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Performance Modelling of Nonhomogeneous Unreliable Multiserver Systems Using MOSEL

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Abstract—In this paper, we introduce a nonhomogeneous unreliable multiserver system with Markovian arrival, service, breakdown, and repair processes. First, we consider the case with only one queue and different servers and the job is assigned to one server. Then, we extend this model to more than one queue in which the jobs are assigned to different queues. We assume that our system has different servers with different service times and a job is assigned to a server using the following strategies: FFS (fastest free server) or random selection. FFS strategy means that the job is served by the fastest available server, and if this server is busy then the job goes to the next available server and so on. In the random strategy, the job is served by one of the free servers which is chosen randomly. In our problem, we consider a general queuing system ($M/M/n$) with a finite number of jobs K in the whole system. Our system is unreliable; this means that we need to specify the parameters, mtbf and mttr (mean time between failures and mean time to repair), and we need to consider the possibility that a server might be up or down at some point in time. The performance modelling of this type of system is done using the programming language *MOSEL (MOdelling Specification and Evaluation Language)*, which contains several constructs to describe the system, the results (performance parameters), and the graphical representation. © 2003 Elsevier Science Ltd. All rights reserved.

Keywords—Multiserver systems, Markovian processes, Exponential distribution, Performance.

1. PROBLEM DESCRIPTION

The calculation of the performance measure is fairly simple if there is only one server ($M/M/1$), or several servers with identical service time arranged in parallel, and the interarrival and service times are exponentially distributed ($M/M/n$) queuing systems; see, for example, [1–3]. However,

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heterogeneous multiple servers often occur in practice, and here the servers have different service times.

This situation occurs, for example, when machines of different age or manufacturer are running in parallel. So, it is interesting and useful to be able to calculate the performance measures of such systems. To the best knowledge of the authors, there are no exact formulae for such systems. In our problem, we consider a queuing system with a different number of servers and queues, where the servers have different service times. First, we consider a system with only one queue. We assume that the arrival stream of jobs forms a Poisson process with rate λ and that the service time of jobs at the server are independent and exponentially distributed random variables with rate μ_i, $i = 1, \dots, n$. We further assume that the maximum number of jobs is K; that is, if a job arrives while there are already K jobs in the system, the new job is turned away. Such queuing systems are of $(M/M/n/K)$ type.

The problem is divided into two main parts.

(i) Typical $(M/M/n/K)$ reliable queuing system; see [4,5].

(ii) $(M/M/n/K)$ unreliable queuing system. In queuing systems, we may find many situations where the terminals or servers are subject to breakdown and repair; see [2,6,7].

In our model, a server could fail and the failed server could be repaired. It is assumed that the failure and repair times are exponentially distributed with rates depending on the index of the servers and the repair is carried out by a single repairman according to the order of the failure.

In our implementation, we use two strategies for selecting the free server.

- The job is assigned randomly to a free server.
- The fastest free server is selected (FFS).

We already give priority to every server depending on the service rate (see Figure 1). We give the first server with higher service rate a higher priority and the one with less service rate one less priority and so on. Then, we do not assign the job to the server unless we make sure that this server is working; otherwise, we have to repair this server and then assign the job to it. So, we are always interested in the server with higher priority: if it is up, then we assign the job to it; otherwise, we let it go to the repair phase and assign that job to the next server with one less priority. Now, we may face a situation in which the busy server has broken down. Then this server has to go to the repair process and the job that we have already assigned to it has to go to the next higher priority server if it is up and free. We apply this strategy to all servers alike, depending on their priorities.

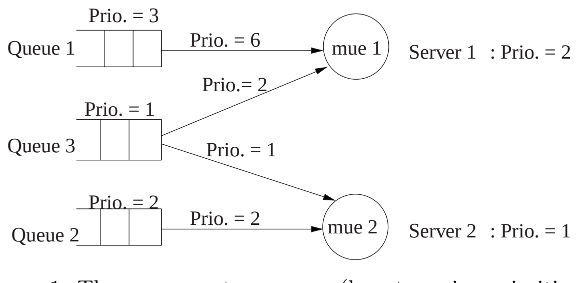


Figure 1. Three queues, two servers (how to assign priorities).

So, in this case we ignore the quality of the server, which is the opposite situation of the homogeneous system (symmetric) in which we should differentiate between the quality of different servers. That was a description of a straightforward implementation in the systems that have only one queue, but if the number of queues is increased in the system (means more than one queue); this also means that we have different types of jobs in each queue. Some of the queues give jobs to one server and others give jobs to more than one server. In this case, we will consider that we have different queues with different classes of jobs in each queue and the jobs inside each queue are in FCFS policy. It is important to mention that in the case where we have only one queue, we only need to be concerned with the priorities of the servers, but in the case of several queues in the system, we also have to give priorities to the different queues. Since we have priorities for the servers and priorities for the queues, we shall find a general rule to decide about the relationship between these two priorities.

We can conclude a general rule for giving priorities to different servers and queues, because in our problem, some queues provide jobs to different servers at the same time, and this causes competition between the queues, so we have to give priorities to different servers and queues.

Please note that we consider different queues with different jobs in each queue, but it is also possible to have the situation that there is a FCFS (first come first service) policy for the different queues.

In general, with respect to our problem, we will consider the following rules to compute the different priorities to the queues and the servers. Note that we will use these rules in the following programs in all situations where we have many different queues and servers.

- Priority of server $j = (n - j + 1)$.
- Priority of queue $l = (Q - l + 1)$, where Q is the number of queues in the system.
- We found by experiment that for every arrow that comes from queue to server, we can multiply the value of the priority of that queue with the priority of the destination server to get the result priority of the arrow between the two. A graphical representation to explain that is in Figure 1.
- Please note the following.
  - (i) The implementation of the computations for the situation where we have only one queue and many servers has been done in a straightforward way. This means, by giving different priorities to the servers, the simulation program assigns jobs to the different servers depending on those priorities.
  - (ii) In the situation where we have many queues and many servers, the implementation has been done by giving priorities to the queues and servers, as well. Then, we compute the priorities to the arrows going from every queue to the destination server, as we mentioned above. The jobs in the first queue have a higher priority; this means that these jobs are going to be served first. If there are no jobs in this queue, then the jobs in the queue with one less priority are served next, and so on. Note that the queues that give jobs to the servers directly have no competition between them. In the simulation program, we have implemented this situation first; then we did the situation where we have competition next. By looking at Figure 1, one can notice the arrows going from the queues to the servers directly without competition and the required priorities to those arrows.

If there are no jobs in the queues that give jobs to the servers directly without competition (queue 1 and queue 2 in Figure 1), then we move one level less priority to this type of queue which gives jobs to different servers, but at the same time it generates competition with other queues that give jobs to the same server (queue 3 in Figure 1). The jobs in our new queue are served depending also on the priorities given by the arrows going from this queue to the different servers. The reader can understand the above situation, either by looking to the program code or watching the graphical representation which describes the above situation in Figure 1.

Of course, you have to know that we consider that the jobs come to the queue and they are served by the server depending on the order in which they come to each queue (FCFS policy); this means that the policy inside each queue is FCFS policy.

