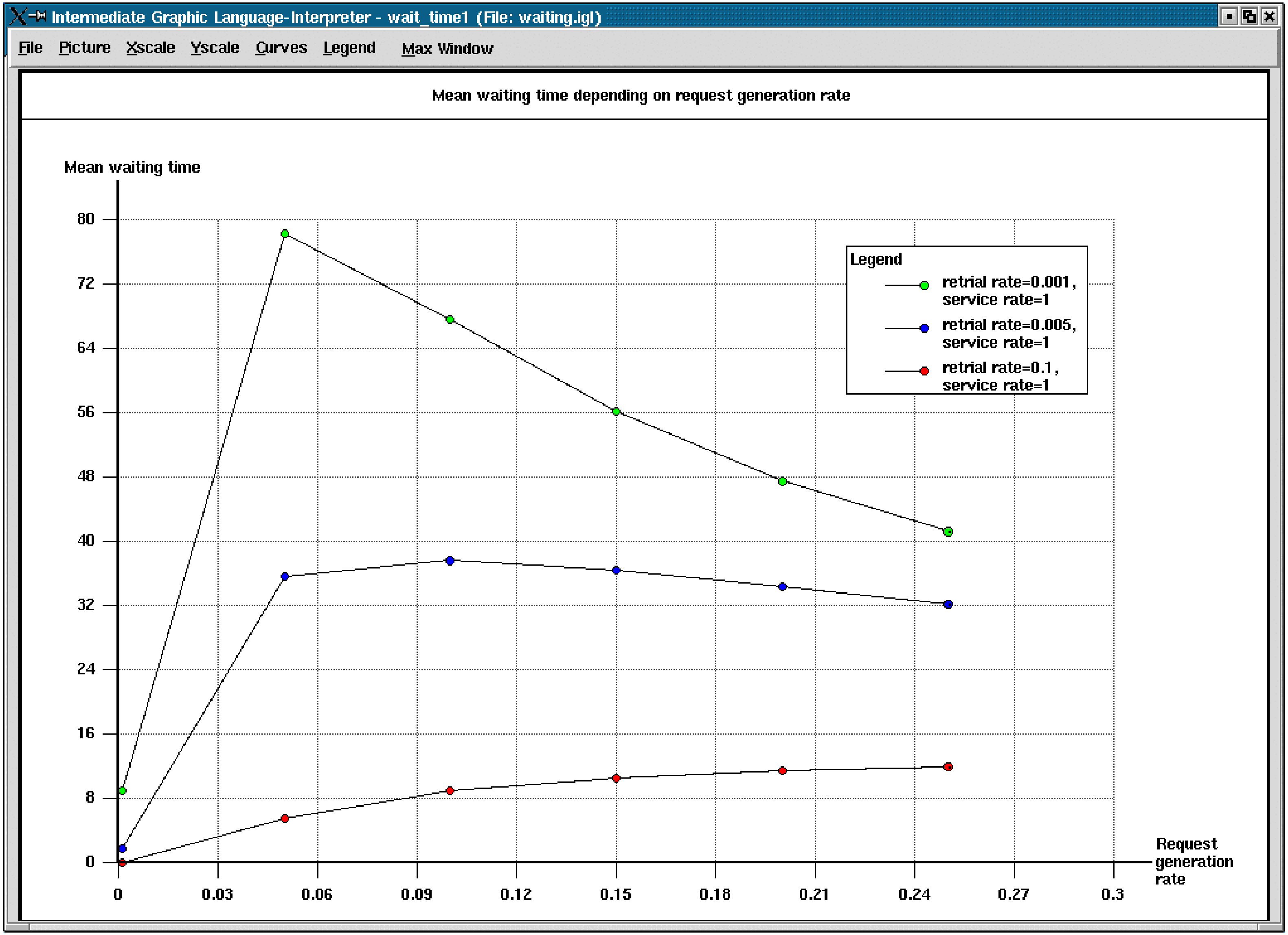

$E[T]$ versus retrial rate



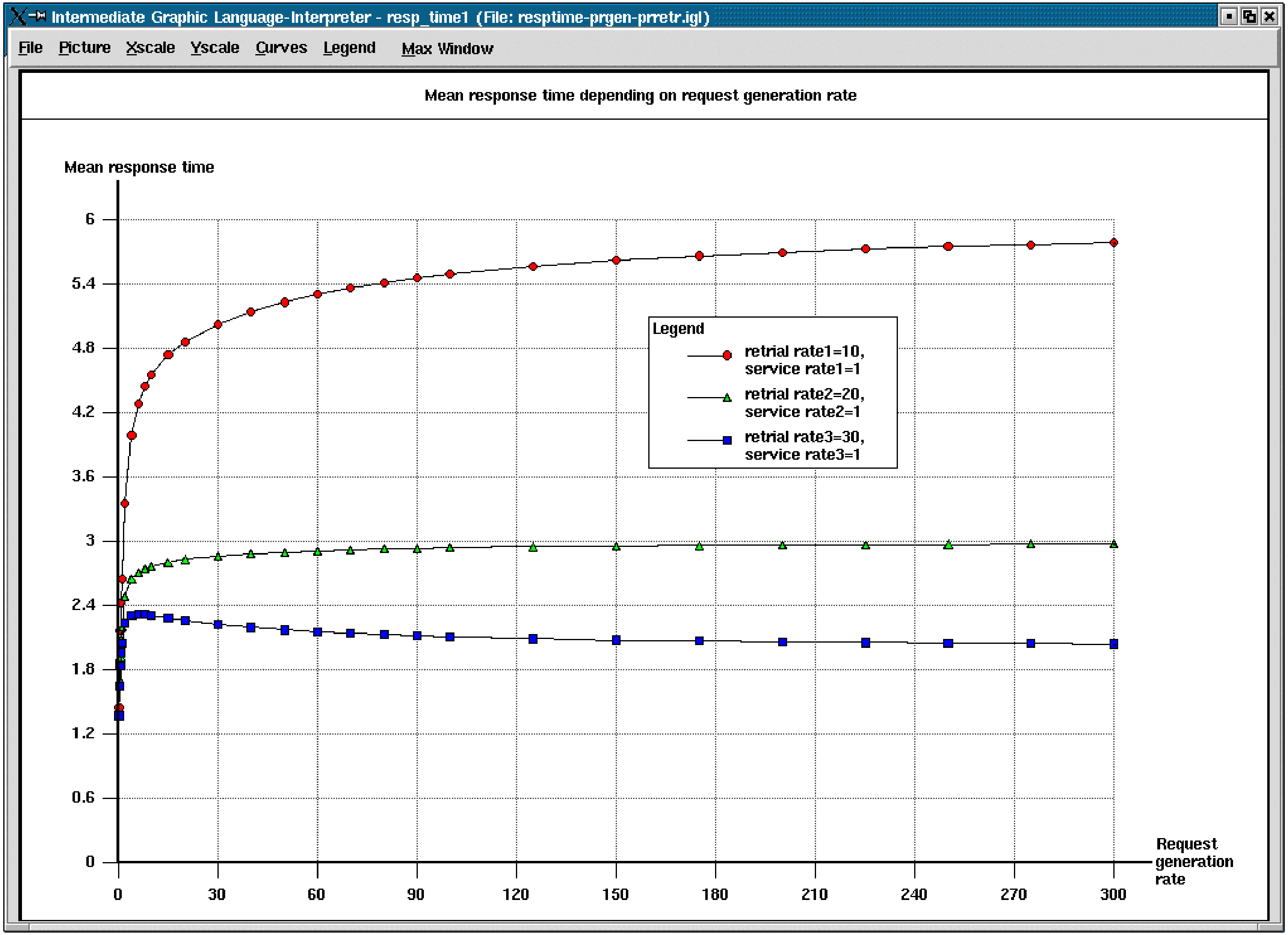

