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Algorithmic Approach to Study the Model of Perishable Inventory System with Repeated Customers

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Abstract

The model of perishable inventory system with repeated customers is examined under (s, S) policy. The stability condition of the system is derived and the joint distribution of the number of customers in orbit and the inventory level is obtained by using matrix-geometric method. Formulas for calculation of the performance measures are developed.

Keywords: perishable inventory system, repeated customers, (s, S) policy, matrix-geometric method, calculation methods, performance measures

1. Introduction

One of the important class of operations management systems is a perishable inventory systems (PIS) in which an inventory life time is a finite random quantity, for example, blood banks, systems of processing an outdated information, food provision systems, etc. In PIS the inventory level decreases not only after its release to a customer but also due to the end of inventory lifetime. Note that works [1] - [4] contain references of numerous literature sources in this direction.

Here we consider PIS models without service facility [5] - [8]. This paper is similar to [8]. The main contributions of this paper are as follows: (i) We extend the model investigated in [8] by considering perishable inventory items. (ii) We assume that arrived primary customers (p-customers) in accordance to Bernoulli scheme either join the orbit or leave the system when the inventory level is zero. (iii) We assume that retrial customers (r-customers) might be impatient, i.e. if the inventory level is zero upon arrival, then the r-customers in accordance to Bernoulli scheme either leave the system or re-join the orbit. (iv) We consider (s, S) replenishment policy, i.e.

when the inventory level hits s or below, an order is placed to make the inventory full.

2. Description of the model

The system has a store house of limited volume S and p-customers forms Poisson input with rate λ. If at the moment of p-customer arrival the inventory level is positive, then it is instantly serviced and leaves the system; otherwise (i.e. when inventory level is zero) it with probability H_p either leaves for infinity orbit to repeat its inquiry, or with complementary probability $1 - H_p$ eventually leaves the system. Only r-customer on the head of the orbit repeat its request at random time which has exponential d.f. with parameter η, i.e. retrial rate is independent on the number of r-customers. If at the moment of a r-customer arrival inventory level is positive, then such customer is instantly serviced and leaves an orbit; otherwise the r-customer either leaves an orbit with probability H_r or with complementary probability $1 - H_r$ stays there to repeat its request. It is assumed that only one item can perish in a very little interval and random life time of each item has an exponential d.f. with mean $\gamma^{-1}, \gamma < \infty$. Here we consider (s, S) policy and assume that lead time is positive random variables that has exponential d.f. with the mean ν^{-1}.

3. Computation of the steady-state probabilities

Mathematical model of the system is 2D MC with states that defined by 2D vectors (n, m), where n is total number of customers in orbit, $n = 0, 1, ...,$ and n indicates the inventory level, $m = 0, 1, ..., S$. State space of the indicated 2D MC is given by

$$E = \bigcup_{n=0}^{\infty} L(n) \tag{1}$$

where $L(n) = \{(n, 0), (n, 1), ..., (n, S)\}$ called the n^{th} level, $n = 0, 1, 2, \ldots$.

The transition rate from the state $(n_1, m_1) \in E$ to the state $(n_2, m_2) \in E$ is denoted by $q((n_1, m_1), (n_2, m_2))$. According to the accepted service scheme and replenishment policy, we obtain the following relations for the determining of the indicated transitions:

$$q((n_1,m_1),(n_2,m_2)) = \begin{cases} \lambda + m_1\gamma, & \text{if } m_1 > 0, (n_2,m_2) = (n_1, m_1 - 1) \\ \eta, & \text{if } n_1 m_1 > 0, (n_2,m_2) = (n_1 - 1, m_1 - 1) \\ \eta H_r, & \text{if } n_1 > 0, m_1 = 0, (n_2,m_2) = (n_2 - 1, m_1) \\ \lambda H_p, & \text{if } m_1 = 0, (n_2,m_2) = (n_1 + 1, m_1) \\ \nu, & \text{if } m_1 \le s, (n_2,m_2) = (n_1, S) \\ 0, & \text{in other cases} \end{cases} \tag{2}$$

Hereinafter, the equality of vectors means that their corresponding components are equal to each other. States from the space E is renumbered in lexicographical order as follows $(0,0),(0,1),...,(0,S),(1,0),(1,1),...,(1,S),....$ Then indicated 2D MC has the following generator:

$$\begin{pmatrix} B & A_0 & . & . & . \\ A_2 & A_1 & A_0 & . & . \\ . & A_2 & A_1 & A_0 & . \\ . & . & . & . & . \end{pmatrix} \tag{3}$$

