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Theory and Methodology

On the heterogeneous machine interference problem with priority and ordinary machines

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Abstract: This paper deals with a heterogeneous machine interference model under the assumption that the priority machines have preemptive priority over the ordinary ones. In each group machines are characterized by exponentially distributed running and repair times with different rates. The failed machines are served by a single server according to the FIFO discipline. Although the system is Markovian, the number of states becomes very large. The main contribution of this paper is to give a computationally tractable method for the solution of steady-state equations. In equilibrium the article is concerned with the main operational characteristics of the system. Finally, an optimization problem is treated and numerical results illustrate the problem in question.

Keywords: Running time, repair time, operative utilization, machine availability, Little's formula

1. Introduction

The machine repair problem has been analysed in many forms over the past 30 years. In its simplest form, where there are exponential running times, exponential repair times, a fixed number of machines in the system and a fixed number of repairman, the problem is frequently used as a textbook example of a continuous-time Markov chain or of limited source exponential queueing systems. Many articles have generalized this basic model by assuming, for example, general repair times and general failure times. For an extensive bibliography on the basic homogeneous finite-source model, reference may be made to Jaiswal [4] or Takács [11]. The machine interference problem with different types of machines has been discussed, for example, by Csige and Tomkó [1], Gross and Ince [3], Kameda [6], Lehtonen [7], and Sztrik [9,10]. The multiple finite-source priority models, especially the double finite-source one of size N_1 and N_2, with exponentially distributed source time and arbitrarily distributed service time were studied by Jaiswal [4], and Jaiswal and Thiruvengadam [5]. In this paper we consider an exponential machine interference situation where the operator looks after two independent sets of machines, each having different failure and repair rates.

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This situation is quite common because new machines are frequently introduced either through expansion or replacement, and since the complete elimination of old machines takes years, the operator is quite likely to have a heterogeneous mixture of old and new ones. The question of deciding which type of machine should be assigned priority so as to minimize the loss due to the interference is obvious importance. If $m = 1$, the model may be interpreted as being the machine interference with limited server's availability. (See Pósafalvi and Sztrik [8].)

The assumptions are the following. There are two groups of machines of which one comprises of m priority machines and the other comprises of n ordinary ones. If at any time t, priority machine i is in working order, the probability that it will call service in the time interval $(t, t+\Delta t)$ is $\gamma_i \Delta t + o(\Delta t)$, $i = 1,\dots, m$. Similarly, ordinary machine j may break down in the interval $(t, t+\Delta t)$ with probability $\lambda_j \Delta t + o(\Delta t)$, $j = 1,\dots, n$. The successive repair times of priority machine i and ordinary machine j are supposed to be exponentially distributed random variables with means $1/\sigma_i$, $1/\mu_j$, respectively. We also assume that the working and service times are mutually independent.

The repairman serves the failed machines according to the following discipline. An ordinary machine will be taken up for service only when there is no priority machine waiting for service and if a priority one breaks down when an ordinary is in the service facility, the former preempts the latter. That is, the service discipline between the groups is preemptively resumed. In the same group the machines are repaired in the order to their arrivals, that is, the service rule is FIFO.

The purpose of the present paper is to generalize the models treated by Elsayed [2], Jaiswal and Thiruvengadam [5], Pósafalvi and Sztrik [8], and Thiruvengadam [12]. Although the system is Markovian, the number of states becomes very large. The emphasis of the article, therefore, is on deriving computationally tractable formulas for the steady-state probabilities. In equilibrium the paper deals with the main operational characteristics of the system, such as operative utilization, mean length of busy period, average number of failed machines, machine availability, expected duration of time while the given machine is not in working order, mean waiting times of failed machines. Finally, an optimization problem is treated and numerical results illustrate the problem in question.

2. The mathematical model

Let the random variables $\chi(t)$, $v(t)$ denote the number of priority and ordinary machines at the service facility at time t, respectively.

