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Сборник содержит избранные материалы XXIII Международной конференции имени А.Ф. Терпугова по следующим направлениям: теория массового обслуживания и ее приложения, прикладной вероятностный анализ, интеллектуальные системы, информационные технологии и программная инженерия, математическое и компьютерное моделирование. Также в сборник вошли материалы международного симпозиума "Современные стохастические модели и проблемы актуарной математики"(МАМОНТ-2024).

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This volume presents selected papers from the 23rd International Conference named after A.F. Terpugov. The papers are devoted to new results in the following areas: queuing theory and its applications, applied probabilistic analysis, intelligent systems, information technology and software engineering, mathematical and computer modeling. The collection also presents the proceedings of symposium "Modern Stochastic Models and Problem of Actuarial Mathematics"(MAMMOTH-2024).

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Математическая теория телетрафика и теория массового обслуживания

INVESTIGATION OF M/G/1//N SYSTEM WITH IMPATIENT CUSTOMERS, UNRELIABLE PRIMARY AND A BACKUP SERVER

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This paper investigates a finite-source retrial queueing system characterized by request collisions, primary server unreliability, and the inclusion of a backup server. In cases of collisions, when a new job arrives while the service facility is occupied, both jobs are sent to a virtual waiting area called the orbit. Customers in the orbit make further attempts to access the server after a random interval. During server breakdowns, the customer at the server is transferred to the orbit. The system consists of a backup facility when the primary server is unreachable to process requests while the main service unit is under repair. The novelty of this study lies in the implementation of the impatience of the customers and conducting a sensitivity analysis using various service time distributions for the primary customers. We examined a scenario where key performance measures are visually represented, highlighting the observed disparities. **Keywords:** *simulation, queueing system, finite-source model, sensitivity analysis, backup server, collisions, unreliable operation, impatience.*

Introduction

In the current era of increasing traffic volumes and growing user bases, analyzing communication systems and designing optimal configurations present significant challenges. Information exchange plays a crucial role in all aspects of life, making it essential to develop or adapt mathematical and simulation models for telecommunication systems to meet these evolving demands. Retrial queues are particularly effective and suitable for modeling real-world scenarios encountered in telecommunication systems, networks, mobile networks, call centers, and similar fields. Numerous scholarly works, such as those referenced in [2] and [3], have extensively investigated various aspects of retrial queuing systems characterized by retrial calls.

In certain contexts, researchers assume the perpetual availability of service units; however, operational interruptions or unexpected events may occur, leading to the rejection of incoming customers. Devices across various industries are prone to malfunctions, making the presumption of their infallible operation overly optimistic and impractical. Similarly, in wireless communication environments, diverse factors can affect transmission rates, causing interruptions during packet delivery. The inherent unreliability of retrial queuing systems significantly impacts system functionality and performance metrics. Additionally, halting production entirely is unfeasible, as it may cause delays in order fulfillment. Therefore, during such occurrences, machines or operators with lower processing capacities may continue operating to maintain smoother functionality. Furthermore, the authors explore the feasibility of incorporating a backup server capable of providing services at a reduced rate when the primary server is unavailable. Numerous recent studies have extensively examined retrial queuing systems with unreliable servers, as demonstrated by references such as [5].

Waiting is a ubiquitous phenomenon in various aspects of life, often leading to dissatisfaction due to the time spent in queues. This dissatisfaction can result in early departures of requests from the system without being served, a behavior termed impatience. Such behavior is observed in diverse domains including healthcare applications, call centers, and telecommunication networks. The impatience mechanism is a crucial aspect of the model, as it influences the overall system performance by potentially reducing the number of customers waiting in the system and affecting the service dynamics. Studies examining these behaviors include [6].

In technological contexts such as Ethernet networks or constrained communication sessions, job collisions are likely to occur. Multiple entities within the source may initiate asynchronous attempts, leading to signal interference and necessitating retransmissions. Consequently, it is crucial to incorporate this phenomenon into research aimed at developing effective strategies to mitigate conflicts and the resultant message delays. Publications that address findings related to collisions include [7].

The objective of this study is to perform a sensitivity analysis using various service time distributions for the primary server, in order to evaluate the main performance metrics in scenarios that incorporate the feature of impatience of the customers. When the primary server fails, customer service is transferred to the backup facility. During this period, new customers are directed to the backup unit or to the orbit if the backup unit is busy. Our investigation focuses on the impact of the impatient feature, with results obtained through simulation using Simpack [4]. The simulation program is developed based on fundamental code elements that facilitate the computation of desired metrics across a range of input parameters. Graphical representations are provided to illustrate the effects of different parameters and distributions on key performance indicators.

1. System model

We investigate a finite-source retrial queueing system denoted as type $M/G/1//N$ (as depicted in Figure 6), which integrates an unreliable primary service unit, collision occurrences, and a backup service unit. This model features a finite-source, where each of the N individuals generates requests to the system following an exponential distribution with parameter λ. Arrival times adhere to an exponential distribution with a mean of $\lambda * N$. In the absence of queues, arriving jobs are serviced immediately based on a gamma, hypo-exponential, hyper-exponential, Pareto, or log-normal distribution, each characterized by distinct parameters but sharing equivalent mean and variance values (η).

In cases of server busyness, an arriving customer causes a collision with the customer currently being serviced, resulting in both customers being transferred to the orbit. Jobs in the orbit subsequently initiate further attempts to access the server after an exponentially distributed random time with parameter σ. Additionally, random breakdowns occur, with failure times modeled by exponential random variables. The failure time parameter is γ_0 when the server is occupied and γ_1 when idle.