All block matrices in (3) are square matrices of dimension $S+1$. From relations (2) we conclude that entities of the block matrices $B = ||b_{ij}||$ and $A_k = ||a_{ij}^{(k)}||, i,j = 0,1,...,S$ are determined as follows:

$$b_{ij} = \begin{cases} \nu, & \text{if } i \le s, j = S \\ \lambda + i\gamma, & \text{if } i > s, j = i - 1 \\ -(\nu + \lambda H_p), & \text{if } i = j = 0 \\ -(\nu + i\gamma + \lambda), & \text{if } 0 < i \le s, j = i \\ -(i\gamma + \lambda), & \text{if } s < i \le S, j = i \\ 0, & \text{in other cases} \end{cases} \tag{4}$$

$$a_{ij}^{(0)} = \begin{cases} \lambda H_p, & \text{if } i = j = 0 \\ 0, & \text{in other cases} \end{cases} \tag{5}$$

$$a_{ij}^{(1)} = \begin{cases} \nu, & \text{if } i \le s, j = S \\ \lambda + i\gamma, & \text{if } i > s, j = i - 1 \\ -(\nu + \lambda H_p + \eta H_r), & \text{if } i = j = 0 \\ -(\nu + i\gamma + \lambda + \eta), & \text{if } 0 < i \le s, j = i \\ -(i\gamma + \lambda + \eta), & \text{if } s < i, j = i \\ 0, & \text{in other cases} \end{cases} \tag{6}$$

$$a_{ij}^{(2)} = \begin{cases} \eta H_r, & \text{if } i = j = 0 \\ \eta, & \text{if } 1 < i, j = i-1 \\ 0, & \text{in other cases} \end{cases} \tag{7}$$

Let $A = A_0 + A_1 + A_2$. Stationary distribution that correspond to the generator A is denoted by $\pi = (\pi(0), \pi(1), ..., \pi(S))$, i.e. we have

$$\pi A = 0, \pi e = 1 \tag{8}$$

where 0 is null row vector of dimension $S+1$ and e is column vector of dimension $S+1$ that contains only 1's.

From relations (5)-(7) we obtain that entities of generator $A = ||a_{ij}||, i, j = 0, 1, ..., S$, are determined as

$$a_{ij}^{(1)} = \begin{cases} -\nu, & \text{if } i = j = 0 \\ \nu, & \text{if } 0 \le i \le s, j = S \\ \lambda + i\gamma, & \text{if } i > s, j = i-1 \\ -(\nu + \lambda H_p + \eta H_r), & \text{if } i = j = 0 \\ -(\nu + i\gamma + \lambda + \eta), & \text{if } 0 < i \le s, j = i \\ -(i\gamma + \lambda + \eta), & \text{if } s < i, j = i \\ 0, & \text{in other cases} \end{cases} \tag{9}$$

Proposition. The system is ergodic if and only if the following relation is fulfilled:

$$\lambda H_p \pi(0) < \eta(1 - (1 - H_r)\pi(0)) \tag{10}$$

Proof: From relations (9) we obtain that system of equations (8) has following form:

$$(\nu + (m\gamma + \lambda + \eta)(1 - \delta_{m,0}))\pi(m) = ((m+1)\gamma + \lambda + \eta)(\pi(m+1), 0 \le m \le s \tag{11}$$

$$\begin{aligned} (m\gamma + \lambda + \eta)\pi(m) = ((m+1)\gamma + \lambda + \eta)(\pi(m+1)\chi(s+1 \le m \le S-1) + \\ +\nu \sum_{m=0}^{s} \pi(m)\delta_{m,S}, s+1 \le m \le S \end{aligned} \tag{12}$$

Here $\delta_{x,y}$ denotes Kronecker delta and $\chi(A)$ is indicator function of event A.

From (11) and (12) all values $\pi(m), m = 1, .., S$ are expressed by as follows:

$$\pi(m) = \begin{cases} a_m \pi(0), & \text{if } 1 \le m \le s+1 \\ b_m \pi(0), & \text{if } s+1 < m \le S \end{cases} \tag{13}$$

where $a_m = \prod\limits_{i=1}^{m} \frac{\Lambda_{i-1}+\nu}{\Lambda_i}; b_m = \frac{\Lambda_{s+1}}{\Lambda_m} \prod\limits_{i=1}^{s+1} \frac{\Lambda_{i-1}+\nu}{\Lambda_i}; \Lambda_i = \lambda + \eta + i\gamma, i = 1, 2, ..., S.$

The probability $\pi(0)$ is determined from normalizing condition, i.e.

$$\pi(0) = \left(1 + \sum_{m=1}^{s+1} a_m + \sum_{m=s+2}^{S} b_m\right)^{-1}$$

In accordance to [9] (chapter 3, pages 81-83) investigated 2D MC is ergodic if and only if the following condition is fulfilled:

$$\pi A_0 e < \pi A_2 e \tag{14}$$

By taking into account (5), (7) and (13) after some algebras from (14) we obtain that relation (10) is true.