The index of the i-th priority machine and the j-th ordinary machine in the queue (in FIFO order) at time t is $\alpha_i(t)$, $\beta_j(t)$ respectively. Since all the distributions are exponential, the process

$$X(t) = \left(\chi(t),\, \alpha_1(t),\dots,\alpha_{\chi(t)}(t);\; v(t),\, \beta_1(t),\dots,\beta_{v(t)}(t)\right) \tag{1}$$

is a finite-state, continuous-time, irreducible Markov chain with state space

$$\bigcup_{k=0}^{m} \bigcup_{s=0}^{n} (i_1,\dots, i_k;\; j_1,\dots, j_s),$$

where

$$(i_1,\dots, i_k) \in V_k^m, \qquad (j_1,\dots, j_s) \in V_s^n, \qquad k = 0,\dots, m, \qquad s = 0,\dots, n,$$

and V_l^r is the set of all variations of order l of the integers $1,\dots, r$.

By definition, let $V_0^r = i_0 = j_0 = \{0\}$ denote the event that $\chi(t)$ or $v(t)$ equals to zero. Then it is easy to see that if γ_i, $\sigma_i > 0$, $i = 1,\dots, m$ and λ_j, $\mu_j > 0$, $j = 1,\dots, n$, then $(X(t),\ t \geqslant 0)$ possesses a unique ergodic distribution which can be obtained as the solution of the steady-state balance equations satisfying the normalizing condition.

Let us denote the steady-state distribution by

$$p(0;\, 0) = \lim_{t\to\infty} P(\chi(t) = 0;\ v(t) = 0), \tag{2a}$$

$$p(0;\, j_1,\dots, j_s) = \lim_{t\to\infty} P(\chi(t) = 0;\ v(t) = s,\ \beta_1(t) = j_1,\dots, \beta_s(t) = j_s), \tag{2b}$$

$$p(i_1,\dots, i_k;\, 0) = \lim_{t\to\infty} (\chi(t) = k,\ \alpha_1(t) = i_1,\dots, \alpha_k(t) = i_k;\ v(t) = 0), \tag{2c}$$

$$\begin{aligned} &p(i_1,\dots, i_k;\ j_1,\dots, j_s) \\ &\quad= \lim_{t\to\infty} P(\chi(t) = k,\ \alpha_1(t) = i_1,\dots, \alpha_k(t) = i_k;\ v(t) = s,\ \beta_1(t) = j_1,\dots, \beta_s(t) = j_s). \end{aligned}$$

Then it is easy to verify that the stationary equations are

$$\left(\sum_{i=1}^{m} \gamma_i + \sum_{j=1}^{n} \lambda_j\right) p(0;\, 0) = \sum_{j=1}^{n} \mu_j p(0;\, j) + \sum_{i=1}^{m} \sigma_i p(i;\, 0), \tag{3}$$

$$\begin{aligned} &\left(\sum_{i=1}^{m} \gamma_i + \sum_{v\neq j_1,\dots,j_s} \lambda_v + \mu_{j_1}\right) p(0;\, j_1,\dots, j_s) \\ &\quad= \sum_{i=1}^{m} \sigma_i p(i;\, j_1,\dots, j_s) + \sum_{r\neq j_1,\dots,j_s} \mu_r p(0;\, r,\, j_1,\dots, j_s) + \lambda_{j_s} p(0;\, j_1,\dots, j_{s-1}) \\ &\qquad \text{for } 1 \leqslant s < n, \end{aligned} \tag{4}$$

$$\left(\sum_{i=1}^{m} \gamma_i + \mu_{j_1}\right) p(0;\, j_1,\dots, j_n) = \sum_{i=1}^{m} \sigma_i p(i;\, j_1,\dots, j_n) + \lambda_{j_n} p(0;\, j_1,\dots, j_{n-1}), \tag{5}$$

$$\begin{aligned} &\left(\sum_{q\neq i_1,\dots,i_k} \gamma_q + \sum_{j=1}^{n} \lambda_j + \sigma_{i_1}\right) p(i_1,\dots, i_k;\, 0) \\ &\quad= \gamma_{i_k} p(i_1,\dots, i_{k-1};\, 0) + \sum_{q\neq i_1,\dots,i_k} \sigma_q p(q,\, i_1,\dots, i_k;\, 0) \quad \text{for } 1 \leqslant k < m, \end{aligned} \tag{6}$$

$$\left(\sum_{j=1}^{m} \lambda_j + \sigma_{i_1}\right) p(i_1,\dots, i_m;\, 0) = \gamma_{i_m} p(i_1,\dots, i_{m-1};\, 0), \tag{7}$$

