Hierarchical Space Merging Algorithm for Analysis of Two Stage Queueing Network with Feedback

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Abstract. A Markov model of two stage queuing network with feedback is proposed. Poisson flows arriving to both stages from outside and part of already serviced calls in the first node instantaneously enter to the second node (if there is free space here) while the remaining part leaves the network. At the completion of call processing in the second node there are three possibilities: (1) it leaves the network; (2) it instantaneously feeds back to the first node (if there is free space here); (3) it feeds back to the first node after some delay in orbit. All feedbacks are determined by known probabilities. Both nodes have finite capacities. The mathematical model of the investigated network is a three dimensional Markov chain (3-D MC) and hierarchical space merging algorithm to calculate its steady-state probabilities is developed. This algorithm allows asymptotic analysis of the quality of service (QoS) metrics of the investigated network as well.

Keywords: Two stage queueing network · Instantaneous and delayed feedback · Three-dimensional Markov chain · Space merging algorithm

1 Introduction

In the queueing networks, upon the completion of service in a particular node each call either is instantly sent to another node, or returns to the same node according to the routing matrix. Namely, if the network allows re-serving of

A. Dudin et al. (Eds.): ITMM 2016, CCIS 638, pp. 238–249, 2016.
DOI: 10.1007/978-3-319-44615-8_21

the calls in the same node, then we can assume that this network is an instantaneous feedback one. However the concept of "feedback" initially (historically) was introduced for single-station queueing systems [1], and they have been intensively studied in recent years (for further literature on work in this area, see, e.g., [2,3]).

There are several studies which investigated the two-phase model of an open queueing network with instantaneous feedback [4–8]. In these studies, the authors show that analysis of the characteristics of the integrated cellular networks and WLANs (Wireless Local Area Network, WLAN) needs to investigate similar models. However, in the queueing networks the delayed feedback is not taken into account when the already-served calls return back for repeating service after some random delay in orbit. In the available literature almost all the authors did not analyze the mentioned models in queueing networks with both types of feedback – instantaneous and delayed.

In this paper we study the model of a two-phase open queueing network with instantaneous and delayed feedback. It should be noted that taking into account the delayed feedback leads to the necessity of analyzing the three-dimensional Markov chain (3-D Markov Chain, 3-D MC). We developed an effective method for the calculation of state probabilities of the large dimension 3-D MC. There is much research on solving similar problems for the large dimension 2-D MC [9–20]. However, almost all of them have similar problems related to ill-conditioned matrices which arise during computational procedures. In this regard, we have developed a new method that uses simple explicit formulas for the calculation of the state probabilities of the constructed 3-D MC. The proposed method is based on the fundamental ideas of the theory of phase merging of stochastic systems [21]. Furthermore its counterpart for the 2-D MC was successfully used in the study of various models of single-phase queueing systems [22–25].

2 The Model

The structure of a two stage queueing network with feedback is shown in Fig. 1. For simplicity in the model it is assumed that both nodes of the network contain a single channel (server) but their service rates are not identical, i.e. the channel occupancy times of calls in node i are assumed to be independent and have identical exponential distribution with a mean $1/\mu_i, i = 1, 2$ and generally speaking $\mu_1 \neq \mu_2$. Total capacity of node i (the total number of calls in the channel and buffer) is R_i and to node i from outside arrives a Poisson flow of calls with intensity $\lambda_i, i = 1, 2$. If the node $i, i = 1, 2$, is full upon the arrival of a call then the arrived call will be lost with probability 1.