Upon the primary service unit's failure, repair begins immediately, with the repair duration following an exponential distribution with parameter γ_2. If the server fails while busy, the customer is promptly moved to the orbit. During the primary server's unavailability, all customers in the source continue to generate requests, which are then directed to the backup server. The backup server operates at a reduced rate, described by an exponentially distributed random variable with parameter μ, and is assumed to be reliable, functioning only when the primary server is unavailable. Incoming requests are directed to the orbit if the backup server is busy, and collisions do not occur at the backup service unit.

Each primary customer in the system is characterized by an impatience property, which reflects their potential decision to leave the system if not served within a certain time frame. This decision to abandon the system is made after a random time period, which follows an exponential distribution with rate parameter τ.

The model presumes complete independence among all random variables in its formulation.

2. Simulation results

We utilized a statistical module class equipped with an advanced statistical analysis tool to quantitatively estimate the mean and variance of observed variables via the batch mean method. This technique involves ag-

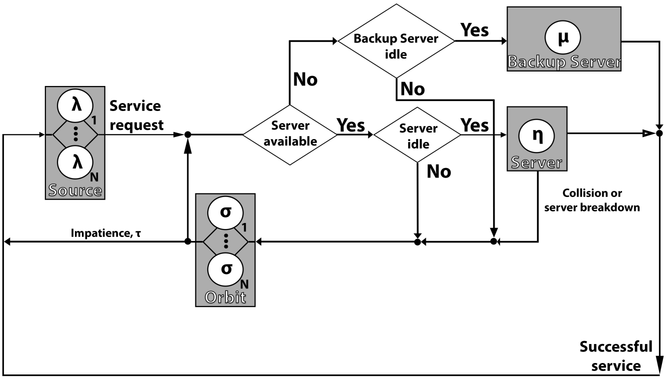

Figure 1. System model

gregating n successive observations from a steady-state simulation to generate a sequence of approximately independent samples. The batch mean method is widely recognized for its effectiveness in establishing confidence intervals for the steady-state mean of a process. To ensure the sample averages are approximately independent, large batch sizes are essential. Detailed information on the batch mean method is provided in [1]. Our simulations were conducted with a confidence level of 99.9%, and the simulation run was terminated when the relative half-width of the confidence interval reached 0.00001.

Table 1

Numerical values of model parameters

N	γ_0	γ_1	γ_2	σ	μ	τ
100	0.1	0.1	1	0.05	0.1	0.01

In this section, our goal was to establish service time parameters for each distribution such that they exhibit equivalent mean values and variances. Four distinct distributions were analyzed to evaluate their impact on performance metrics. The hyper-exponential distribution was specifically chosen to ensure a squared coefficient of variation exceeding one. The input parameters for the various distributions are detailed in Table 2, and Table 1 enumerates the values of other pertinent parameters.

Figure 2 illustrates the relationship between the mean response time of successfully served customers and the arrival intensity. Under successfully served customers we mean those customers which do not depart from the

Table 2

Parameters of service time of primary customers

Distribution	Gamma	Hyper-exponential	Pareto	Lognormal
Parameters	$\alpha = 0.011$ $\beta = 0.011$	$p = 0.494$ $\lambda_1 = 0.989$ $\lambda_2 = 1.011$	$\alpha = 2.005$ $k = 0.501$	$m = -2.257$ $\sigma = 2.125$
Mean	1			
Variance	90.25			
Squared coefficient of variation	90.25			

system earlier because of impatience. The Pareto distribution demonstrates the highest mean response time, whereas the differences among the other distributions become more pronounced. Notably, the gamma distribution results in the lowest mean response time.

An interesting observation is that as the arrival intensity increases, the mean response time initially rises but subsequently decreases after reaching a specific threshold. This phenomenon is characteristic of retrial queuing systems with a finite-source and tends to manifest under appropriate parameter configurations. In the extended version of our paper, we plan to present results utilizing an alternative parameter configuration where the squared coefficient of variation is less than one and focusing more on how impatience has a role in modifying the performance measures.

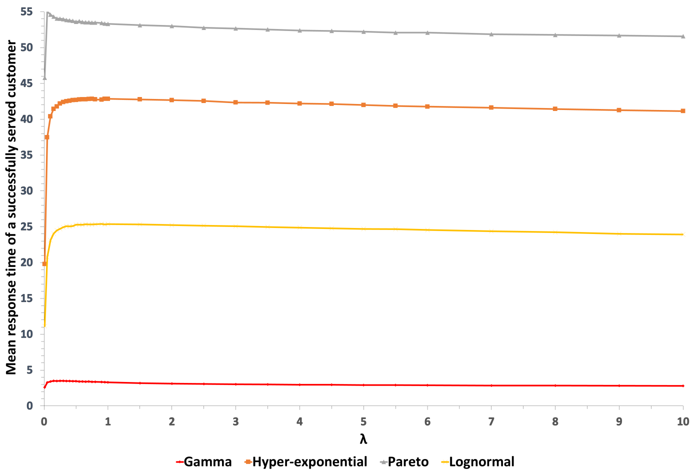

Figure 2. Mean response time vs. arrival intensity

3. Conclusion

We conducted simulations of a retrial queuing system based on the $M/G/1//N$ model, which incorporates an unreliable primary server and a backup service unit. Our program facilitated a sensitivity analysis on various performance metrics, including the mean response time of a successfully served customer. Notably, when the squared coefficient of variation exceeds one, significant deviations are observed among distributions across multiple aspects of the investigated metrics. Future studies will aim to further explore the impact of server blocking, other types of customer impatience in alternative models, and conduct sensitivity analyses for other variables, such as failure rates.

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