$$\begin{aligned} &\left(\sum_{q\neq i_1,\dots,i_k} \gamma_q + \sum_{v\neq j_1,\dots,j_s} \lambda_v + \sigma_{i_1}\right) p(i_1,\dots, i_k;\ j_1,\dots, j_s) \\ &\quad= \gamma_{i_k} p(i_1,\dots, i_{k-1};\ j_1,\dots, j_s) + \lambda_{j_s} p(i_1,\dots, i_k;\ j_1,\dots, j_{s-1}) \\ &\qquad+ \sum_{q\neq i_1,\dots,i_k} \sigma_q p(q,\, i_1,\dots, i_k;\ j_1,\dots, j_s) \quad \text{for } 1 \leqslant k < m,\ 1 \leqslant s \leqslant n, \end{aligned} \tag{8}$$

$$\begin{aligned} &\left(\sum_{v\neq j_1,\dots,j_s} \lambda_v + \sigma_{i_1}\right) p(i_1,\dots, i_m;\ j_1,\dots, j_s) \\ &\quad= \gamma_{i_m} p(i_1,\dots, i_{m-1};\ j_1,\dots, j_s) + \lambda_{j_s} p(i_1,\dots, i_m;\ j_1,\dots, j_{s-1}) \\ &\qquad \text{for } 1 \leqslant s \leqslant n,\quad \sum_{v\neq j_1,\dots,j_n} \lambda_v = 0. \end{aligned} \tag{9}$$

Analytic solution of (3)–(9) appears to be formidable, if not impossible. Therefore, the problem was approached numerically.

Let us denote by $\| V_l^r \|$ the cardinality of V_l^r and let $Z^{(k,s)}$ be the vector

$$Z^{(k,s)} = \begin{pmatrix} p(1,\dots,k;\ 1,\dots,s) \\ \vdots \\ p(1,\dots,k;\ j_1,\dots,j_s) \\ \vdots \\ p(1,\dots,k;\ n,\dots,n-s+1) \\ \vdots \\ p(i_1,\dots,i_k;\ j_1,\dots,j_s) \\ \vdots \\ p(m,\dots,m-k+1;\ n,\dots,n-s+1) \end{pmatrix}$$

of dimension $\| V_k^m \| \cdot \| V_s^n \|$, $k = 0,\dots,m$, $s = 0,\dots,n$. The components $p(i_1,\dots,i_k;\ j_1,\dots,j_s)$ are listed in the lexicographical order of the elements $(i_1,\dots,i_k;\ j_1,\dots,j_s)$. Notice that Eqs. (3)–(9) can be written in the following matrix form:

$$Z^{(0,0)} = A_{0,0} Z^{(1,0)} + B_{0,0} Z^{(0,1)}, \tag{10}$$

$$Z^{(0,s)} = A_{0,s} Z^{(1,s)} + B_{0,s} Z^{(0,s+1)} + C_{0,s} Z^{(0,s-1)} \quad \text{for } 1 \leqslant s < n, \tag{11}$$

$$Z^{(0,n)} = A_{0,n} Z^{(1,n)} + C_{0,n} Z^{(0,n-1)}, \tag{12}$$

$$Z^{(k,0)} = D_{k,0} Z^{(k-1,0)} + A_{k,0} Z^{(k+1,0)} \quad \text{for } 1 \leqslant k < m, \tag{13}$$

$$Z^{(m,0)} = D_{m,0} Z^{(m-1,0)}, \tag{14}$$

$$Z^{(k,s)} = A_{k,s} Z^{(k+1,s)} + D_{k,s} Z^{(k-1,s)} + C_{k,s} Z^{(k,s-1)} \quad \text{for } 1 \leqslant k < m,\ 1 \leqslant s \leqslant n, \tag{15}$$

$$Z^{(m,s)} = D_{m,s} Z^{(m-1,s)} + C_{m,s} Z^{(m,s-1)} \quad \text{for } 1 \leqslant s \leqslant n, \tag{16}$$

where the involved matrices are of the following order:

$$A_{k,s} - \| V_k^m \| \cdot \| V_s^n \| \times \| V_{k+1}^m \| \cdot \| V_s^n \|; \qquad B_{0,s} - \| V_s^n \| \times \| V_{s+1}^n \|,$$

$$C_{k,s} - \| V_k^m \| \cdot \| V_s^n \| \times \| V_k^m \| \cdot \| V_{s-1}^n \|; \qquad D_{k,s} - \| V_k^m \| \cdot \| V_s^n \| \times \| V_{k-1}^m \| \cdot \| V_s^n \|.$$

The entries of the above matrices can be determined by the help of Eqs. (3)–(9). First we give the solution to (13) and (14).