After completion of the service of the call in the first node it either leaves the network with probability θ_1 or it enters the second node with probability $\theta_2 = 1 - \theta_1$. If at the moment of completion of the service of the call in the

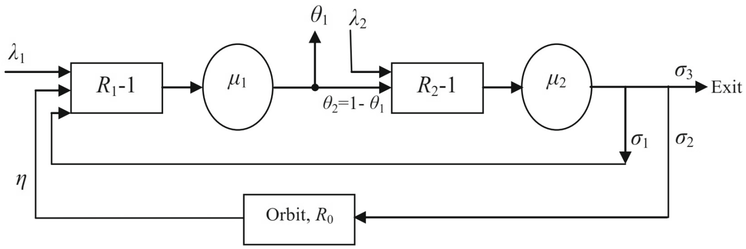

Fig. 1. The structure of the proposed model

first node the second node is full then this call will leave the network with probability 1.

After completion of the service of the call in the second node the following decisions might be made: (i) it feeds back instantaneously to the first node with probability σ_1; (ii) it enters orbit with probability σ_2; (iii) it leaves the network with probability $\sigma_3 = 1 - \sigma_2 - \sigma_1$ The orbit size is $R_0, 0 < R_0 < \propto$. It means that call arriving to orbit from the second node will be accepted if upon its arrival the number of calls in orbit is less than R_0; otherwise an arriving call will be lost. The sojourn times of calls in the orbit are independent and identically distributed random variables and they have common exponential distribution with mean $1/\eta$. It is assumed that calls from the orbit are not persistent, i.e. if upon arrival of the call the first node is full then this call is eventually lost.

3 Proposed Method for the Calculation of Steady-State Probabilities

The investigated network is described by the three-dimensional MC (3-D MC) and its states are described as 3-D vector $\boldsymbol{n} = (n_1, n_2, n_3)$ where the first (n_1) and second components (n_2), respectively, indicate the number of calls in the first and second nodes, and the third component (n_3) indicates the number of calls in orbit. The state space of the given 3-D MC is defined as follows:

$$S = \{\boldsymbol{n} : n_1 = 0, 1, ..., R_1; n_2 = 0, 1, ..., R_2; n_3 = 0, 1, ..., R_0\}. \tag{1}$$

Therefore, the geometric form of state space is depicted with integer values inside the parallelepiped whose base is a rectangle with length of sides (R_1) and (R_2); the height of the parallelepiped equal to (R_0) (see. Fig. 2).

The intensity of transition from $\boldsymbol{n}$ one state to $\boldsymbol{n\prime}$ is denoted as $q(\boldsymbol{n}, \boldsymbol{n\prime})$, $\boldsymbol{n}, \boldsymbol{n\prime} \in S$. These parameters involve the generating matrix (Q-matrix) of the given 3-D MC. They are determined as follows:

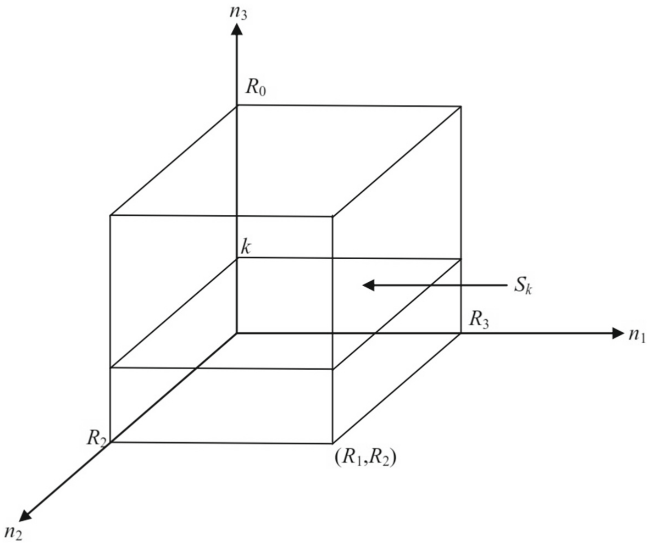

Fig. 2. The state space of the proposed model

$$q(\boldsymbol{n},\boldsymbol{n}') = \begin{cases} \lambda_1, & \text{if } \boldsymbol{n}' = \boldsymbol{n} + \boldsymbol{e_1} \\ \mu_1\theta_1, & \text{if } \boldsymbol{n}' = \boldsymbol{n} - \boldsymbol{e_1} \\ \mu_1\theta_2, & \text{if } \boldsymbol{n}' = \boldsymbol{n} - \boldsymbol{e_1} + \boldsymbol{e_2} \\ \lambda_2, & \text{if } \boldsymbol{n}' = \boldsymbol{n} + \boldsymbol{e_2} \\ \mu_2\sigma_3, & \text{if } \boldsymbol{n}' = \boldsymbol{n} - \boldsymbol{e_2} \\ \mu_2\sigma_1, & \text{if } \boldsymbol{n}' = \boldsymbol{n} - \boldsymbol{e_2} + \boldsymbol{e_1} \\ \mu_2\sigma_2, & \text{if } \boldsymbol{n}' = \boldsymbol{n} - \boldsymbol{e_2} + \boldsymbol{e_3} \\ \eta, & \text{if } n_1 < R_1, \boldsymbol{n}' = \boldsymbol{n} - \boldsymbol{e_3} + \boldsymbol{e_1} \text{or } n_1 = R_1, \boldsymbol{n}' = \boldsymbol{n} - \boldsymbol{e_3} \\ 0, & \text{in other cases.} \end{cases} \tag{2}$$

where $\boldsymbol{e}_1 = (1,0), \boldsymbol{e}_2 = (0,1)$.

Here e_i is the i-th unit vector of the 3-D Euclidean space, $i = 1,2,3$. The given 3-D MC with finite-state is irreducible since a stationary regime exists. Let $p(\boldsymbol{n})$ mean a steady-state probability of the state $\boldsymbol{n} \in S$. These probabilities are uniquely determined by solving the appropriate system of equilibrium equations (SEE) completed with a normalization condition (due to evidence of constructing the explicit form of SEE not being shown here). Unfortunately, due to the complex structure of the Q-matrix it is too complicated to find an analytical solution to the above indicated SEE. The dimension of the SEE is determined based on the dimension of the state space (1) which consists of $(R_0+1)(R_1+1)(R_2+1)$ states. Therefore, the above-given exact method makes it possible to calculate the steady-state probabilities only in moderate dimensions of state space (1), but in its large values it encounters great computational difficulties. Therefore the only way to solve them is to use numerical methods of linear algebra (for computational difficulties of these methods see the Introduction).

Here a hierarchical space merging algorithm (HSMA) for calculating the steady-state probabilities of the studied 3-D MC subject to the following condition is proposed: $\sigma_3 << \sigma_1 + \sigma_2$. In other words, it is assumed that upon completion of service in the second node, the call rarely (in comparison with the intensity of leaving the system and instantaneous return to the node 1) goes into orbit. In other words, the intensity of the call from the orbit is substantially less rather than the intensity of calls from the outside to the network nodes, i.e., $\eta << \min\{\lambda_1, \lambda_2\}$. Then, having this assumption we can say that the transition intensity between states inside the planes that are parallel to the base of the parallelepiped is much greater than the transition intensity between states of different planes (see Fig. 2). In that case we can consider the following splitting of the state space (1):

$$S = \bigcup_{k=0}^{R_0} S_k, S_k \bigcap S_{k\prime} = \varnothing, k \neq k\prime, \tag{3}$$

where $S_k = \{\boldsymbol{n} \in S : n_3 = k\}, k = 0, 1, 2, ..., R_0$. In other words, it is considered that the entire state space of the network is split into different planes that are parallel to the base of the parallelepiped (see Fig. 2).

The merge function is determined based on the splitting (3) as follows:

$$U_1(\boldsymbol{n}) = <k>, \text{if } \boldsymbol{n} \in S_k \tag{4}$$

where $<k>$ is a merged state, which includes all states of class S_k. Let $\Omega_1 = <k> : k = 0, 1, ..., R_0$.