In the following, we introduce different problems and the solution of them using MOSEL. Please note that in our implementation, we are using MOSEL, see, for example [8] or [4], to have an idea about MOSEL, which required the user just to know a little about C programming language, because MOSEL is the C language plus an additional functionality necessary for the tools we are using (for example, SPNP tool).

2. ONE QUEUE, THREE SERVERS PROBLEM

Now, for this type of problem, we will consider the two strategies (FFS, random) mentioned earlier. We will show different results by using the graphical representation provided by MOSEL, and we display the performance measures changing under different values for the parameters $(\mu_1, \mu_2, \ldots, \mu_n)$ and λ.

Strategy 1: FFS (Fastest Free Server)

In all cases, we try to cover the transient solution which is very important to show the behavior of the system over time very clearly. You can see the results by looking at the related graphical representations provided by MOSEL.

Transient solution

We will show our implementation for this type of system using MOSEL.

MOSEL description

This model could be described in MOSEL as it is shown in the following listing (`q1s3ffs_loopfailure.msl`).

```
//PROGRAM q1s3ffs_loopfailure.msl
//==============================
//DEFINITIONS
#define Q 10 // The number of jobs in the queue
#define n 3 // The number of servers

#define mue1 1.9
#define mue2 1.7
#define mue3 1.1
#define lambda 3

#define mtbf1 500.0 //failure time to server1
#define mtbf2 100.0 //failure time to server2
#define mtbf3 10.0 //failure time to server3

#define mttr1 10.0 //repair time to server1
#define mttr2 5.0 //repair time to server2
#define mttr3 5.0 //repair time to server3

//HELP Variables
HELP int K= Q + n;
<1...n> HELP int prio# = n - # + 1;

//NODES
enum down_up {down, up}; //enumerated data type
       NODE p1[Q];
<1...3> NODE m#[1];
<1...3> NODE state#[down_up] = up;
       NODE num[K];

//RULES
FROME TO p1,num w lambda;
```

```
<1...3> FROM state#[up] TO state#[down] W 1.0/mtbf#;
<1...3> FROM state#[down] TO state#[up] W 1.0/mttr#;

<1...n> IF (state#==up) FROM p1 TO m# PRIO prio#;
<1...3> FROM m#, num TOE W mue#;

//RESULTS
RESULT>>IF (num==0) prob_system_idl = PROB;
RESULT>>IF (num==K) prob_system_reject = PROB;
RESULT>>Mean_q_length = MEAN p1;
RESULT>> rho1 = UTIL m1;
RESULT>> rho2 = UTIL m2;
RESULT>> rho3 = UTIL m3;
RESULT>> throughput = rho1*mue1+rho2*mue2+rho3*mue3;

//====PICTURE============
PICTURE "Mean Queue Length"
-TITLE "Mean_q_length...over time"
      -FONT utopia -FONTSIZE 22
CURVE TIME Mean_q_length, throughput, prob_system_idl,
      prob_system_reject
      -DIFF THICK,POINT,COLOR,FILL
XSCALE LOOKOUT -GRID
YSCALE LOOKOUT -GRID
//=======================
```

The results

We start MOSEL with the following command:

```
mosel -cst0.5,15,0.5 q1s3ffs_loopfailure.msl,
```

in which we use the option *t* for the transient solution.

The following result file:

```
q1s3ffs_loopfailure.res
```

is provided by this run of the MOSEL program.

```
 =====================================
  Results provided by the tool 'SPNP'
=====================================
Constants:
      Q = 10
      n = 3
      mue1 = 1.9
      mue2 = 1.7
      mue3 = 1.1
      lambda = 3
      mtbf1 = 500.0
      mtbf2 = 100.0
      mtbf3 = 10.0
      mttr1 = 10.0
      mttr2 = 5.0
      mttr3 = 5.0
      down = 0
      up = 1
- - - - - - - - - - - - - - - - -
Results:
      K = 13 (Time:  0.5)
         ...............
        = 13 (Time:  15)
      prio1 = 3 (Time:  0.5)
         ...............
        = 3 (Time:  15)
      prio2 = 2 (Time:  0.5)
         ...............
        = 2 (Time:  15)
      prio3 = 1 (Time:  0.5)
         ...............
        = 1 (Time:  15)
prob_system_idl = 0.376902577082 (Time:  0.5)
               = 0.247519142478 (Time:  1)
               ...................
               = 0.124825112647 (Time:  15)
prob_system_reject = 1.23327243737e-09 (Time:  0.5)
                  = 5.98866164356e-07 (Time:  1)
                  ...................
                  = 0.00179488676598 (Time:  15)
Mean_q_length = 0.0332511182898 (Time:  0.5)
             = 0.138010324771 (Time:  1)
             .....................
             = 1.23224007021 (Time:  15)
rho1 = 0.563570015612 (Time:  0.5)
     = 0.631413945771 (Time:  1)
     ...................
     = 0.741660982655 (Time:  15)
rho2 = 0.292915413727 (Time:  0.5)
     = 0.455775191912 (Time:  1)
     ...................
     = 0.636379905075 (Time:  15)
rho3 = 0.108220521489 (Time:  0.5)
     = 0.262912073193 (Time:  1)
     ...................
     = 0.424162882093 (Time:  15)
throughput = 1.68778180664 (Time:  0.5)
           = 2.26370760373 (Time:  1)
           ...............
           = 2.95758087597 (Time:  15)
Help variables:   K=13, prio1=3; prio2=2; prio3=1;
                K=13; prio1=3; prio2=2;
                .....................
                prio2=2; prio3=1; K=13; prio1=3
                prio2=2; prio3=1;
============================================
```

The graphical representation of the file `q1s3ffs_loopfailure.igl` is also provided by the run of the MOSEL program and it is shown in Figure 2. By seeing the above results and looking to the graphical representations in Figure 2, one can notice that the performance measures (mean queue length, throughput, rho1, rho2, rho3) are increasing at the beginning and they start to reach a steady state at the end. We would expect this result from this type of system since we have just one queue and many servers, and the servers have no problem to cover the needs of the queue.

Next, we would like to show the changes of the operating parameters; for example, *lambda* (arrival rate) or *mtbf* (mean time between failure) in the previous program (one queue, three