It is easy to see that $Z^{(k,0)}$ can be obtained in an iterative manner, viz.

$$Z^{(k,0)} = G_{k,0} Z^{(k-1,0)}, \quad k = 1,\dots,m, \tag{17}$$

where

$$G_{m,0} = D_{m,0}, \qquad G_{k,0} = (I - A_{k,0} G_{k+1,0})^{-1} D_{k,0}.$$

In the following let the vector $Z^{(s)}$ be

$$Z^{(s)} = \begin{pmatrix} Z^{(0,s)} \\ \vdots \\ Z^{(k,s)} \\ \vdots \\ Z^{(m,s)} \end{pmatrix}$$

of dimension $\| V_s^n \| \cdot (\sum_{k=0}^{m} \| V_k^m \|)$, $s = 1, \ldots, n$. By the help of these vectors Eqs. (10), (13) and (14) can be written as

$$Z^{(0)} = A_0 Z^{(0)} + B_0 Z^{(1)}. \tag{18}$$

Similarly, from (11), (15) and (16) we get

$$Z^{(s)} = A_s Z^{(s)} + B_s Z^{(s+1)} + C_s Z^{(s-1)}, \quad 1 \leqslant s < n. \tag{19}$$

Finally, from (12), (15) and (16) we obtain

$$Z^{(n)} = A_n Z^{(n)} + C_n Z^{(n-1)}. \tag{20}$$

The involved matrices A_s, B_s, C_s are of the following structure:

$$A_s = \begin{pmatrix} 0 & A_{0,s} & & 0 \\ D_{1,s} & & \ddots & \\ \vdots & \ddots & & \\ & & \cdot & A_{m-1,s} \\ 0 & & D_{m,s} & 0 \end{pmatrix}, \quad B_s = \begin{pmatrix} B_{0,s} & 0 \\ 0 & 0 \end{pmatrix}, \quad C_s = \begin{pmatrix} C_{0,s} & & 0 \\ & \ddots & \\ 0 & & C_{m,s} \end{pmatrix}$$

Then it can be easily verified that $Z^{(s)}$ can be evaluated recursively by

$$Z^{(s)} = F_s Z^{(s-1)}, \quad s = 1, \ldots, n, \tag{21}$$

where

$$F_n = (I - A_n)^{-1} C_n, \qquad F_s = (I - A_s - B_s F_{s+1})^{-1} C_s, \quad s = 1, \ldots, n-1.$$

Since (21) is started by $Z^{(0)}$ which is calculated by (17), hence the whole system depends on $Z^{(0,0)} = p(0;\, 0)$. Starting with any $p(0;\, 0)$ after normalizing by

$$\sum_{k=0}^{m} \sum_{s=0}^{n} \sum_{(i_1,\ldots,i_k) \in V_k^m} \sum_{(j_1,\ldots,j_s) \in V_s^n} p(i_1, \ldots, i_k;\, j_1, \ldots, j_s) = 1,$$

the steady-state probabilities (2) are obtained.

3. The main operational characteristics

In this section the Markov chain (1) is supposed to be in equilibrium. We study the following measures.

(i) Operative utilization. By operative utilization we mean the long-run fraction of time during which the repairman is occupied serving either priority or ordinary machines. Clearly, if this measure is denoted by U_s, then we have

$$U_s = 1 - p(0;\, 0) = M\delta \Big/ \left(M\delta + 1 \Big/ \left(\sum_{i=1}^{m} \gamma_i = \sum_{j=1}^{n} \lambda_j \right) \right),$$

where $M\delta$ denotes the expected busy period length. Thus,

$$M\delta = (1 - p(0;\, 0)) \Big/ \left(p(0;\, 0) \left(\sum_{i=1}^{m} \gamma_i + \sum_{j=1}^{n} \lambda_j \right) \right).$$