According to SMA [21] the steady-state probabilities of the given model are defined as follows

$$p(\boldsymbol{n}) \approx \rho^k(n_1, n_2)\pi_1(<k>), \tag{5}$$

where $\rho^k(n_1, n_2)$ denote the probability of the state (n_1, n_2) within the splitting model with state space S_k, and $\pi(<k>)$ denote the probability of the merged state $(<k>) \in \Omega_1$.

From (5) we conclude that for the calculation of the steady-state probabilities given 3-D MC we need to find probability distributions of 2-D MCs (its number is $R_0 + 1$) and one 1-D MC. For large capacities of the nodes computational difficulties arise when calculating the stationary distribution of these 2-D MC with state space $S_k, k = 0, 1, ..., R_0$. Therefore in order to calculate stationary distributions within the classes $S_k, k = 0, 1, ..., R_0$, it is necessary to apply SMA to each class, in other words, we consider the hierarchy of the merged models.

All the splitting models with state spaces $S_k, k = 0, 1, ..., R_0$ involve identical 2-D MCs (see Fig. 3) since below the value of k is fixed and a splitting model with state space S_k is analyzed.

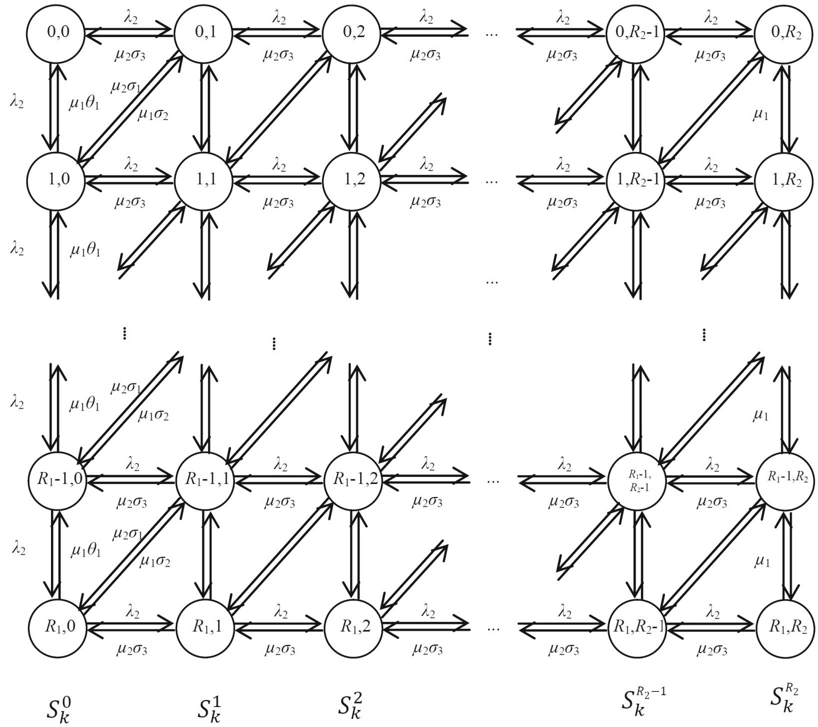

Fig. 3. The state diagram of the split model with state space S_k

The proposed method allows us to find the approximate values of state probabilities in the splitting model with state space $S_k, k = 0, 1, ..., R_0$, with asymmetric load, i.e., we distinguish two cases: (1) $\lambda_1 >> \lambda_2$, $\mu_1 >> \mu_2$; (2) $\lambda_1 << \lambda_2$, $\mu_1 << \mu_2$.

First of all let us analyze case 1. In this case, we can split the state space of S_k into the columns, i.e., in the state space S_k following splitting is considered (see Fig. 3):

$$S = \bigcup_{k=0}^{R_2} S_k^i, S_k^i \bigcap S_k^j = \varnothing, i \neq j, \tag{6}$$

where $S_k^i = \{\boldsymbol{n} \in S_k : n_2 = i\}, i = 0, 1, 2, ..., R_2$.