$E[T]$ versus service rate



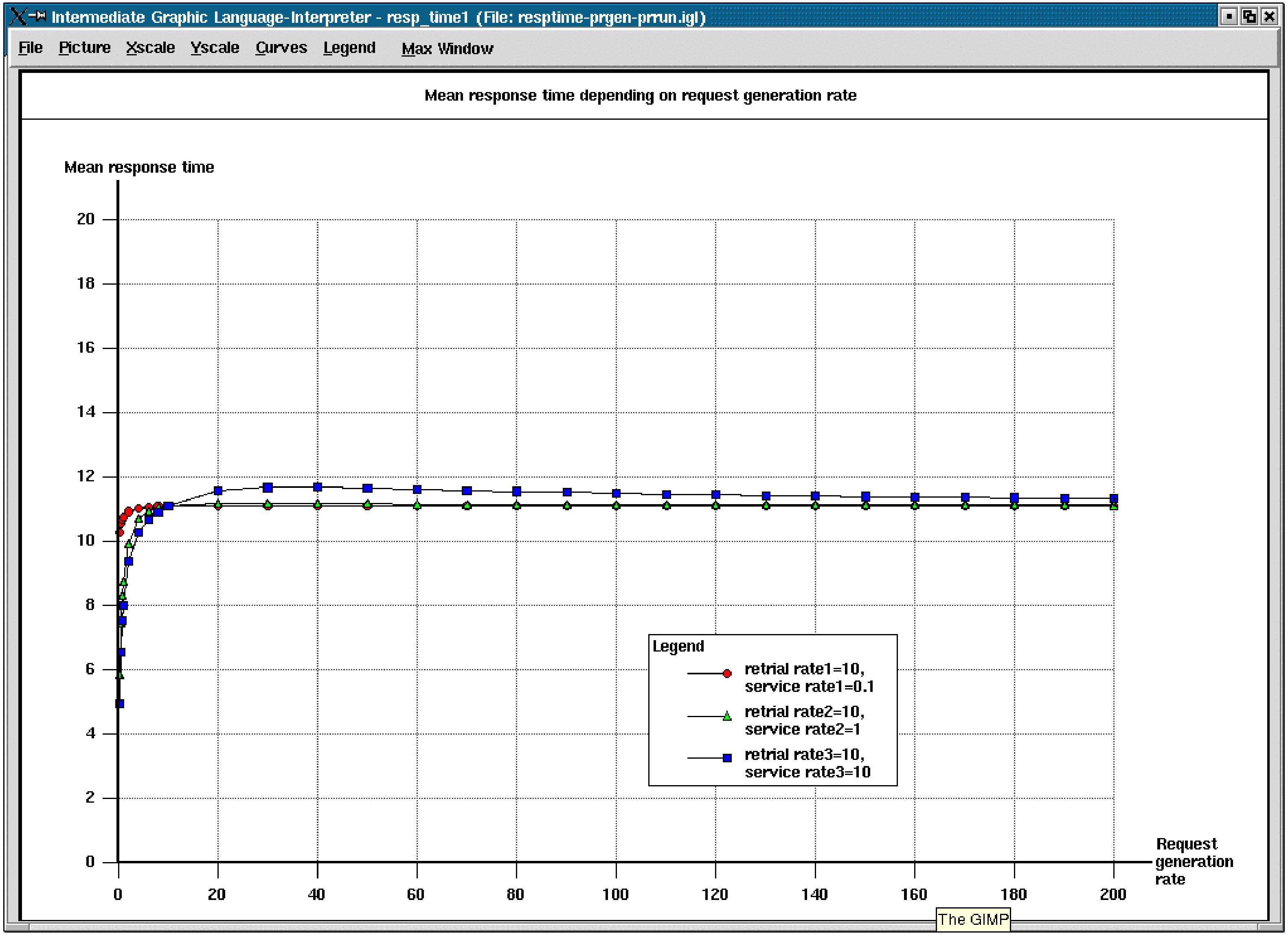