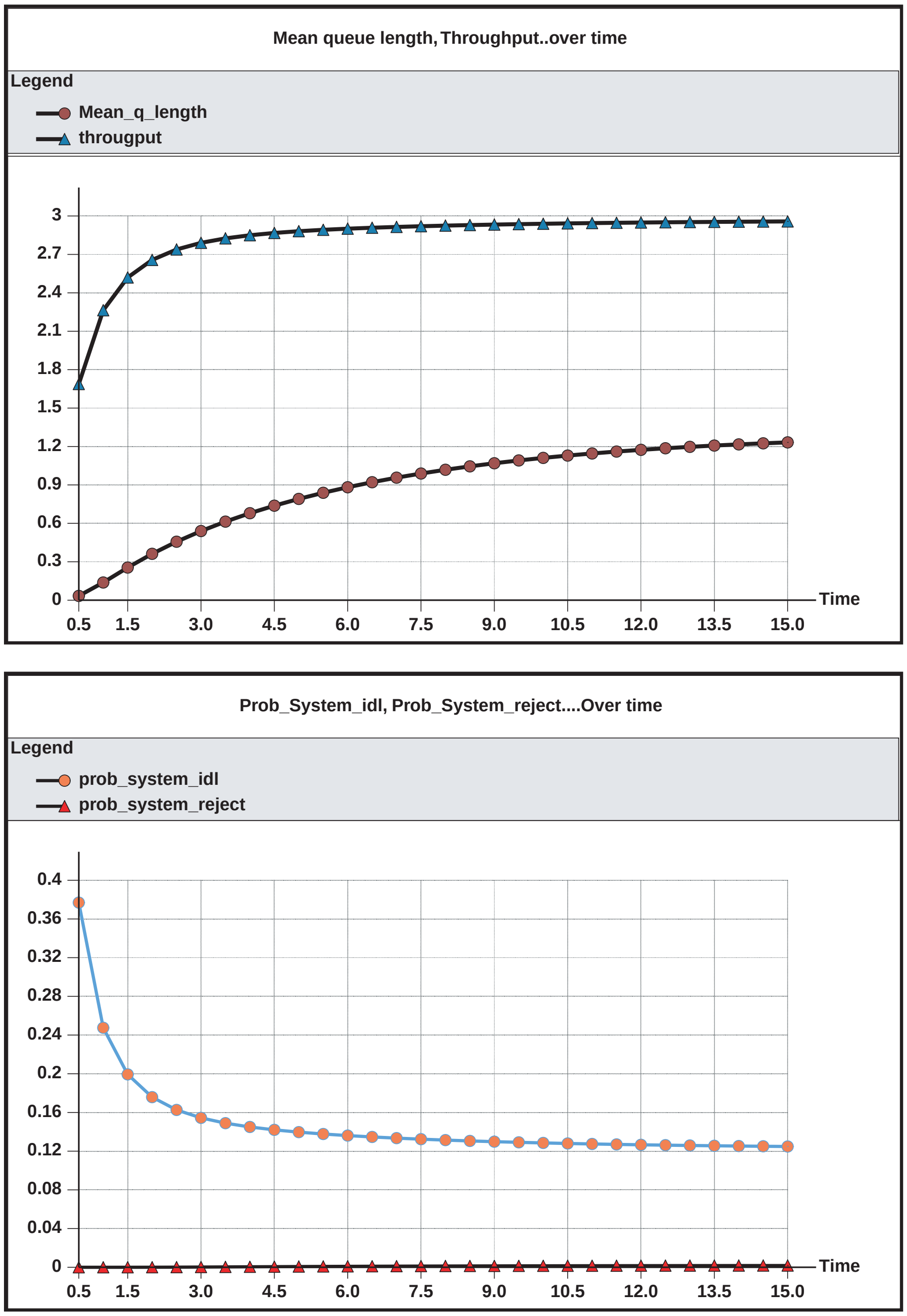


Figure 2. (IGL.) One queue, three servers (FFS-loop-transient-unreliable system, lambda = 3).

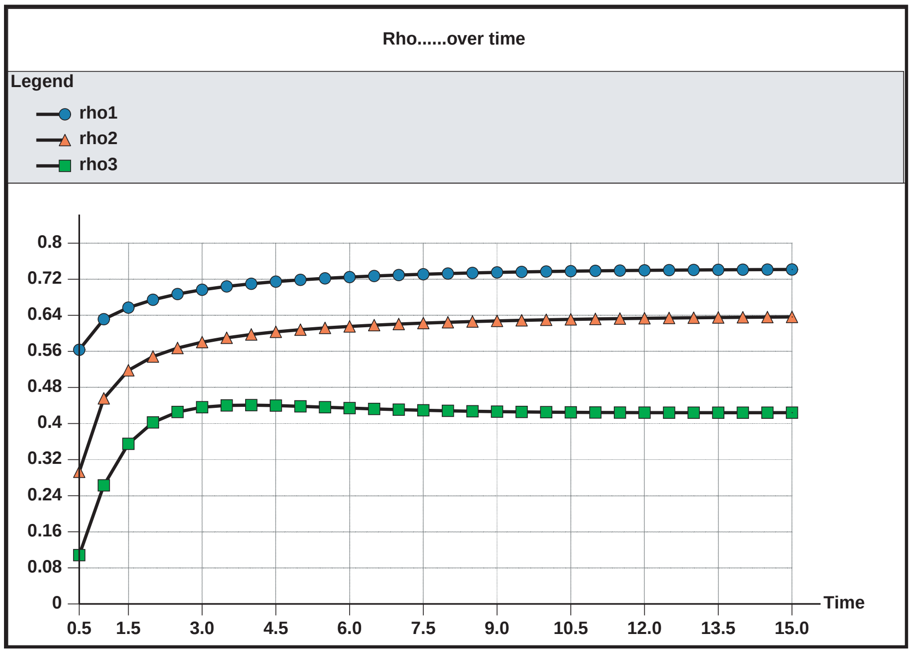


Figure 2. (cont.)

servers) will affect the results. For example, with lambda = 3, we obtained the following:

```
 mean queue length at time 1 was equal 0.13
mean queue length at time 15 was equal 1.23
also
throughput at time 1 was 2.26
throughput at time 15 was 2.95
```

and now, we increased the parameter lambda from 3 to 8 and we obtained the following results:

```
 mean queue length at time 1 equal 2.24
mean queue length at time 15 equal 8.80
also
throughput at time 1 equal 4.18
throughput at time 15 equal 4.29
```

It is very interesting also to look at the graphical representation that gives the probability that the system will reject in Figures 2 and 3; when lambda was small, the probability was small and tends to a constant value by time. After we increase lambda, the probability that the system will reject is also very small in the beginning, but it starts to increase in time. The above statement results, because in any system if the arrival rate increases, the mean queue length and throughput also increase dramatically.

You can see the new results after the above modification of lambda by looking at the graphical representation provided by the run of the same MOSEL program with the new value of lambda in Figure 3.

- The next modification of the operating parameters is just to try to increase the value of the *mtbf* and see its effect on the *utilization* of the three servers.

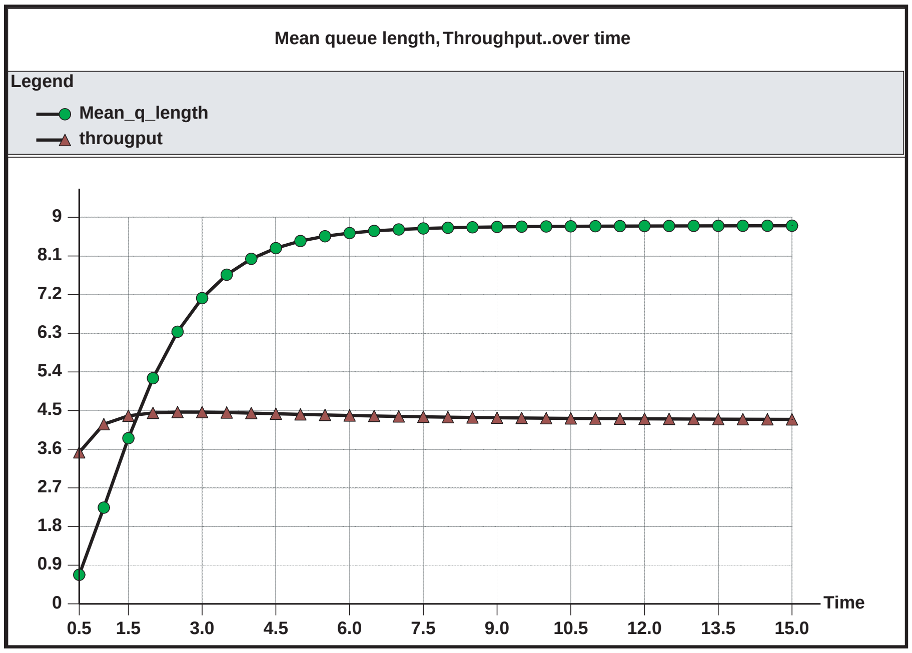
For example, in the previous program with: mtbf1 = 500, mtbf2 = 100, mtbf3 = 10, we obtained the following results for the *rho1, rho2, rho3*:

```
for example, rho1 at time 15 was equal 0.74
             rho2 at time 15 was equal 0.63
```

```
                rho3 at time 15 was equal 0.42
   the probability that the server is idle was at time 15 is 0.12
```

