НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ
ТОМСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ
САРАТОВСКИЙ НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ
ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ ИМ. Н. Г. ЧЕРНЫШЕВСКОГО
РОССИЙСКИЙ УНИВЕРСИТЕТ ДРУЖБЫ НАРОДОВ
ИНСТИТУТ ПРОБЛЕМ УПРАВЛЕНИЯ
им. В.А. ТРАПЕЗНИКОВА РАН

ИНФОРМАЦИОННЫЕ ТЕХНОЛОГИИ И МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ (ИТММ-2019)

МАТЕРИАЛЫ
XVIII Международной конференции
имени А. Ф. Терпугова
26–30 июня 2019 г.

Часть 2

ТОМСК
«Издательство НТЛ»
2019

УДК 519
ББК 22.17
И74

И74 Информационные технологии и математическое моделирование (ИТММ-2019): Материалы XVIII Международной конференции имени А.Ф. Терпугова (26–30 июня 2019 г.). – Томск: Изд-во НТЛ, 2019. – Часть 2. – 376 с.

ISBN 978-5-89503-629-7

Сборник содержит избранные материалы XVIII Международной конференции имени А.Ф. Терпугова по следующим направлениям: теория массового обслуживания и телетрафика, графы и их применение в задачах анализа дискретных автоматов, прикладной вероятностный анализ.

Для специалистов в области информационных технологий и математического моделирования.

УДК 519
ББК 22.17

ISBN 978-5-89503-629-7

Simulation of finite-source retrial queues with two-way communications to the orbit[1]

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Finite-source retrial queues are effective and commonly used systems for modeling arising real life problems in main telecommunication systems like cellular mobile communication networks, computer networks, local-area networks with random access protocols, call centers and CSMA-based wireless networks. With the decrement of the rate of generation of new calls the number of customers in the system increases in case of many practical situation. This can be performed with the help of finite-source or quasi-random input models. Their importance can be viewed by the reader in the following works, for example [4, 10, 11, 14, 15].

Systems with retrial feature are identified by a specific feature of arriving customers when the server is occupied. These customers stay in the system and spend their time in a virtual waiting room called orbit. Customers in the orbit attempt to be served after a random time. Because the number of calls are finite, the assumption of working with finite-source queuing systems follows real circumstances.

In this paper we examine two-way communication retrial queuing system which is quite popular topic in the recent years. This can be explained by the fact that using two-way communication scheme is very helpful in many application fields modelling occurring real life problems. Especially in case of call centers where service unit can perform certain other work in idle state like selling, advertising and promoting products including serving incoming calls. In such systems utilization of the service unit is always pivotal, see for example in [1, 2, 6, 13, 20, 22, 9, 16].

Once the server becomes idle it calls for customers inside and outside of the system which is called an outgoing call. This is a typical feature of two-way communication system. In our investigated model the idle service unit can generate a call only from the orbit which arrives after a random time. It will only be served if no customers from the finite source or from the orbit

[1] The research was financed by the Higher Education Institutional Excellence Programme of the Ministry of Human Capacities in Hungary, within the framework of the **20428-3/2018/FEKUTSTRAT** thematic programme of the University of Debrecen.

come. Otherwise this outgoing call will be canceled. Papers dealing with two-way communication systems by the help of retrial queues, where the source is infinite, are found in [3, 5, 7, 8, 17, 18, 19, 21].

Our aim is to study the operation of the system where the service unit is reliable and can perform outgoing call from the orbit. The novelty of this paper is to compare this system with the common finite source retrial system using various distribution of service on performance measures like mean waiting time of an incoming call or utilization of the server. We are mainly interested in how the different distributions modify the characteristics of the system. To achieve this goal a simulation program has been developed using the base of SimPack [12] which contains a number of C/C++ libraries and executable programs. One of the main reasons for its usage is that the user has the freedom what performance measure are calculated and how the model is built up. SimPack toolkit also provides a set of utilities that demonstrate how to build a working simulation from a model description.

System model

We consider a retrial queuing system of type M/G/1//N with a reliable server which is capable to produce outgoing calls to the customers residing in the orbit. N customers are located in the source, where all of them can generate incoming, primary calls towards the server. The distribution of the inter-request times is exponential with rate λ/N. In default of waiting queue an incoming customer either from the source or orbit finds the server in an idle state then its service begins instantly. The service times of incoming customers are assumed to be gamma, hypo-exponentially, hyper-exponentially, Pareto and lognormal distributed with different parameters but with the same mean value. Customers return to the source after their service is terminated. If the server is busy, meaning that a request is under service, an incoming customer remains in the system and enters into the orbit. Customers located in the orbit are able to attempt to access the server again after an exponentially distributed time with parameter σ/N.