$E[T]$ versus primary request generation rate



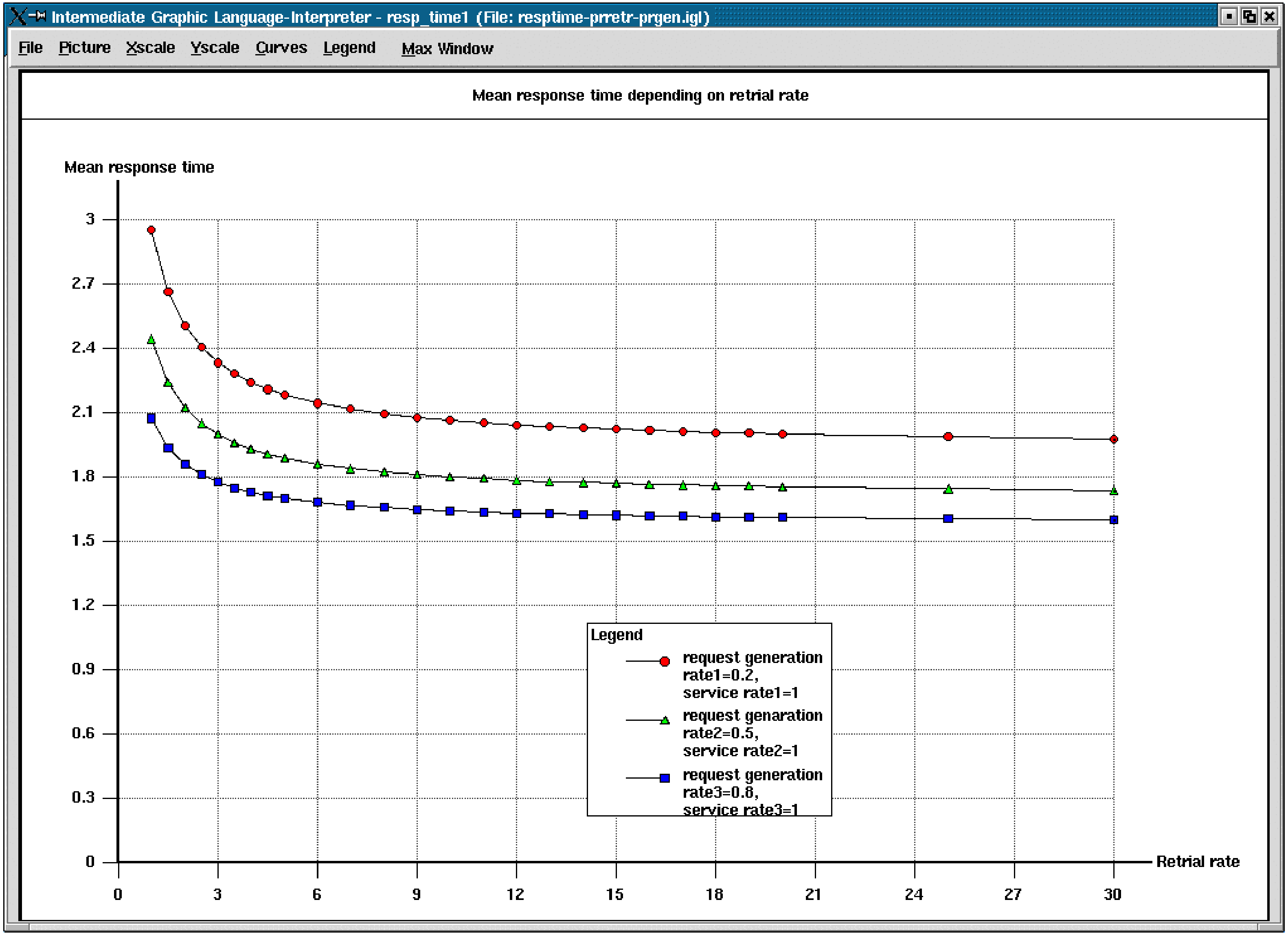