and now, we double the values of mtbf1, mtbf2, mtbf3 to be: mtbf1 = 1000, mtbf2 = 200, mtbf3 = 20, and we left all the other parameters without any change; then we obtained the following results for the *utilization*:

```
    for example, rho1 at time 15 was equal 0.73
                 rho2 at time 15 was equal 0.62
                 rho3 at time 15 was equal 0.47
```



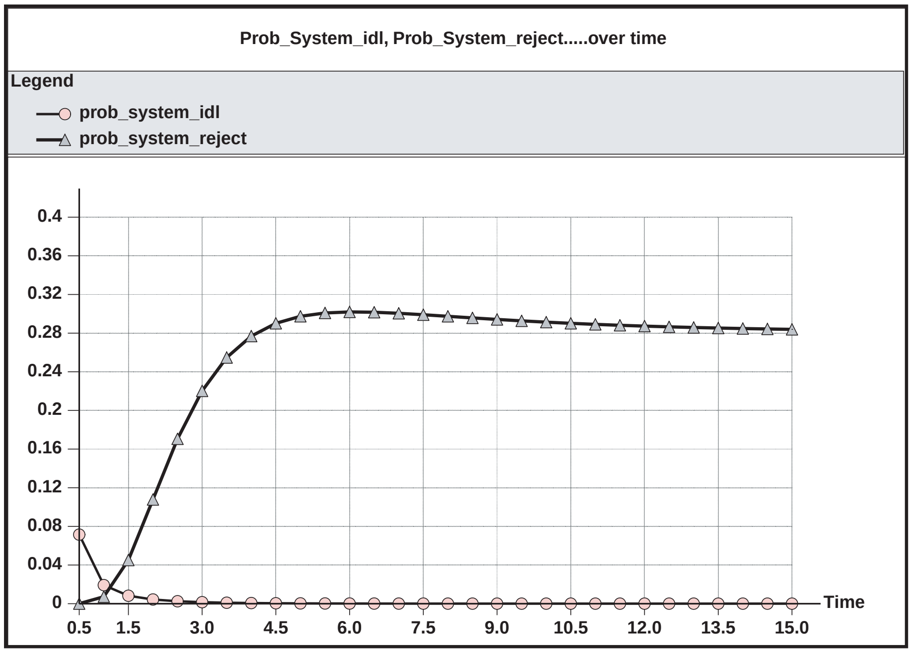


Figure 3. (IGL.) One queue, three servers (transient-FFS-loop-unreliable system, lambda = 8).

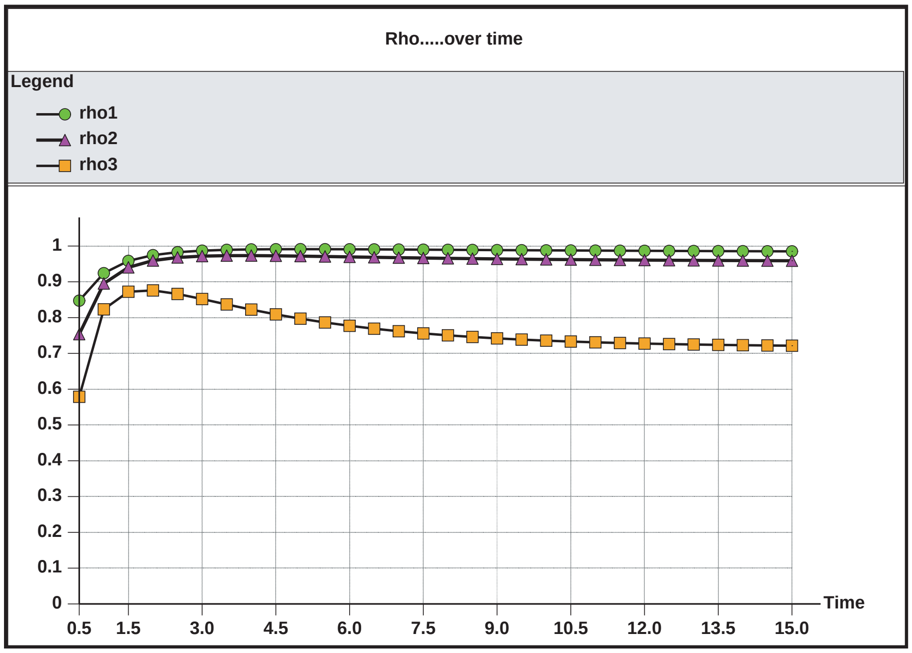


Figure 3. (cont.)

and, for example, the probability that the system is idle at time 15 is 0.09. This means that the *utilization* of the first two servers decreased and the *probability that the system is idle* increased. This is what we should expect from this type of system. We have also noticed that the *utilization* of the third server is increased. This was an interesting result because we gave priorities to the servers, and server3 had the lower priority to get a job, and now it has a higher priority to get jobs while the other servers are failed.

Please note that we tried in the above graphical representations to combine two or even more graphs in one graphical representation, which can be done easily by MOSEL, but the result of this combination made the graph unclear, misleading, and difficult to understand. So we agree that it is better to make separation between the performance measures because they have different scales.

Next, we will consider the random case. Actually, with MOSEL it is possible to use a loop for abbreviation, i.e., replace a set of statements with one statement containing a loop.

Strategy 2: Random

We usually try to show the reader the transient solution, so he or she can watch the behavior of the system over time. Please note that you can also see the steady-state solution for these problems using MOSEL. Next, we will show you the transient solution.