(ii) Machine availability. If U_i, $\hat{U}_j$ denote the long-run fraction of time during which priority machine i and ordinary machine j are in working order, respectively, clearly we have

$$U_i = 1 - \sum_{k=1}^{m} \sum_{s=0}^{n} \sum_{i \in (i_1, \dots, i_k)} \sum_{(j_1, \dots, j_s)} p(i_1, \dots, i_k;\, j_1, \dots, j_s),$$

$$\hat{U}_j = 1 - \sum_{k=0}^{m} \sum_{s=1}^{n} \sum_{(i_1, \dots, i_k)} \sum_{j \in (j_1, \dots, j_s)} p(i_1, \dots, i_k;\, j_1, \dots, j_s).$$

(iii) Mean waiting times. Applying Theorem 3 in Tomkó [13], namely

$$U_i = (1/\gamma_i)(S_i + 1/\gamma_i)^{-1}, \quad i = 1, \dots, m; \qquad \hat{U}_j = (1/\lambda_j)(\hat{S}_j + 1/\lambda_j)^{-1}, \quad j = 1, \dots, n, \tag{22}$$

for the average time while the machine i and j are not in working order, we obtain

$$S_i = (1 - U_i)/(\gamma_i U_i); \qquad \hat{S}_j = (1 - \hat{U}_j)/(\lambda_j \hat{U}_j).$$

Since the average number of failed machines (Q) is

$$Q = \sum_{i=1}^{m} (1 - U_i) + \sum_{j=1}^{n} (1 - \hat{U}_j),$$

from (22) we have

$$Q = \sum_{i=1}^{m} \gamma_i U_i S_i + \sum_{j=1}^{n} \lambda_j \hat{U}_j \hat{S}_j,$$

which is Little's formula for an $\langle (m, n)/\vec{M}/\vec{M}/1 \rangle$ queueing system. Clearly, the mean waiting times are

$$D_i = S_i - 1/\sigma_i, \quad i = 1, \dots, m; \qquad \hat{D}_j = \hat{S}_j - 1/\mu_j, \quad j = 1, \dots, n.$$

4. Numerical results

(i) As an illustration consider the following optimization problem. An operator looks after two independent sets of machines, old and new ones. The question is, which type of machines should be assigned priority so as to minimize the total loss per unit time, i.e. to minimize

$$L = \sum_{i=1}^{m} C_i(1 - U_i) + \sum_{j=1}^{n} \hat{C}_j(1 - \hat{U}_j) + Bp(0; 0),$$

where

C_i = profit per unit time for priority machine i,
$\hat{C}_j$ = profit per unit time for ordinary machine j,
B = wage of the operator per unit time.

Input parameters for old and new machines are shown in Table 1.

In Case 1 new machines have priority, while in Case 2 old machines preempt the new ones. The derived results can be seen in Table 2. As it can be observed assuming priority to the old ones is preferable. However, it should be noted that more numerical work is needed to make specific comments about the effect of different parameters.

(ii) In this section our aim is to show, how the different rates influence the operational characteristics. We consider six cases, the input parameters are depicted in Table 3, 5, 7, 9, 11 and 13 while the resulting measures are collected in Table 4, 6, 8, 10, 12 and 14.

Table 1
Input parameters [a]

Old machines				New machines			
Machine	Failure rate	Repair rate	Profit	Machine	Failure rate	Repair rate	Profit
1	0.5	0.8	4	1	0.	0.3	2
2	0.6	0.9	5	2	0.2	0.4	3
3	0.7	1.1	6				

[a] Wage of the operator is 7.

Table 2

Case	U_s	$M\delta$	S_1	S_2	S_3	D_1	D_2	D_3
1	0.9551	10.1481	4.19	3.27	–	0.85	0.77	–
2	0.9857	32.9199	2.23	2.11	1.96	0.98	1.00	1.05
	Q	L	$\hat{S}_1$	$\hat{S}_2$	$\hat{S}_3$	$\hat{D}_1$	$\hat{D}_2$	$\hat{D}_3$
1	3.0400	1.3871	6.46	6.09	5.70	5.21	4.98	4.79
2	3.3413	1.2709	39.97	34.78	–	36.64	32.28	–

