Multiserver Retrial Queues with Finite Number of Heterogeneous Sources

J. Roszik, B. Almási, J. Sztrik
Institute of Informatics, University of Debrecen
Debrecen, Hungary

e-mail: jroszik@inf.unideb.hu
www: http://irh.inf.unideb.hu/user/jroszik/
OUTLOOK

• The queueing model
• Applications
• Mathematical model
• The MOSEL tool
• Case studies
• References
The queueing model
Applications

- magnetic disk memory systems
- local area networks with CSMA/CD protocols
- cellular mobile networks
Mathematical model

- The system state at time $t$ can be described with the process

$$X(t) = (\alpha_1, \ldots, \alpha_c, \beta_1, \ldots, \beta_{N(t)})$$

- where
  - $c$ is the number of servers
  - $N(t)$ is the number of sources of repeated calls at time $t$
  - the indices of the sources at the servers are denoted by $\alpha_i$, $i=1, \ldots, c$
  - the indices of the sources in the orbit are denoted by $\beta_j$, $j=1, \ldots, N(t)$
We define the stationary probabilities:

\[
P(i_1, \ldots, i_c, 0) = \lim_{t \to \infty} P\{\alpha_1 = i_1, \ldots, \alpha_c = i_c, N(t) = 0\}
\]

\[
P(i_1, \ldots, i_c, j_1, \ldots, j_k) = \lim_{t \to \infty} P\{\alpha_1 = i_1, \ldots, \alpha_c = i_c, \beta_1 = j_1, \ldots, \beta_k = j_k\},
\]

\[k = 1, \ldots, K - c\]

Once we have obtained these limiting probabilities the system performance measures can be derived in the following way.
1. The probability of the $i$-th source is sending repeated calls

$$N_i = \sum_{i_1, \ldots, i_c} \sum_{k=1}^{K-c} \sum_{\substack{j_1, \ldots, j_k \notin \{i_1, \ldots, i_c\} \atop i \in \{j_1, \ldots, j_k\}}} P(i_1, \ldots, i_c, j_1, \ldots, j_k), \quad i = 1, \ldots, K.$$ 

2. The probability of the $i$-th source is under service

$$Y_i = \sum_{i_1, \ldots, i_c} P(i_1, \ldots, i_c, 0) + \sum_{i_1, \ldots, i_c} \sum_{i \in \{i_1, \ldots, i_c\}} \sum_{k=1}^{K-c} \sum_{\substack{j_1, \ldots, j_k \notin \{i_1, \ldots, i_c\} \atop i \in \{i_1, \ldots, i_c\}}} P(i_1, \ldots, i_c, j_1, \ldots, j_k), \quad i = 1, \ldots, K.$$
3. The mean rate of generation of primary calls of the $i$-th source

$$\overline{\lambda_i} = \lambda_i (1 - Y_i - N_i), \quad i = 1, \ldots, K.$$ 

4. The mean waiting time of the $i$-th source

$$\overline{W_i} = \frac{N_i}{\overline{\lambda_i}}, \quad i = 1, \ldots, K.$$ 

5. The mean response time of the $i$-th source

$$\overline{T_i} = \frac{N_i}{\overline{\lambda_i}} + \frac{1}{\mu_i}, \quad i = 1, \ldots, K.$$ 

6. The utilization of the $i$-th source

$$U_i = 1 - N_i - Y_i, \quad i = 1, \ldots, K.$$
The MOSEL (Modeling, Specification and Evaluation Language) tool

1. The system to be modelled
2. Description in MOSEL
3. Translation

- MOSES
- SPNP
- TimeNET
- PEPSY
- ...

The MOSEL tool generates result and IGL files
Validations in the homogeneous multiserver case

<table>
<thead>
<tr>
<th></th>
<th>MOSEL program</th>
<th>Pascal program</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean no. of busy servers:</td>
<td>1.2263326334</td>
<td>1.2263325906</td>
</tr>
<tr>
<td>Mean no. of retrying sources:</td>
<td>0.7078359870</td>
<td>0.7078359330</td>
</tr>
<tr>
<td>Mean response time:</td>
<td>0.5771974881</td>
<td>0.5771973594</td>
</tr>
</tbody>
</table>

The results are the same at least up to the 6th decimal digit.
Validations in the heterogeneous case

<table>
<thead>
<tr>
<th></th>
<th>repeated attempts</th>
<th>FIFO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utilization of the server:</td>
<td>0.5454545965</td>
<td>0.5454545840</td>
</tr>
<tr>
<td>Utilizations of the sources:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. source:</td>
<td>0.8636363668</td>
<td>0.8636362907</td>
</tr>
<tr>
<td>2. source:</td>
<td>0.7727272562</td>
<td>0.7727274804</td>
</tr>
<tr>
<td>3. source:</td>
<td>0.6090908485</td>
<td>0.6090907576</td>
</tr>
<tr>
<td>Mean response times:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. source:</td>
<td>1.5789473265</td>
<td>1.5789483457</td>
</tr>
<tr>
<td>2. source:</td>
<td>1.4705883740</td>
<td>1.4705864961</td>
</tr>
<tr>
<td>3. source:</td>
<td>1.2835824163</td>
<td>1.2835829064</td>
</tr>
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</table>

The results are the same at least up to the 5th decimal digit.
Utilizations versus primary request generation rate

Legend

1. source (retrial rate = 0.5)
2. source (retrial rate = 1)
3. source (retrial rate = 2)
4. source (retrial rate = 3)
5. source (retrial rate = 5)
Mean response times versus primary request generation rate

Legend
- 1. source (retrial rate = 0.5)
- 2. source (retrial rate = 1)
- 3. source (retrial rate = 2)
- 4. source (retrial rate = 3)
- 5. source (retrial rate = 5)

Mean response time versus primary request generation rate.
Mean response times versus retrial rate

Legend

- 1. source (primary request generation rate = 0.1)
- 2. source (primary request generation rate = 0.2)
- 3. source (primary request generation rate = 0.5)
- 4. source (primary request generation rate = 0.8)
- 5. source (primary request generation rate = 1)

Mean response time

Retrial rate

Mean response times versus retrial rate
Mean number of retrying sources versus retrial rate

Mean number of retrying sources

Retrial rate
References