Transient solution

MOSEL description

This model could be described in MOSEL as it is shown in the following list (`q1s3random_loop failure.msl`).

```
 //PROGRAM: q1s3random_loopfailure.msl
//================================
//DEFINITIONS
#define mtbf1 500.0 //failure time to server1
```

```
#define mtbf2 100.0 //failure time to server2
#define mtbf3 10.0 //failure time to server3
#define mttr1 10.0 //repair time to server1
#define mttr2 5.0 //repair time to server2
#define mttr3 5.0 //repair time to server3
#define Q 10 //The number of jobs in the queue
#define n 3 //The number of servers

#define mue1 1.1
#define mue2 1.9
#define mue3 1.7
#define lambda 3

//HELP Variable
HELP int K = Q + n;

//NODES
enum down_up {down, up}; //enumerated data type
       NODE p1[Q];
<1...3> NODE state#[down_up] = up;
<1...3> NODE m#[1];
       NODE num[K];

//RULES
FROME TO p1, num W lambda;

<1...3> FROM state#[up] TO state#[down] W 1.0/mtbf#
<1...3> FROM state#[down] TO state[up] W 1.0/mttr#

//CASE1
IF (state1 == up) AND (state2 == up) AND
(state3 == up) AND (m1 == 0) AND (m2 == 0)
AND (m3 == 0)
<1...3> FROM p1 TO m# P 1.0/3.0;

//CASE2
IF (state1 == up) AND (state2 == up) AND
(state3 == down) AND (m1 == 0) AND (m2 == 0)
AND (m3 == 0)
<1...2> FROM p1 TO m# P 1.0/2.0;

//CASE3
IF (state1 == up) AND (state2 == down) AND
(state3 == up) AND (m1 == 0) AND (m2 == 1)
AND (m3 == 0)
<1,3> FROM p1 TO m# P 1.0/2.0;

//CASE4
IF (state1 == down) AND (state2 == up) AND
(state3 == up) AND (m1 == 0) AND (m2 == 0)
AND (m3 == 0)
```

```
<2,3> FROM p1 TO m# P 1.0/2.0;

FROM p1 TO m1 IF (state1 == up) AND (m2 == 1)
AND (m3 == 1);
FROM p1 TO m2 IF (state2 == up) AND (m1 == 1)
AND (m3 == 1);
FROM p1 TO m3 IF (state3 == up) AND (m1 == 1)
AND (m2 == 1);

<1...3> FROM m#, num TOE W mue#;

//RESULTS
RESULT>>IF (num==0) prob_system_idl = PROB;
RESULT>>IF (mum==K) prob_system_reject = PROB;
RESULT>>Mean_q_length = MEAN p1;
RESULT>> rho1 = UTIL m1;
RESULT>> rho2 = UTIL m2;
RESULT>> rho3 = UTIL m3;
RESULT>> throughput = rho1*mue1+rho2*mue2+rho3*mue3;

//========PICTURE=======
PICTURE "Mean Queue Length"
-TITLE "prob.system_idl, prob.sys_reject..over time"
      -FONT utopia -FONTSIZE 22
CURVE TIME prob_system_idl, prob_system_reject
      -DIFF THICK,POINT,COLOR,FILL
XSCALE LOOKOUT -GRID
YSCALE LOOKOUT -GRID
//===============================
```

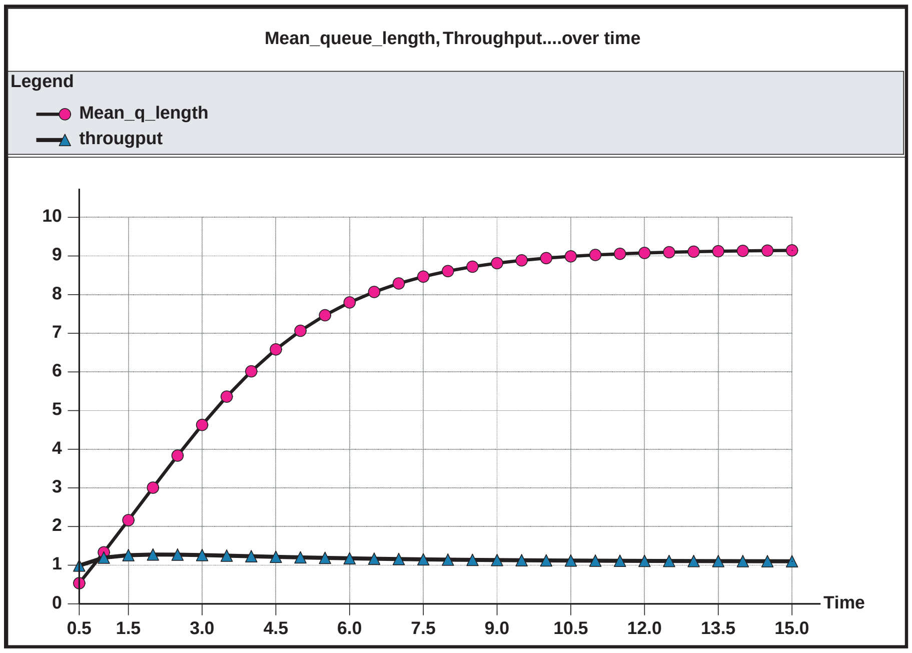
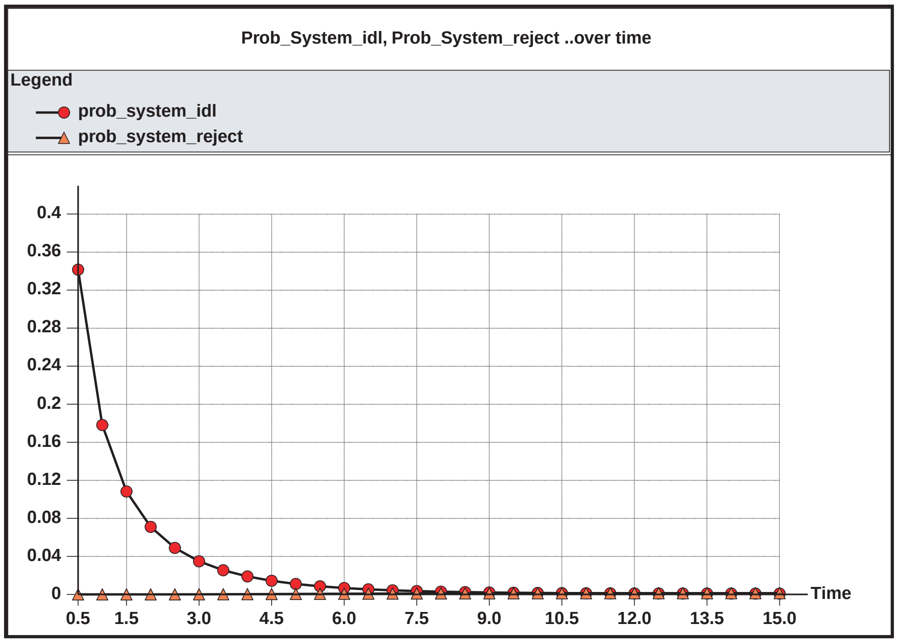


Figure 4. (IGL.) One queue, three servers (transient-random-loop-unreliable system).