In the other hand, when the server becomes idle it can make outgoing call towards the customers in the orbit. It is performed after an exponentially distributed time with parameter ν. The service time of these outgoing customers follows gamma distribution with parameters α_1 and β_1. In a consecutive paper we aim to investigate the same system by the help of asymptotic methods when N tends to infinity and that is the reason we use λ/N and σ/N parameters. All the random variables involved in the model construction are assumed to be totally independent of each other.

Simulation results

The values of the input parameters are shown in Table 1. In this section these results are in connection with the effect of different service time distributions of incoming customers where the mean and variance are equal. We use hyper-exponential distribution if the squared coefficient of variation is greater than one, Table 2 shows the exact values of parameters of service time of incoming customers.

Table 1

Numerical values of model parameters

N	σ/N	ν	α_1	β_1
100	0.01	0.02	1	0.5

Table 2

Parameters of service time of incoming customers

Distribution	Gamma	Hyper-exponential	Pareto	Lognormal
Parameters	$\alpha = 0.04$ $\beta = 0.04$	$p = 0.48$ $\lambda_1 = 0.961$ $\lambda_2 = 1.04$	$\alpha = 2.02$ $k = 0.505$	$m = -1.629$ $\sigma = 1.805$
Mean	1			
Variance	25			
Squared coefficient of variation	25			

Besides hyper-exponential, gamma, lognormal and Pareto distributions are also used for comparison. The case when the squared coefficient of variation is less than one was also investigated and will be published in the extended version of the paper. But this case is less interesting that is why we present the former one.

Figure 1 shows the mean waiting time in function of arrival intensity of incoming customers. For these values of parameters regardless of the applied distribution a maximum value of the mean waiting time can be seen. This maximum feature occurs for finite-source retrial queues, see for example [4, 9, 10, 16]. Differences can be observed among the values of mean waiting time especially in the case of using gamma and Pareto distribution, despite the fact that the mean and variance are the same. On this figure the effect of different distibutions is clearly observable.

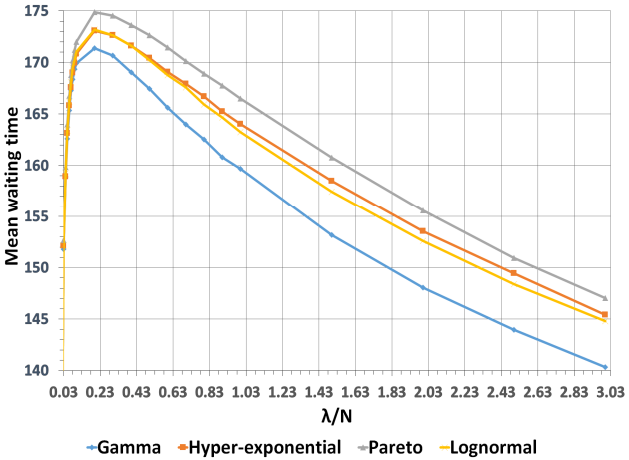

Fig. 1. Mean waiting time vs. arrival intensity using various distributions

Figure 2 illustrates how the utilization of the server grows with the increment of the arrival intensity of incoming customers. The highest values can be found at gamma distribution but the differences of the applied distributions are not as much as in case of Fig. 1.

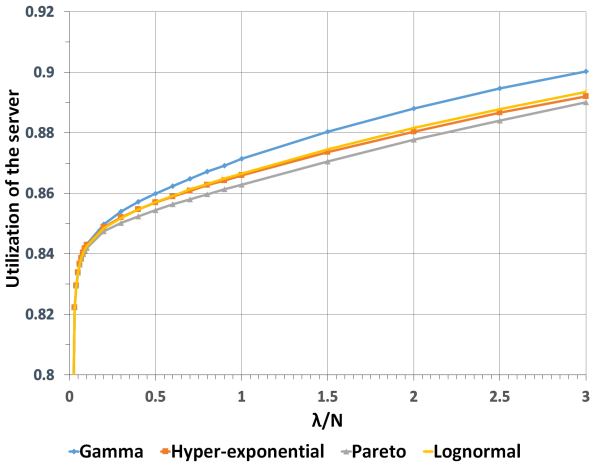

Fig. 2. Utilization of server vs. arrival intensity using various distributions

Conclusion

A finite-source retrial queueing system is introduced where the server can produce outgoing calls towards the customers of the orbit. Several figures present the effect of the applied distributions on the mean waiting time and on the utilization of the server. Using stochastic simulation method results clearly indicate that when the squared coefficient of variation is greater than one then the contrast of the values of the performance measures is quite high having the same mean and variance.

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