# Reliability Analysis of Cognitive Radio Networks

Mohamed Hedi Zaghouani
Department of IT Systems and Networks
University of Debrecen
Debrecen, Hungary
zaghouani.hedi@inf.unideb.hu

Janos Sztrik
Department of IT Systems and Networks
University of Debrecen
Debrecen, Hungary
sztrik.janos@inf.unideb.hu

Agassi. Z. Melikov
Institute of Control Systems
ANAS
Baku, Azerbaijan
Agassi.melikov@gmail.com

***Abstract*—The current paper deals with the simulation of a queuing model developed to evaluate the performance of a cognitive radio network and its reliability.**

**We take into consideration two interconnected subsystems, the first one is dedicated to primary users (PU). The number of sources is finite, and each source generates a primary request after an exponentially distributed interval of time, these requests then are then sent to a single server called Primary Channel Service (PCS), under the assumption that the service times are distributed exponentially as well. The service is carried out in the order of the arrivals of primary calls. The second part of the model is associated to secondary users (SU), which has also a finite source with exponentially distributed request generation time and it is assumed that the service time at the Secondary Channel Service (SCS) with a single server is exponentially distributed. However, the two subsystems operate in the following way. Each generated primary request is directed to the primary server in order to check its accessibility, in case that the service unit is free, the service starts instantly. If the primary unit is already busy with another primary request, the call joins a FIFO queue. However, if the primary unit is busy by processing a service for a secondary user, this latter service stops immediately and should be sent back to the (SCS), based on the availability of the secondary server this postponed task either starts the service again or joins the orbit.**

**In the other hand, the secondary requests are directed to the secondary server to verify its availability, if the aimed server is accessible, the service starts immediately, otherwise these secondary requests try to join the (PSU) and if it is idle the service starts. If not, they join to the orbit at SCS. Postponed requests in the orbit retry to be served after an exponentially distributed interval of time.**

**In the current work both service units are subject to some random breakdowns, in such case the interrupted requests are sent either to the queue or to the orbit, respectively. It is assumed that the operation and repair times of the given server are generally distributed. We use Hypo-Exponential, Hyper-Exponential and Gamma distributed times because by assuming the same means and variances the effect of the distribution could be analyzed. Due to the page limitations we deal with the effect of the distribution of operation and service time distributions, only on the mean response time of the secondary server.**

**Assuming that all the random times concerned in this model are independent of each other the primary aim of the present paper is to show the impact of the distribution of the operation and repair time on the mean response time of the secondary users as a function of the failure and repair intensity, respectively Several Figures are generated to visualize the difference due to the distributions.**

## I. Introduction

Recent years have seen a significant increase in the demand for radio spectrum. As this kind of networks provide a effective usage for customers, by allowing unlicensed (Secondary) users to process their services, while there is no licensed (Primary) user in the spectrum.

Cognitive Radio (CR) is an intelligent technology that can sense automatically the available channels in a wireless spectrum and modify transmission parameters allowing more communications to establish and also improve the network's behavior. Their major feature is to exploit the free sections of the primary frequency bands for the benefit of unlicensed customers, without any disadvantage for the licensed users. For a more detailed discussion and a survey, see for example [1].

As the idea of the cognitive radio was introduced to the research community of wireless communications, the investigation of Cognitive Radio Networks (CRNs) became more and more popular, see for example [2],[3],[4],[5],[6].

There are two types of CRN, the first is known as (underlay network) in which unlicensed users are entitled at the same time to use the primary channels with the PUs, depending on some predefined conditions. The second type is called (overlay networks), where the secondary customers are allowed to use the Primary Service Unit only if this unit is free and there are no licensed customers.

This paper is going to deal with the second mentioned type of CRN (overlay), by modeling a CRN that uses a retrial finite source queuing system with two non-reliable servers (Primary and Secondary) exposed to breakdowns and repairs.

In an earlier work [7] the authors investigated a similar system when all random times were assumed to be exponentially distributed and the service was not reliable. By introducing a multidimensional Markov chain several figures were presented to show the impact of different rates on the main performance measures of the system.

In [8] an investigation of a CRN that uses two finite sources (Primary and Secondary) and a retrial queueing system at the secondary service unit was carried out by the help of simulation, in order to show the effect of the distribution on the main characteristic of the system. The two servers were reliable, however the transmissions into the frequency channel were unreliable.