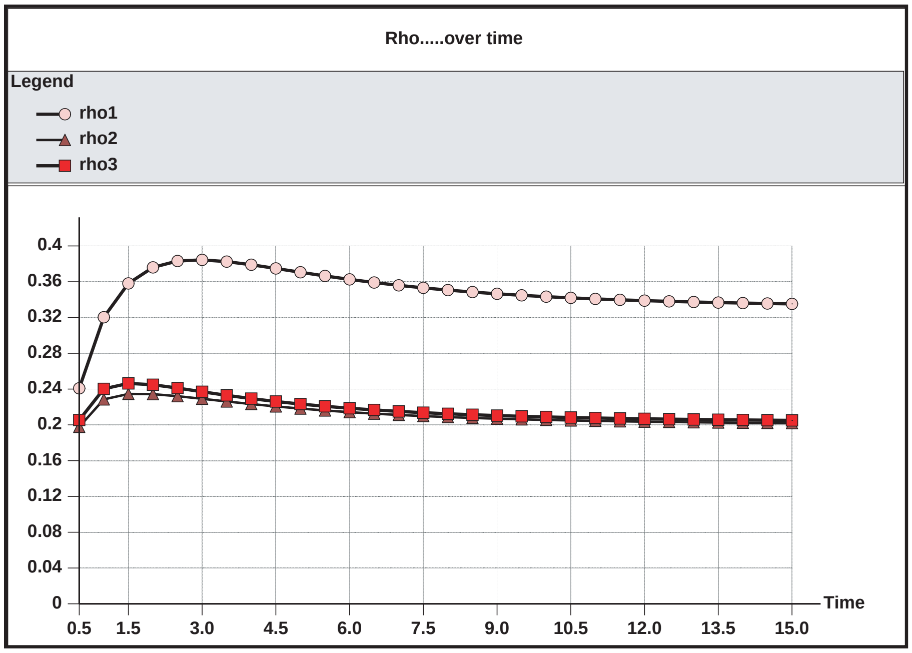


Figure 4. (cont.)

The results

The graphical representation of the file `q1s3random_loopfailure.igl` is also provided by the run of the MOSEL program and it is shown in Figure 4.

Comments

If we look at the graphical representations which represent the performance measures: mean queue length, throughput, probability that the system is idle, and probability that the system is reject in Figures 2 and 4, you will notice that the behavior is almost the same with respect to the two strategies: FFS and random. On the other hand, if you compare the graphical representations which represent the utilization of the servers: rho1, rho2, rho3, you will notice that there is a clear difference; the utilization of the servers in the FFS strategy is higher than those values in the random strategy. This is expected in this type of system.

One Queue, n Servers Problem

Now, the number of servers is not fixed which means that, in MOSEL, one can give any value to the parameter (n). So, just do not define the parameter n in your program, and then MOSEL will automatically ask you about the value of n during the execution time. So, we could construct the program 'in general' and at the same time we give the user an opportunity to define the number of servers during the execution.

In the following, we will consider problems of different type; we will have a different number of queues and a different number of servers. Please note that we assumed in our implementations that every queue has different types of jobs from other queues; this means that we may assign priorities not just to the servers, but also to the queues.

3. THREE QUEUES, TWO SERVERS PROBLEM

Please note that the priorities in the program above are given by an expression, and this expression is a function that creates a relationship between the queue priorities and the server priorities. It multiplies the priority of each queue with the priority of each server and produces the result which is the priority of the arrow going from the queue to server (see Figure 1). Actually, we can guess the behavior of this system before we look at the results. So, we expect that in this system, the mean queue length and the mean number of jobs in the whole system will increase, and we expect that the two servers will be utilized and heavily loaded all the time.

Strategy 1: FFS (Fastest Free Server)

Please note that we try to get the transient solution in order to watch the solution over time; we suppose it is very useful to say something about the system behavior.

Transient solution

We give the reader the implementation (code) using MOSEL. We can also say something about the system behavior by looking at the graphical representation (see [8] for more details).

MOSEL description

This model can be described in MOSEL as it is shown in the following (`q3s2ffsfailure.msl`).

```
 //PROGRAM q3s2ffsfailure.msl
//========================
//DEFINITIONS
#define Q 3 //number of queues
#define n 2 //number of servers
#define 1 4 //max number of jobs in every queue
#define mtbf1 500.0 //failure time to server1
#define mtbf2 100.0 //failure time to server2
#define mttr1 10.0 //repair time to server1
#define mttr2 5.0 // repair time to server2

<1...Q>#define lambda# #
<1...n>#define mue# 1.<3-#>

//Macro definition
```

```
#string rho_mue + rho#*mue#:
```

//HELP Variables

```
//HELP int K=(Q*1)+n;
<1...n> HELP int a# = (n - #+1) * (Q - #+1);
<1...m> HELP int b# = (n - #+1) * (Q - 3+1);
```

//NODES

```
enum down_up {down, up}; //enumerated data type
<1...Q> NODE p#[1];
<1...n> NODE state#[down_up]=up;
<1...n> NODE m#[1];
        NODE num[K];
```

//RULES

```
<1...Q> FROME TO p#, num W lambda#;
<1...n> FROM state#[up] TO state#[down] Q 1.0/mtbf#;
<1...n> FROM state#[down] TO state#[up] Q 1.0/mttr#;
<1...n> IF (state#==up) FROM p# TO m# PRIO a#;
<1...n> IF (state#==up) FROM p3 TO m# PRIO b#;
<1...n> FROM m#, num TOE W mue#;
```

//RESULTS

```
RESULT>>IF (num == 0) prob_system_idle = PROB;
RESULT>>IF (num == K) prob_system_reject = PROB;
RESULT>>Mean_noj = MEAN num;
<1...n>RESULT>> rho# = UTIL m#;
RESULT>> throughput = rho1*mue1$rho_mue(<2...n>); // $ is to call Macro
//========================================
```

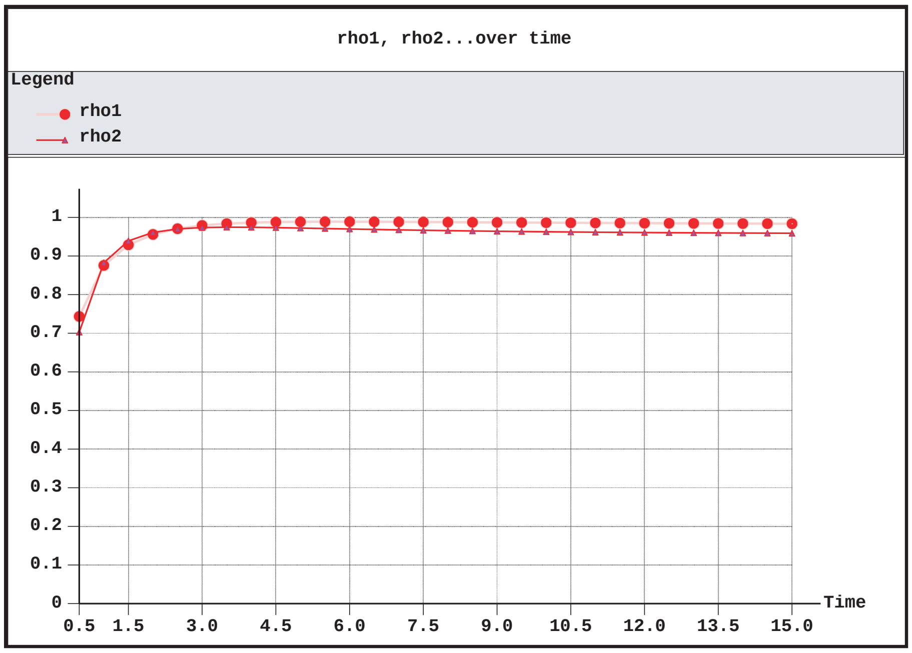
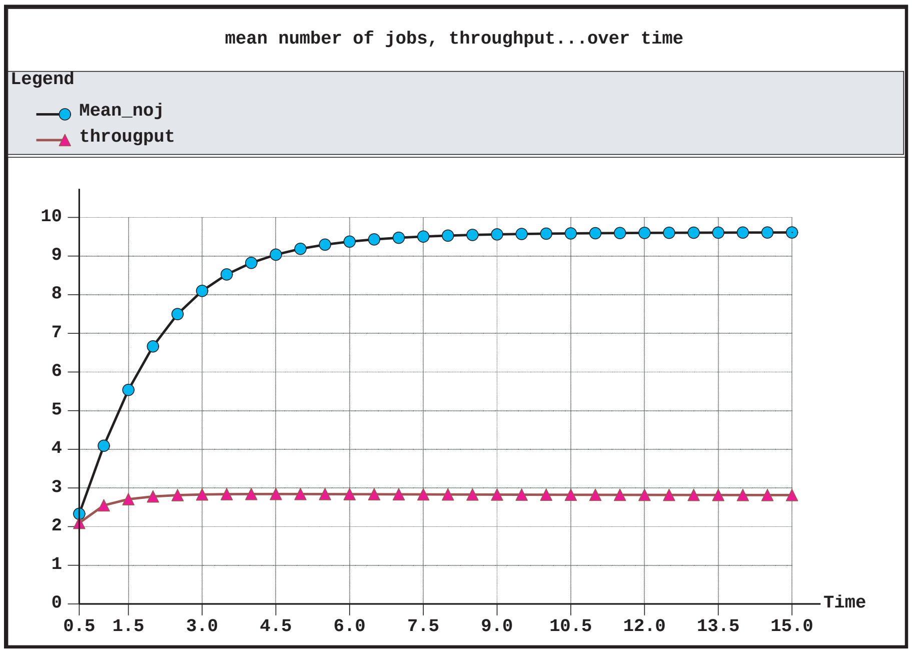