Based on the splitting (6) in the state space S_k the new merged function is determined:

$$U_2(\boldsymbol{n}) = <i>, \text{if } \boldsymbol{n} \in S_k^i \tag{7}$$

where $<i>$ is a merged state, which includes all states of class S_k^i. Let $\Omega_2 = <i> : i = 0, 1, ..., R_2$.

According to the SMA we have:

$$\rho^k(n_1, n_2) \approx \rho^k_{n_2}(n_1)\pi^k_2(<n_2>), \tag{8}$$

where $\rho^k_{n_2}(n_1)$ is the state probability of (n_1, n_2) within a splitting model with state space $S^{n_2}_k$, and $\pi^k_2(<n_2>)$ is the probability of the merged state $<n_2> \in \Omega_2$.

Let us consider the problem of calculating the state probabilities within the classes S^i_k. In the class S^i_k the second component is constant since the micro-state $(n_1, i) \in S^i_k$ can be defined only by the first component, i.e. $(n_1, i) \in S^i_k$ micro-state is just referred as $n_1, n_1 = 0, 1, ..., R_1$. From (2) we conclude that the transition intensities between the states n_1 and $n_1\prime$ of the splitting model with state space S^i_k does not depend on k. Therefore in the remaining part of the paper the subscription k is omitted in the notation of the state probabilities.

Also, from (2) we conclude that the probabilities $\rho^{n_1}_{n_2}, n_2 = 0, 1, ..., R_2 - 1$ coincide with the state probabilities of the model $M(\lambda_1)/M(\mu_1\theta_1)/1/R_1$, and when $n_2 = R_2$ then these probabilities coincide with the state probabilities of the model $M(\lambda_1)/M(\mu_1)/1/R_1$ (here and later we used a modified version of the Kendall notation where the values in brackets denote appropriate intensities).

Hence, the desired state probabilities $\rho_i(j), j = 0, 1, ..., R_1$ are calculated as follows:

$$\rho_i(j) = \begin{cases} \frac{1-\nu_1}{1-\nu_1^{R_1+1}}\nu_1^j, & \text{if } i = 0, 1, ..., R_2 - 1 \\ \frac{1-\nu_2}{1-\nu_2^{R_1+1}}\nu_2^j, & \text{if } i = R_2 \end{cases} \tag{9}$$

where $\nu_1 = \lambda_1/\mu_1\theta_1, \nu_2 = \lambda_1/\mu_1$.

Then after certain mathematical transformations we find the following relations to calculate the transition intensities $q(<i>, <j>), <i>, <j> \in \Omega_2$.

$$q(<i>, <j>) = \begin{cases} \lambda_2 + \mu_1\theta_2(1 - \rho_i(0)), & \text{if } i = 0, 1, ..., R_2 - 1, j = i + 1 \\ \mu_2(\sigma_3 + \sigma_1(1 - \rho_i(R_1))), & \text{if } i = 0, 1, ..., R_1, j = i - 1 \\ 0, & \text{in other cases.} \end{cases} \tag{10}$$

where $\Lambda_1 = \mu_2\sigma_2(1 - \pi_2(<0>))$.

Note 1. In calculating the stationary distribution of the splitting model with the state space $S^i_{R_0}$ it is necessary to set $\sigma_2 = 0$.

Thus, from (10) we get the following expression in order to calculate the probabilities of merged states $\pi_2(<n_2>), <n_2> \in \Omega_2$.

$$\pi_2(<n_2>) = \prod_{i=0}^{n_2-1} \frac{q(<i>, <i+1>)}{q(<i+1>, <i>)}\pi_2(<0>), n_2 = 1, ..., R_2, \tag{11}$$

where $\pi(<0>)$ is derived from the normalizing condition, i.e.
$\pi(<0>) = \left(\sum_{n_2=0}^{R_2} \pi_2(<n_2>)\right) = 1$.