Steady-state probabilities corresponding to the generator matrix G we denote by $p = (p_0, p_1, ...)$, where $p_n = (p(n,0), p(n,1), ..., p(n,S)), n = 0, 1, ...$ Under the ergodicity condition (10) steady-state probabilities are determined from the following equations:

$$p_n = p_0 R^n, n \geq 1 \tag{15}$$

where R is nonnegative minimal solution of the following quadratic matrix equation:

$$R^2 A_2 + R A_1 + A_0 = 0$$

Bound probabilities p_0 are determined from the normalizing condition:

$$\begin{aligned} p_0(B + RA_2) &= 0 \\ p_0(I - R)^{-1} e &= 1 \end{aligned} \tag{16}$$

where I is indicated identity matrix of dimension $S + 1$.

4. Performance measures

Performance measures are calculated via steady-state probabilities as follows.

Average inventory level: $S_{av} = \sum\limits_{m=1}^{S} m \sum_{n=0}^{\infty} p(n, m)$

Average order size under (s, S) policy: $V_{av} = \sum\limits_{m=S-s}^{S} m \sum_{n=0}^{\infty} p(n, S - m)$

Average number of customers in orbit: $L_o = \sum\limits_{n=1}^{\infty} m \sum_{m=S}^{\infty} p(n, m)$

Average reorder rate: $RR = (\lambda + (s + 1)\gamma) \sum\limits_{n=0}^{\infty} p(n, s + 1) + \eta \sum\limits_{n=1}^{\infty} p(n, s + 1)$

Loss probability of p-customers: $P_p = (1 - H_p) \sum_{n=0}^{\infty} p(n,0)$

Loss probability of r-customers: $P_r = H_r \sum_{n=1}^{\infty} p(n,0)$

5. Conclusion

In this paper, the Markovian model of PIS without service facility and with repeated customers under (s, S) policy is proposed. It is assumed that arrived primary customer in accordance Bernoulli scheme either go to infinity orbit or leaves the system when inventory level is zero. By similar way, if upon arrival of a r-customer inventory level is zero then customer either leaves an orbit or stays to repeat its request. The stability condition of the system is derived and the joint distribution of the number of customers in orbit and the inventory level is obtained by using matrix-geometric method. Formulas to calculating the performance measures are developed. These formulas allow solve the design problems as well as optimization problems of the investigated PIS. Similar model of PIS under (s, Q) replenishment policy can be investigated by using developed here approach.

REFERENCES

1. Goyal S., Giri B. Recent trends in modeling of deteriorating inventory. European Journal of Operations Research. 2001. Vol. 134, Issue 1. P. 1-16.
2. Karaesmen I., Scheller-Wolf A., Deniz B. Managing perishable and aging inventories: Review and future research directions. Planning production and inventories in the extended enterprise. A state of the art handbook. (Eds. Kempf K., Keskinocak P, Uzsoy P.). Vol. 1. Springer, 2011. P. 393-438.
3. Bakker M., Riezebos J., Teunter R. H. Review of inventory systems with deterioration since 2001. European Journal of Operation Research. 2012. Vol. 221. P. 275-284.
4. Nahmias S. Perishable inventory theory. Heidelberg: Springer. 2011.
5. Liu L. An (s, S) continuous review models for inventory with random lifetimes. Operations Research Letters. 1990. Vol. 9. Issue 3. P. 161-167
6. Liu L., Yang T. An (s, S) random lifetimes inventory model with positive lead time. European Journal of Operations Research. 1999. Vol. 113. Issue 1. P. 52-63
7. Kalpakam S., Sapna K.P. Continuous review (s, S) inventory system with random lifetimes and positive lead times. Operations Research Letters. 1994. Vol. 16. Issue 2. P. 115-119
8. Anbazhagan N., Wang J., Gomathi D. Base stock policy with retrial demands. Applied Mathematical Modelling. 2013. Vol. 37. P. 4464–4473
9. Neuts M.F. Matrix-geometric solutions in stochastic models: An algorithmic approach. Baltimore: John Hopkins University Press. 1981. 332 p.