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A new model of finite-source retrial queues with multi-state server's breakdown

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Abstract. This paper deals with unreliable systems, where the performance of the system can be experienced in different levels of health between the perfect and the failed states.

The underlying model is a single server retrial queue with a finite number of homogeneous sources of requests and a single non-reliable server. The server is subject to random deteriorations and breakdowns depending on whether it is busy or idle. The operations of the system might be blocked or unblocked by the failure of the server and the service of the interrupted request may be resumed or the request can be transmitted to the orbit. All random variables involved in the model constructions are supposed to be exponentially distributed and totally independent of each other.

The novelty of the investigation is the levels of goodness of a system in case of finite sources of requests. The transitions of states of the server depend on its working status (ie., busy or idle), which makes the system rather complicated. The MOSEL-2 tool was used to formulate and solve the problem. The main performance and reliability measures were calculated.

1.1. Introduction. Retrial queues have been widely investigated and used to model many problems arising in telephone switching systems, telecommunication networks, computer networks, optical networks, etc. The main characteristic of a retrial queue is that a customer who finds the service facility busy upon arrival is obliged to leave the service area, but some time later he comes back to re-initiate his demand. Between the trials a customer is said to be in the orbit. The literature on retrial queueing systems is very extensive. For a recent account, readers may refer to the recent books of Falin and Templeton [1], Artalejo and Gomez-Corral [7] that summarize the main models and methods. Since in practice some components of the systems are subject to random breakdowns (see for example [2], [3], [4]) it is basically important to study the reliability of retrial queues with server breakdowns and repairs because of limited ability of repairs and heavy influence of the breakdowns on the performance measures of the system. However, so far the repairable retrial queues were analyzed only in the case when the server could be in two states: operable or failed. For related literature the reader is referred to the works [5], [6] where infinite-source non-reliable retrial queues were treated.

In general, systems used in industries always suffer from stochastic deterioration and may undergo many deteriorating states before failure.

In this paper, a finite-source system is investigated with the following assumptions. Consider a single server queueing system, where the primary calls are generated by K , $1 < K < \infty$ homogeneous sources. We assume that the system deterioration condition can be described by a finite number of levels $1, 2, \dots, m+1$. Level 1 denotes the best health condition meaning that the system is as good as a new one, while m denotes the most deteriorated condition. Finally, $m+1$ indicates that the system has failed.

The server can be in three states:

- *idle state:* The server level is between $1..m$ and the server can start serving the arriving requests.
- *busy state:* The server is in busy state, when the server is in available state (the server level is between $1..m$) and one request is under service.
- *failed state:* The server is in failed state, when the server level is $m+1$. It can not start serving any arriving requests until it is repaired.

We assume, that the service times are exponential distributed with parameters μ_1, \dots, μ_m , corresponding to server levels $1, \dots, m$

If a source is free at time t , it can generate a primary call during interval $(t, t+dt)$ with probability $\lambda dt + o(dt)$. If the server is idle at the time of arrival of a call, then the service of the call starts immediately, the source moves into the under service state and the server moves into busy state. The service is finished during the interval $(t, t+dt)$ with probability $\mu_i dt + o(dt)$, $i = 1..m$, according to the server level. If the server is busy, then the source starts generation of a Poisson flow of repeated calls with rate ν until it finds the server idle. After service the source becomes free, and it can generate a

new primary call, and the server becomes idle so it can serve a new call. The server can step into the next level during the interval $(t, t + dt)$ with probability $\delta dt + o(dt)$ if it is idle, and with probability $\gamma dt + o(dt)$ if it is busy. If $\delta = 0, \gamma > 0$ or $\delta = \gamma > 0$ *active or independent breakdowns* can be discussed, respectively. If the server step reaches the level $m + 1$, that is the server is failed, two different cases can be treated. The first one is the *blocked sources case*, when all the operations are stopped, that is no new calls are generated. The second one is the *unblocked sources case*, when only the service is interrupted but all the other operations are continued. In this paper we investigate only the unblocked sources case. If the server fails in a busy state, the interrupted request returns to the sources. The repair times are exponentially distributed with a finite mean $1/\tau$. All the times involved in the model are assumed to be mutually independent of each other.

Because of the fact, that the state space of the describing Markov chain is very large, it is rather difficult to calculate the system measures in the traditional way of writing down and solving the underlying steady-state equations. To simplify this procedure we used the software tool MOSEL (Modeling, Specification and Evaluation Language), see Begail et al. [8], to formulate the model and to obtain the performance measures.

1.2. The $M/M/1//K$ retrial queueing model with multi-state unreliable server. To maintain theoretical manageability, the distributions of inter-event times (i.e., request generation time, service time, retrial time, available state time, failed state time) presented in the network are by assumption exponential and totally independent.

We introduce the following notations:

- $y(t)$ represents the server states at time t , $y(t) = 1 \dots m$ if the server is up with the corresponding service rates μ_1, \dots, μ_m and $y(t) = m + 1$ if the server is failed.
- $c(t) = 0$ if the server is idle, $c(t) = 1$ if the server is busy
- $o(t)$ is the number of jobs in the orbit at time t .

The system state at a time t corresponds to a Continuous Time Markov Chain (CTMC) with 3 dimensions:

$$X(t) = (y(t); c(t); o(t))$$

The steady-state distributions are denoted by

$$P(y; c; o) = \lim_{t \rightarrow \infty} P(y(t) = y, c(t) = c, o(t) = o), \quad y = 1 \dots m + 1, \quad c = 0, 1, \quad o = 0, \dots, K - 1.$$

Note, that the state space of this Continuous Time Markovian Chain is finite, so the steady-state probabilities surely exist. For computing the steady-state probabilities and the system characteristics, we use the MOSEL software tool in this paper.

As soon as we have calculated the distributions defined above, the most important steady-state system characteristics can be obtained in the following way:

- *Utilization of the server*

$$U_S = \sum_{y=1}^m \sum_{o=0}^{K-1} P(y, 1, o)$$

- *Availability of the server*

$$A_S = \sum_{y=1}^m \sum_{c=0}^1 \sum_{o=0}^{K-1} P(y, c, o)$$

- *Average number of jobs in the orbit*

$$\begin{aligned} \bar{O} &= E(o(t)) = \\ &= \sum_{y=1}^{m+1} \sum_{c=0}^1 \sum_{o=0}^K o P(y, c, o) \end{aligned}$$

- The mean number of calls staying in the orbit or in service

$$M = E[o(t) + c(t)] = \bar{O} + \sum_{q=1}^{m+1} \sum_{o=0}^{K-1} P(q, 1, j).$$

- The mean rate of generation of primary calls

$$\bar{\lambda} = \begin{cases} \lambda E[K - c(t) - o(t); y(t) \text{ in } (1, \dots, m)], & \text{for blocked case,} \\ \lambda E[K - c(t) - o(t)], & \text{for unblocked case.} \end{cases}$$

- The mean response time

$$E[T] = M/\bar{\lambda}.$$

- The mean waiting time

$$E[W] = N/\bar{\lambda}.$$

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