Table 3

Case	γ_1	γ_2	σ_1	σ_2	λ_1	λ_2	μ_1	μ_2
1	0.3	0.3	0.2	0.5	0.3	0.3	0.7	1.1
2	0.3	0.3	0.2	0.7	0.3	0.3	0.5	1.1
3	0.3	0.3	0.2	1.1	0.3	0.3	0.5	0.7
4	0.3	0.3	0.5	0.7	0.3	0.3	0.2	1.1
5	0.3	0.3	0.5	1.1	0.3	0.3	0.2	0.7
6	0.3	0.3	0.7	1.1	0.3	0.3	0.2	0.5

Table 4

Case	Q	U_s	$M\delta$	S_1	S_2	D_1	D_2	$\hat{S}_1$	$\hat{S}_2$	$\hat{D}_1$	$\hat{D}_2$
1	1.80^+	0.9396	12.9707	5.87^+	4.57^+	0.87^+	2.57^+	13.37	12.28^+	11.94^+	11.38^+
2	2.74	0.9396	12.9707	5.52	3.86	0.52	2.43	13.74	11.86	11.74	10.95
3	2.67	0.9396	12.9707	5.25	3.18	0.25	2.27	12.72	11.93	10.72	10.50
4	2.40	0.9396	12.9707	2.45	2.13	0.45	0.70	15.17^+	11.17	10.17	10.26
5	2.29	0.9396	12.9707	2.22	1.57	0.22	0.66	13.68	10.89	8.68	9.46
6	2.16^-	0.9396	12.9707	1.63^-	1.30^-	0.20^-	0.40^-	12.61^-	10.67^-	7.61^-	8.67^-

Table 5

Case	γ_1	γ_2	σ_1	σ_2	λ_1	λ_2	μ_1	μ_2
1	0.2	0.5	0.7	0.7	0.7	1.1	0.7	0.7
2	0.2	0.7	0.7	0.7	0.5	1.1	0.7	0.7
3	0.2	1.1	0.7	0.7	0.5	0.7	0.7	0.7
4	0.5	0.7	0.7	0.7	0.2	1.1	0.7	0.7
5	0.5	1.1	0.7	0.7	0.2	0.7	0.7	0.7
6	0.7	1.1	0.7	0.7	0.2	0.5	0.7	0.7

Table 6

Case	Q	U_s	$M\delta$	S_1	S_2	D_1	D_2	$\hat{S}_1$	$\hat{S}_2$	$\hat{D}_1$	$\hat{D}_2$
1	2.45^-	0.9615^-	9.9970^-	2.02	1.74	0.59	0.31	6.70^-	6.19^-	5.27^-	4.76^-
2	2.54	0.9656	11.2567	2.14	1.74	0.71	0.31	8.24	7.19	6.81	5.76
3	2.68	0.9730	14.4516	2.30	1.74	0.87	0.31	10.53	9.96	9.10	8.53
4	2.73	0.9715	13.6437	2.14	2.02	0.71	0.69	12.93	9.81	11.50	8.38
5	2.90	0.9785	18.2366	2.30	2.02	0.87	0.59	16.74	13.82	15.31	12.39
6	3.01^+	0.9820^+	21.9083^+	2.30	2.14	0.87	0.71	19.92^+	17.38^+	18.49^+	15.95^+

Table 7

Case	γ_1	γ_2	σ_1	σ_2	λ_1	λ_2	μ_1	μ_2
1	0.3	0.4	0.2	0.5	0.6	0.8	0.7	1.1
2	0.3	0.6	0.2	0.7	0.4	0.8	0.5	1.1
3	0.3	0.8	0.2	1.1	0.4	0.6	0.5	0.7
4	0.4	0.6	0.5	0.7	0.3	0.8	0.2	1.1
5	0.4	0.8	0.5	1.1	0.3	0.6	0.2	0.7
6	0.6	0.8	0.7	1.1	0.3	0.4	0.2	0.5