Figure 5. (IGL.) Three queues, two servers (transient-FFS-unreliable system).



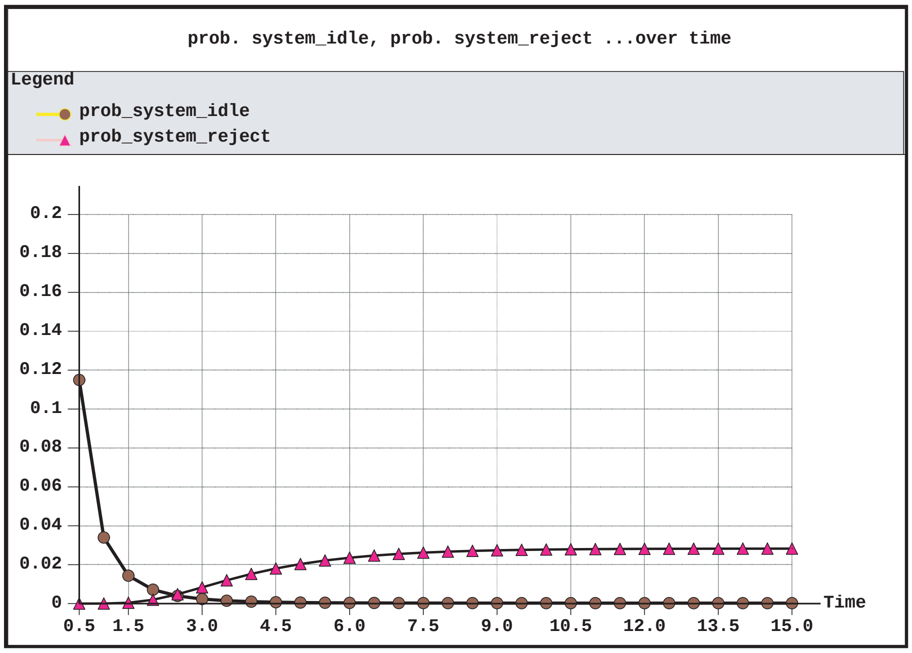


Figure 5. (cont.)

The results

The graphical representation of the file `q3s2ffsfailure.igl` is also provided by running the MOSEL program, and it is shown in Figure 5. In Figure 5, we notice that the probability of the system to be idle is high in the beginning, and it starts to decrease in time until it reaches the steady-state value. On the other hand, the probability that the system rejects a job is very low in the beginning and it starts to increase until it reaches also (steady) stable state. In the above system, we have three queues and two servers, which means that the servers have to be busy all the time.

For the same problem, we will consider next the random strategy, where the way to assign jobs to the servers is difficult.

For the random case, we also use the transient solution, which gives us three graphical representations to describe the behavior of the system (see Figure 6).

Strategy 2: Random

Following is the transient solution.

Transient solution

MOSEL description

This model could be described in MOSEL as it is shown in the following.

```
 //PROGRAM q3s2random_failurenew.msl
//============================
//DEFINITIONS
#define Q 3 //number of queue
#define n 2 //number of servers
#define 1 4 //max number of jobs in every queue
#define mtbf1 500.0 //failure time to server1
#define mtbf2 100.0 //failure time to server2
#define mttr1 10.0 //repair time to server1
#define mttr2 5.0 //repair time to server2

<1...Q>#define lambda# #
<1...n>#define mue# 1.<#+3>
#define p1 1.0
#define p2 1.0

//Macro Definition
#string rho_mue + rho#*mue#:

//HELP Variables
HELP int K=(Q*1)+n;
HELP double p31 = 0.7;
HELP double p32 = 1 - p31;
```

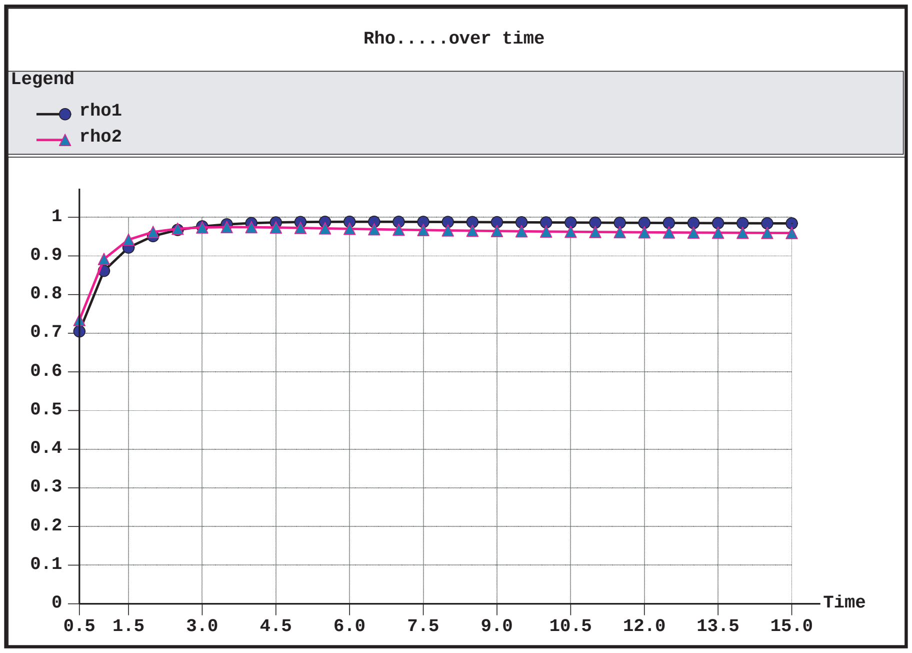
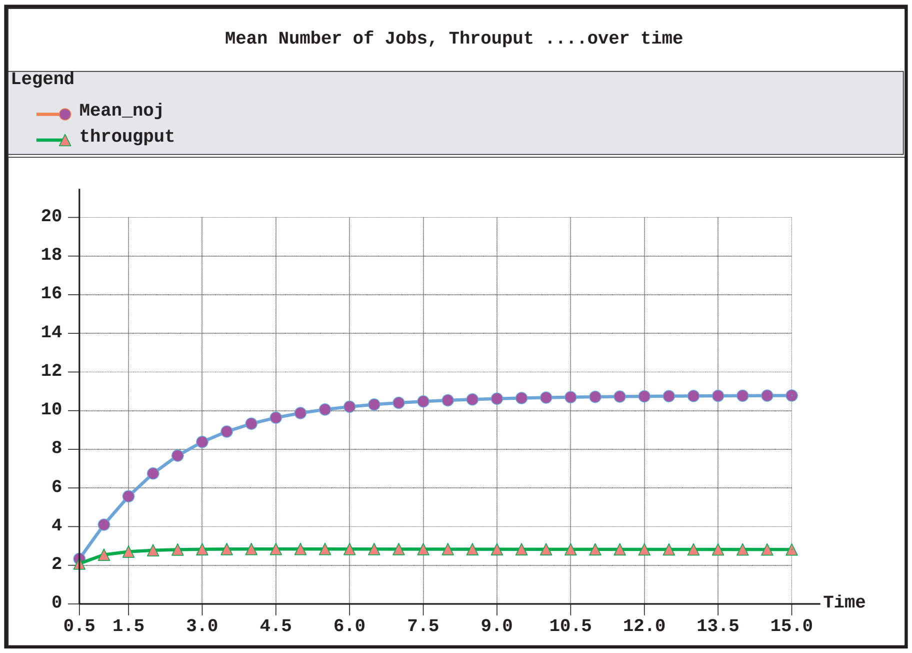