The transition intensities between classes S_k and $S_{k'}$ are determined by the relations (2), (9) and (11) and after certain mathematical transformations we get:

$$q(S_k S_{k'}) = \begin{cases} \Lambda_1, & \text{if } k' = k+1 \\ k\eta, & \text{if } k' = k-1 \\ 0, & \text{in other cases.} \end{cases} \tag{12}$$

where $\Lambda_1 = \mu_2 \sigma_2 (1 - \pi_2(<0>))$.

Hence, from (12) we see that the required probabilities of the merged states $\pi(<k>), <k> \in \Omega_1$ are defined as state probabilities of a classical Erlang's model $M(\Lambda_1)/M(\eta)/R_0/0)$, i.e.,

$$\pi_1(<k>) = \frac{\phi^k}{k!} \left(\sum_{i=0}^{R_0} \frac{\phi^i}{i!} \right)^{-1}, k = 1, ..., R_0, \tag{13}$$

where $\phi = \Lambda_1/\eta$.

Finally, the state probabilities of the given 3-D MC are determined as follows:

$$p(n_1, n_2, n_3) \approx \rho_{n_2}(n_1) \pi_2(<n_2>) \pi_1(<n_3>) \tag{14}$$

Likewise, we study case 2, where we considered $\lambda_1 << \lambda_2$, $\mu_1 << \mu_2$ assumption. In this case it is necessary to split the state space S_k into rows, i.e., in the state space S_k following splitting is considered (see Fig. 3):

$$S = \bigcup_{i=0}^{R_1} S_k^i, S_k^i \bigcap S_k^j = \varnothing, i \neq j, \tag{15}$$

where $S_k^i = \{\boldsymbol{n} \in S_k : n_1 = i\}, i = 0, 1, 2, ..., R_1$.

Note 2. Here for simplicity of presentation it is better to use the same notation as used in case 1.

Next, we can implement all of the steps in the above-developed algorithm. Without repeating these steps, we describe briefly the key point of the calculation of the state probabilities. Thus, the state probabilities inside all of classes S_k^i coincide with the state probabilities of the model $M(\lambda_2)/M(\mu_2\sigma_3)/1/R_2$. Transition intensities $q(<i>, <j>), i, j \in \{0, 1, ..., R\}$ in this case are calculated as follows:

$$q(<i>, <j>) = \begin{cases} \lambda_1 + \mu_2\sigma_1(1 - \rho(0)), & \text{if } j = i+1 \\ \mu_1(\rho(R_2) + \theta_1(1 - \rho(R_2))), & \text{if } j = i-1 \\ 0, & \text{in other cases.} \end{cases} \tag{16}$$

where $\rho(0)$ and $\rho(R_2)$ denotes the probability that system $M(\lambda_2)/M(\mu_2\sigma_3)/1/R_2$ is empty and fully respectively.

So, from (16) we conclude that the probabilities of merged states in the second stage of hierarchy $\pi_2(<n_1>), n_1 \in 0, 1, ..., R_1$ are calculated as state probabilities of the model $M(\lambda_1 + \mu_2\sigma_1(1 - \rho(0)))/M(\mu_1(\rho(R_2)) + \theta_1(1 - \rho(R_2))))/1/R_1$.

The transition intensities between classes S_k and $S_{k'}$ in this case are determined similarly to (12), but in this case the quantity Λ_1 is substituted by the quantity $\Lambda_2 = \mu_2\sigma_2(1-\rho(0))$. In other words, the probabilities of the merged states in the first stage of hierarchy $\pi(<k>), k \in \{0, 1, ..., R_0\}$ are calculated by using the Erlang's formula (13).