Table 8

Case	Q	U_s	$M\delta$	S_1	S_2	D_1	D_2	$\hat{S}_1$	$\hat{S}_2$	$\hat{D}_1$	$\hat{D}_2$
1	3.09	0.9736^-	17.5973^-	6.00^+	4.50^+	1.00^+	2.50^+	14.70^-	12.74^-	13.27^-	11.83^-
2	3.11^+	0.9766	19.8899	5.73	3.61	0.73	2.18	18.50	14.05	16.50	13.14
3	3.08	0.9785	21.7719	5.42	2.72	0.42	1.81	17.99	15.84	15.99	14.41
4	2.90	0.9801	23.4628	2.68	2.25	0.68	0.82	24.78	16.13	19.78	15.22
5	2.84^-	0.9821	26.1671	2.41	1.63	0.41	0.72	23.51	17.83	18.51	16.41
6	2.85	0.9860^+	33.6255^+	1.83^-	1.50^-	0.40^-	0.60^-	26.24^+	22.61^+	21.24^+	20.61^+

Table 9

Case	γ_1	γ_2	σ_1	σ_2	λ_1	λ_2	μ_1	μ_2
1	0.3	0.3	0.35	0.35	0.3	0.3	0.9	0.9
2	0.3	0.3	0.45	0.45	0.3	0.3	0.8	0.8
3	0.3	0.3	0.65	0.65	0.3	0.3	0.6	0.6
4	0.3	0.3	0.6	0.6	0.3	0.3	0.65	0.65
5	0.3	0.3	0.8	0.8	0.3	0.3	0.45	0.45
6	0.3	0.3	0.9	0.9	0.3	0.3	0.35	0.35

Table 10

Case	Q	U_s	$M\delta$	S	D	$\hat{S}$	$\hat{D}$
1	2.61	0.9253	10.3325	4.17	1.31	10.18	9.07
2	2.38	0.9080	8.2320	3.11	0.88	8.11	6.86
3	2.11	0.8962	7.1967	2.02	0.48	7.02	5.35
4	2.15	0.8962	7.1967	2.22	0.55	7.05	5.51
5	2.04	0.9080	8.2320	1.59	0.34	7.76	5.54
6	2.05	0.9253	10.3325	1.38	0.27	9.23	6.37

Table 11

Case	γ_1	γ_2	σ_1	σ_2	λ_1	λ_2	μ_1	μ_2
1	0.35	0.35	0.7	0.7	0.9	0.9	0.7	0.7
2	0.45	0.45	0.7	0.7	0.8	0.8	0.7	0.7
3	0.65	0.65	0.7	0.7	0.6	0.6	0.7	0.7
4	0.6	0.6	0.7	0.7	0.65	0.65	0.7	0.7
5	0.8	0.8	0.7	0.7	0.45	0.45	0.7	0.7
6	0.9	0.9	0.7	0.7	0.35	0.35	0.7	0.7

Table 12

Case	Q	U_s	$M\delta$	S	D	$\hat{S}$	$\hat{D}$
1	2.51	0.9639	10.6819	1.90	0.47	6.74	5.31
2	2.68	0.9707	13.2820	1.98	0.59	8.53	7.10
3	2.92	0.9792	18.8889	2.11	0.68	12.79	11.36
4	2.87	0.9776	17.4822	2.08	0.65	11.64	10.22
5	3.03	0.9826	22.6992	2.19	0.76	16.54	15.11
6	3.07	0.9838	24.3387	2.23	0.83	19.51	17.82

Table 13

Case	γ_1	γ_2	σ_1	σ_2	λ_1	λ_2	μ_1	μ_2
1	0.35	0.35	0.35	0.35	0.7	0.7	0.9	0.9
2	0.45	0.45	0.45	0.45	0.6	0.6	0.8	0.8
3	0.55	0.55	0.65	0.65	0.5	0.5	0.6	0.6
4	0.5	0.5	0.6	0.6	0.55	0.55	0.65	0.65
5	0.6	0.6	0.8	0.8	0.45	0.45	0.45	0.45
6	0.7	0.7	0.9	0.9	0.35	0.35	0.35	0.35

Table 14

Case	Q	U_s	$M\delta$	S	D	$\hat{S}$	$\hat{D}$
1	2.98	0.9698	15.3366	4.28	1.42	11.65	10.54
2	2.96	0.9733	17.3803	3.33	1.11	12.75	11.50
3	2.84	0.9754	18.8914	2.24	0.70	13.29	11.63
4	2.83	0.9732	17.3158	2.42	0.75	12.17	10.64
5	2.77	0.9794	22.6630	1.78	0.53	15.18	12.96
6	2.90	0.19867	35.3479	1.83	0.71	23.08	20.22

In Tables 4, 6 and 8, '+' and '−' denote the maximal and the minimal value of the corresponding characteristics, respectively.