Figure 6. (IGL.) Three queues, two servers (transient-random-unreliable system).



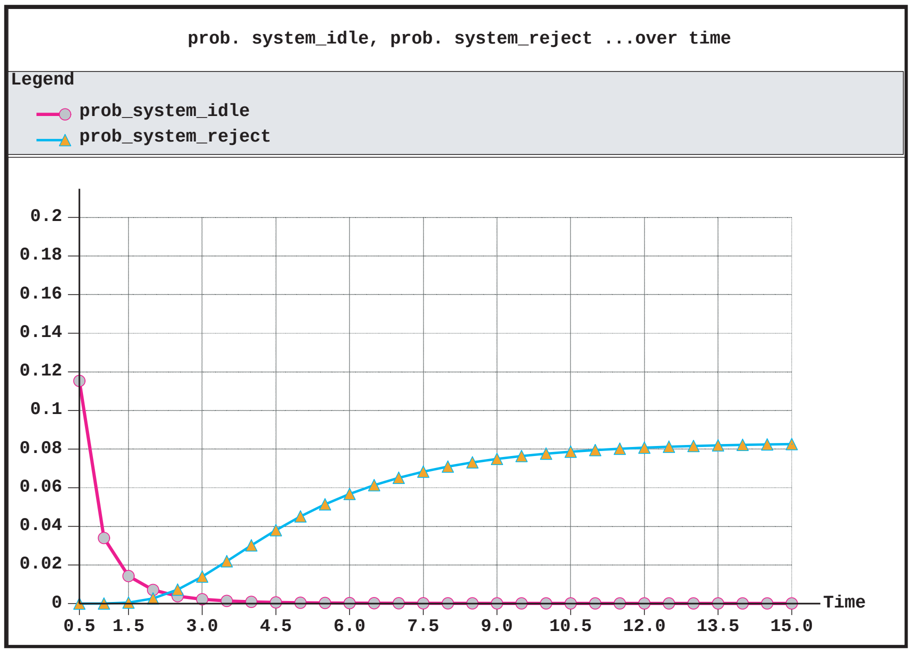


Figure 6. (cont.)

```
  //NODES
enum down_up {down, up}; //enumerated data type
<1...n> NODE state#[down_up] = up;
<1...n> NODE m#[1];
<1...Q> NODE Q#[1];
        NODE num[K];

//RULES
<1...Q> FROME TO Q#, num Q lambda#;
<1...n> FROM state#[up] TO state#[down] W 1.0/mtbf#;
```

```
<1...n> FROM state#[down] TO state#[up] W 1.0/mttr#;
<1...n> IF (state#==up) FROM Q# TO m# P p#;
<1...n> IF (state#==up) FROM Q3 To m# P p3#;
<1...n> FROM m#, num TOE W mue#;

//RESULTS
RESULT>>IF (num== 0) prob_system_idle = PROB;
RESULT>>IF (num== K) prob_system_reject = PROB;
RESULT>>Mean_noj = MEAN num;
<1...n>RESULT>> rho# = UTIL m#;
RESULT>> throughput = rho1*mue$rho_mue(<2...n>); //$ is to call Macro
=========================
```

The results

The three graphical representations for the IGL files are also provided by running the MOSEL program, and they are shown in Figure 6.

We notice that the *probability of the server idle* and *probability that server to be rejected* are very small, because we have here three queues and only two servers, and that keeps the servers busy all the time. One also follows the behavior of the other performance measures: rho1, rho2, mean number of jobs, and throughput.

Comments

We would like to make a comparison between the two strategies, FFS and random, for this type of system (three queues, two servers) with respect to the performance measures. Figures 5 and 6 show that, with respect to the performance measures rho1, rho2, they are almost the same. On the other hand, the mean number of jobs in the whole system is higher in the random case than in the FFS case, but the throughput is the same. This is clear, because in the random case, we choose the available server randomly, but in the FFS, we choose just the fastest server, which means that we expect that the mean number of jobs in any system that uses the random strategy would be higher than the one that uses FFS strategy.

With respect to the third graphical representation which represents the performance measures: the probability that the system is idle and the probability that the system is reject, the probability that the system is idle is higher in the random strategy than that in the FFS strategy while the probability that the system is reject is almost the same. In the case of the system with one queue and three servers in Figure 2 above, you will notice that the probability that the system rejects the jobs is almost zero; on the other hand, in our case here (three queues and two servers), the probability that the system rejects the job was zero at the beginning and it starts to increase until it reaches a stable (steady) state. On the other hand, in the case of the systems which have three queues and two servers, the possibility that the system will be full is very high and we are not surprised that the probability that the system is full and rejects the job is increasing over time.

4. APPLICATIONS

In the case of one queue and n servers, the jobs come and are served by different types of servers. One example is a computer network system in which we have many jobs in one queue (FCFS), and these jobs come to different types of printers (servers). Another example for application is the following situation: some jobs are specialized in just one type of machine (for example, LINUX). These jobs are served only by that machine, while the other class of jobs are served by other machines like UNIX. Some other (third type) jobs could be served either by a machine of type one (LINUX) or a machine of type two (UNIX).

5. CONCLUSION

In this paper, it is shown how the MOSEL language can be used to implement nonhomogeneous unreliable systems very smoothly to solve different problems. We made slight changes on the operating parameters and we have seen how the performance measures changed. The whole time we used the transient solution with useful graphical representations to give the user an opportunity to watch the system behavior over time. Reliable systems can be solved using MOSEL as well; see [5] for more details.

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