4 QoS Metrics

After finding steady-state probabilities of the initial 3-D MC the exact values of QoS metrics of the investigated network might be determined. Thus, since the flow of calls to both nodes are Poisson ones, then the exact values of loss probabilities of calls in node 1 ($P1$) and node 2 ($P2$) are determined as follows:

$$P_i = \sum_{\boldsymbol{n}\in S} p(\boldsymbol{n})\delta(n_i, R_i), i = 1, 2 \tag{17}$$

where $\delta(i, j)$ are Kronecker's symbols.

The exact values of the average number of calls in nodes (L_1 and L_2) and retrial calls in the orbit (L_0) are defined as the expected values of the appropriate discrete random variables:

$$L_i = \sum_{j=1}^{R_i} j\Phi_i(j), \text{where } \Phi_i(j) = \sum_{\boldsymbol{n}\in S} p(\boldsymbol{n})\delta(n_i, j), i = 1, 2. \tag{18}$$

$$L_0 = \sum_{j=1}^{R_0} j\Phi_i(j), \text{where } \Phi_i(j) = \sum_{\boldsymbol{n}\in S} p(\boldsymbol{n})\delta(n_3, j). \tag{19}$$

To calculate the approximate values of the above indicated QoS metrics the following expressions are determined: For case $\lambda_1 >> \lambda_2, \mu_1 >> \mu_2$:

$$P_1 \approx \rho_0(R_1)(1-\pi_2(<R_2>)) + \rho_{R_2}(R_1)\pi_2(<R_2>); \tag{20}$$

$$P_2 \approx \pi_2(<R_2>); \tag{21}$$

$$P_0 \approx \rho_0(R_1)(1-\pi_2(<R_2>)) + \rho_{R_2}(R_1)\pi_2(<R_2>)(1-\pi_1(<0>)); \tag{22}$$

$$L_1 \approx \sum_{k=1}^{R_1} k(\rho_0(k)(1-\pi_2(<R_2>)) + \rho_{R_2}(k)\pi_2(<R_2>); \tag{23}$$

$$L_2 \approx \sum_{k=1}^{R_2} k\pi_2(<k>). \tag{24}$$

For case $\lambda_1 << \lambda_2, \mu_1 << \mu_2$:

$$P_1 \approx \pi_2(<R_1>); \tag{25}$$

$$P_1 \approx \rho(R_2); \tag{26}$$

$$P_0 \approx \pi_2(<R_1>)(1 - \pi_1(<0>)); \tag{27}$$

$$L_1 \approx \sum_{k=1}^{R_1} k\pi_2(<k>). \tag{28}$$

In this case L_2 is calculated as the average queue length in the system $M(\lambda_2)/M(\mu_2\sigma_3)/1/R_2$, i.e.

$$L_2 = \begin{cases} \frac{\omega}{1-\omega} - \frac{R_2+1}{1-\omega^{R_2+1}}\omega^{R_2+1}, & \text{if } \omega \neq 1 \\ \frac{R_2}{2}, & \text{if } \omega = 1. \end{cases} \tag{29}$$

where $\omega = \lambda_2/\mu_2\sigma_3$.

5 Conclusion

The proposed approximate method allows calculating the steady-state probabilities of the network of arbitrary dimension while the exact method might be used only for the models with a moderate size. In addition, the approximate method allows analyzing the behavior of the QoS metrics versus network parameters as well.

Note that the proposed approximate method has high accuracy for a large scale network (in order to be brief here the results which demonstrate the high accuracy of the approximate formulas are not presented). The accuracy of the proposed method is estimated by norm

$$\Delta = \max_{n \in S} |p(\boldsymbol{n}) - \tilde{p}(\boldsymbol{n})|$$

where $\tilde{p}(\boldsymbol{n})$ denotes the approximate value of probability of the state $\boldsymbol{n} \in S$. In a large interval for changing of the parameters of the network the indicated norm is acceptable in engineering practice. Moreover, numerical experiments showed that this norm asymptotically approaches zero as the dimension of the network is increased. The last fact is important since the main advantage of the proposed method is that it is developed especially for large scale networks.

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