Comments. (1) In Table 2 we can see that the costs are very important factors; concerning some characteristics (Q, $\hat{S}_j$, $\hat{D}_j$, $j = 1, 2, 3$), Case 1 is preferable, while respect to the total cost L, Case 2 is better.

The motivation of these input parameters is as follows. As usual, the failure rates of old machines are greater than new ones, but if the operative has no experience in repairing new machines, their repair rates are less than those of old ones.

(2) Table 4 shows that under constant homogeneous failures rates U_s and M_δ are unchanged, which follows from Kameda [6]. The queue length at the service facility is the longest if the priority machines

have the least service rates and vice versa. But this statement does not hold for every measure, for example, for $\hat{S}_1$.

(3) Table 6 demonstrates that under constant homogeneous service rates characteristics Q, U_s, $M\delta$ are closely connected. Their maximal values are obtained if the priority machines possess the greatest failure rates and vice versa.

(4) In Table 8 we can see some interesting facts. Namely, U_s, $M\delta$ are maximal if the priority machines have the greatest failure rates and vice versa, but this observation does not correspond to the queue length. The sojourn and waiting times of the machines belonging to the priority and ordinary groups, respectively, behave just in an opposite way.

(5) Table 10 provides numerical results in the case of homogeneous repairs in both groups, replacing the parameter values of the non-homogeneous case given in Table 3 with their average values. We can observe that the measures Q, U_s, $M\delta$ are less than the corresponding ones in Table 4, furthermore S, D, $\hat{S}$, $\hat{D}$ are also less than the means of the values found in Table 4. The coincidence of U_s, $M\delta$ in Cases 1–6: Cases 2, 5 and Cases 3, 4 follow from Kameda [6], too.

(6) Table 12 demonstrates a surprising fact, namely, replacing the parameter values of the non-homogeneous case given in Table 5 with their average values, the resulting measures are greater than those found in Table 6. So the effect of the replacement in this case is just an opposite than in the earlier one.

(7) Table 14 shows how the system behaves if we replace the parameter values given in Table 7 with their average values. The effect is mixed, since on the one hand measures Q, U_s, $M\delta$ are less than the corresponding ones in Table 8 in Case 1, on the other hand they are greater than the corresponding ones in Case 7.

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References

[1] Csige, L. and Tomkó, J., "The machine interference for exponentially distributed operating and repair times", *Alkalmazott Matematikai Lapok* 8 (1982) 107–124 (in Hungarian).

[2] Elsayed, E.A., "An optimum repair policy for the machine interference problem", *Operational Research Society Journal* 32 (1981) 793–801.

[3] Gross, D., and Ince, J.F., "The machine repair problem with heterogeneous populations", *Operations Research* 29 (1981) 532–549.

[4] Jaiswal, N.K., *Priority Queues*, Academic Press, New York, 1968.

[5] Jaiswal, N.K., and Thiruvengadam, K., "Finite-source priority queues", *SIAM Journal on Applied Mathematics* 15 (1967) 1278–1293.

[6] Kameda, H., "A finite-source queue with different customers, *Journal of the Association for Computing Machinery* 29 (1982) 478–491.

[7] Lehtonen, T., "On the optimal policies of an exponential machine repair problem", *Naval Research Logistics Quarterly* 31 (1984) 173–181.

[8] Pósafalvi, A., and Sztrik, J., "On the heterogeneous machine interference with limited server's availability", *European Journal of Operational Research* 28 (1987) 321–328.

[9] Sztrik, J., "On the machine interference", *Publ. Math.* 30 (1983) 165–175.

[10] Sztrik, J., "On the finite-source $\vec{G}/M/r$ queue", *European Journal of Operational Research* 20 (1985) 261–268.

[11] Takács, L., *Introduction to the Theory of Queues*, Oxford, University Press, New York, 1962.

[12] Thiruvengadam, K., "A priority assignment in machine interference", *Opsearch* (India) 1 (1964) 197–216.

[13] Tomkó, J., "Sojourn time problems for Markov chains", *Alkalmazott Matematikai Lapok* 8 (1982) 91–106 (in Hungarian).