$E[T]$ versus primary request generation rate



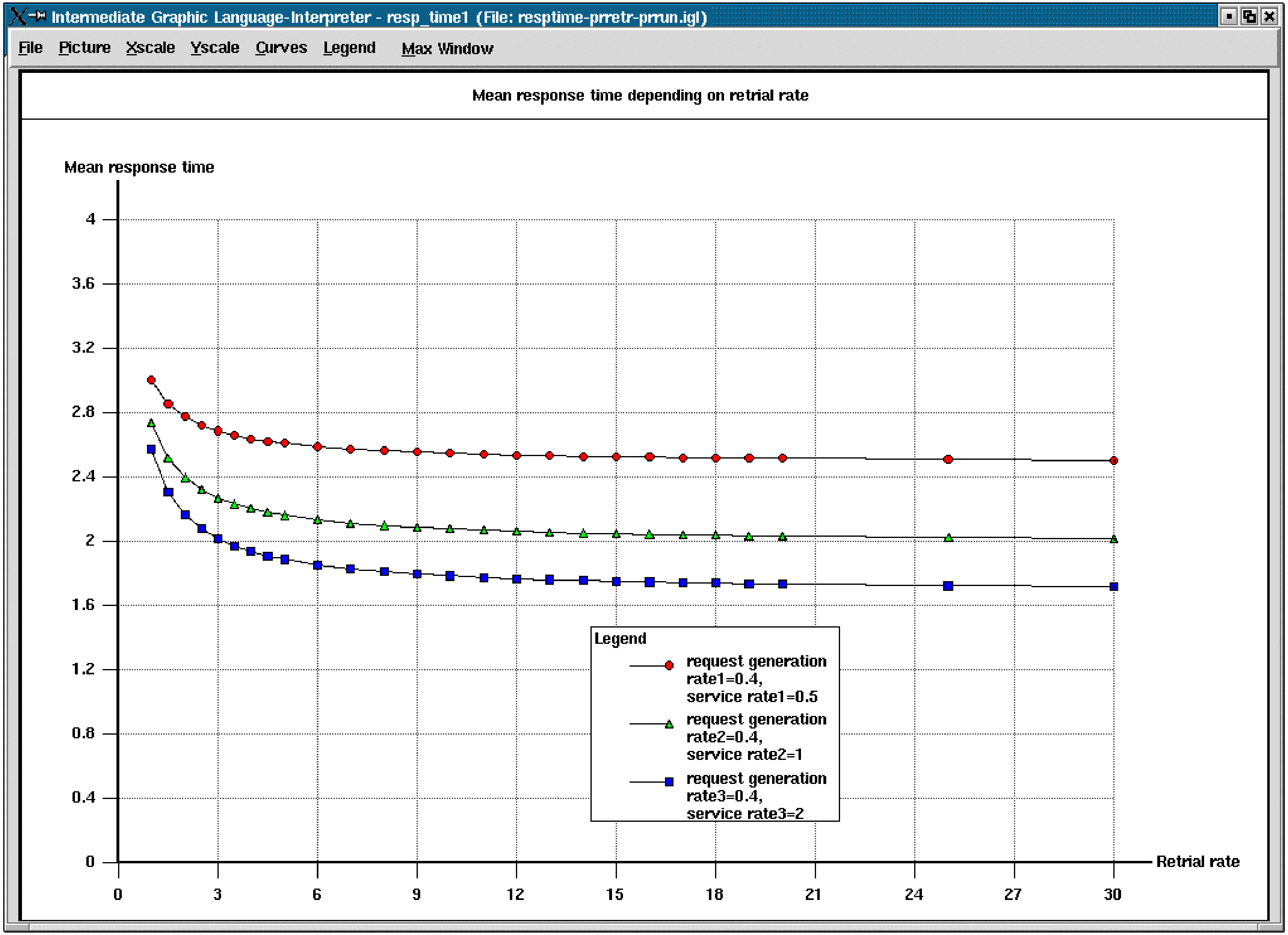

$E[T]$ versus primary request generation rate with homogeneous service and heterogeneous retrial



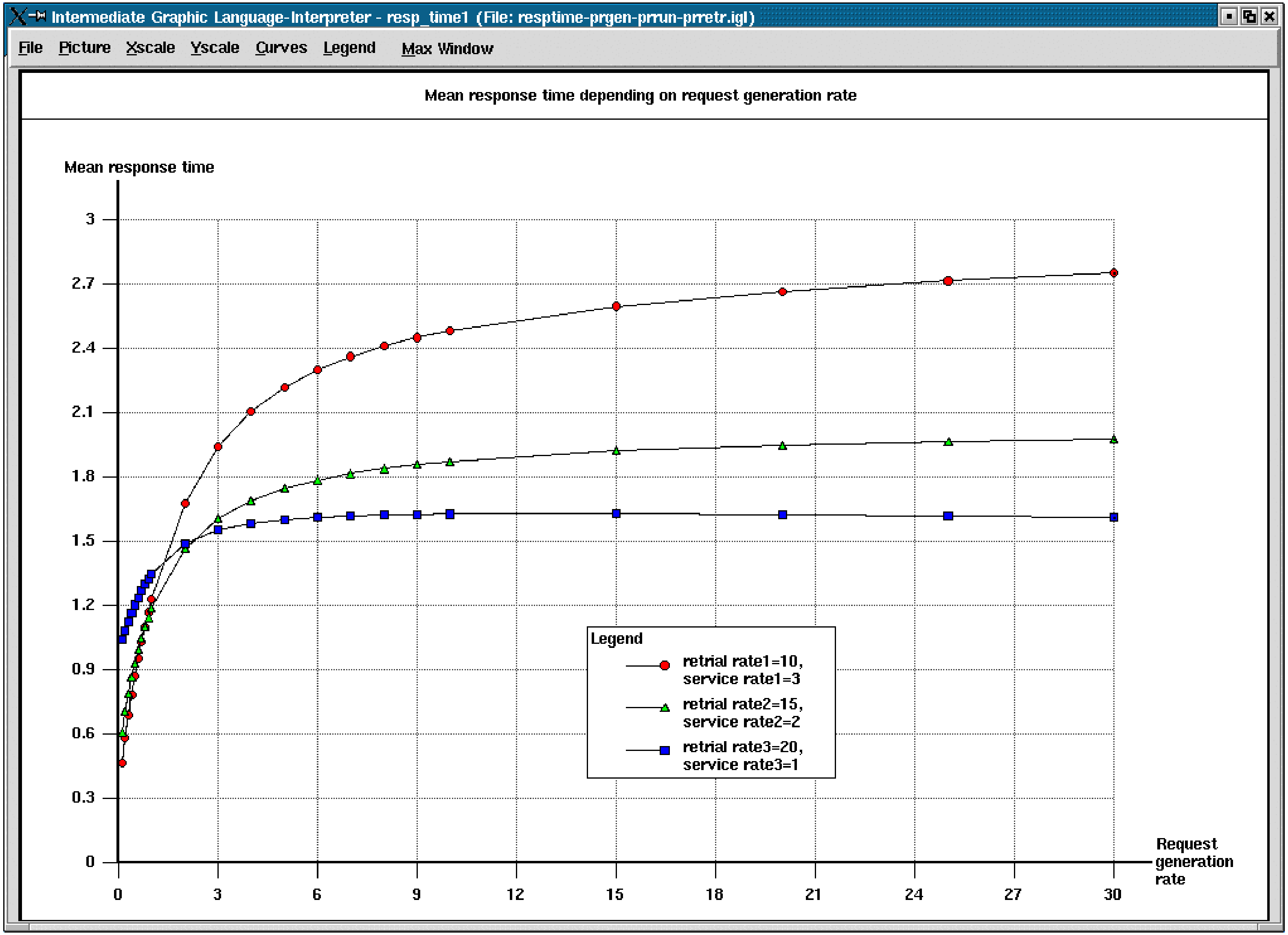

$E[T]$ versus primary request generation rate with homogeneous retrial and heterogeneous service




$E[T]$ versus retrial rate with homogeneous service and heterogeneous primary request generation




$E[T]$ versus retrial rate with homogeneous primary request generation and heterogeneous service




$E[T]$ versus primary request generation rate with heterogeneous service and heterogeneous retrial

## References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