978-1-7281-1401-9/19/$31.00 ©2019 IEEE

The present paper is the continuation of [9] where the servers were subject to random breakdowns and repairs. Under the assumption that the operation and repair times of the server units were Hypo- and Hyper-Exponentially distributed and the all the other the events (services, request generations and retrials) were exponentially distributed, several Figures showed the effect of different input parameters on the main performance measures.

The main aim of this paper is to investigate and to show the impact of the operation and repair time distributions on some performance measures of the system, by allowing Gamma-distributed times with the same mean and variance as the Hypoand Hyper-Exponential distributions, respectively. This enables us to see the effect of the distributions and not only the first two moments.

As it's difficult to get these results analytically, we have created a simulation program written in C language for the investigations. Our experience is that many times it is more effective to write our own program than to use general purpose simulation packages. The simulation was validated by several test results obtained in several special cases. The examples are just illustrations to show the functionality of the simulation program. It has not been applied in any real situation. It is well-known that it is almost impossible to get real data for modeling since providers hide their methods and approaches. With this page limitation there is no room for a case study section, even we have no place to show the effect of distributions on other performance measures, like utilization of the servers, mean number customers in the systems, etc.

#### *1) Abbreviations and acronyms:*

CR: Cognitive Radio
CRN: Cognitive Radio Network
PU: Primary User
SU: Secondary User
PCS: Primary Channel Service
SCS: Secondary Channel Service
PSU: Primary Service Unit
SSU: Secondary Service Unit
FIFO: First In First Out

## II. Applied distributions

This section of the paper deals with the most commonly used distributions in simulations. Gamma, Hyper-exponential, Hypo-exponential, lognormal and Pareto distributions are introduced with their parameters. Procedures for moments fitting are treated as well.

A random variable $X$is Gamma-distributed if its density function is:

$$f(x)=\begin{cases}0 & \text{if } x<0\\ \frac{\beta(\beta x)^{\alpha-1}e^{-\beta x}}{\Gamma(\alpha)} & \text{if } x\geq 0\end{cases}$$

Where $\beta>0$ and $\alpha>0$.

We can calculate the Mean, Variance and the squared coefficient of variation as follows:

$$\overline{X}=\frac{\alpha}{\beta},\qquad Var(X)=\frac{\alpha}{\beta^2},\qquad C_X^2=\frac{1}{\alpha}$$

If the mean and variance are already known we can obtain the parameters using the following formulas:

$$\alpha=\frac{1}{C_X^2},\qquad \beta=\frac{\alpha}{\overline{X}}$$

### *A. Lognormal distribution*

Assuming that, $Y\in N(m,\sigma)$ is a normally distributed random variable, lognormal is a continuous distribution in which $X=e^Y$ is the logarithm of a variable having a normal distribution with two parameters $(m,\sigma)$. Its density function is the following:

$$f_x(x)=\frac{1}{\sigma x}\varphi\Big(\frac{\ln(x)-m}{\sigma}\Big),\quad x>0.$$

Mean, Variance and Squared coefficient of variation can be obtained as:

$$\overline{X}=e^{m+\frac{\sigma^2}{2}},\quad Var(X)=e^{2m+\sigma^2}(e^{\sigma^2}-1),\quad C_X^2=e^{\sigma^2}-1.$$

The below interrelation helps us to calculate the two parameters of a lognormal distribution:

$$\sigma=\sqrt{\ln(1+C_X^2)},\quad m=\ln(\overline{X})-\frac{\sigma^2}{2}$$

### *B. Pareto distribution*

Supposing a random variable $X$ has a Pareto distribution if its density function is:

$$f(x)=\begin{cases}0 & \text{if } x<k\\ \alpha k^{\alpha}x^{-\alpha-1} & \text{if } x\geq k\end{cases}$$

This kind of distribution has two parameters: $\alpha$ which is the shape parameter and $k$, the location parameter. Both $\alpha$ and $k$ are input parameters for the random number generator. We can obtain the Mean, Variance and squared coefficient of variation using the following formulas:

$$\overline{X}=\begin{cases}\frac{k\alpha}{\alpha-1} & \text{if } \alpha>1\\ \infty & \text{if } \alpha\leq 1\end{cases}$$

$$Var(X)=\frac{k^2\alpha}{\alpha-2}-\Big(\frac{k\alpha}{\alpha-1}\Big)^2,\quad C_X^2=\frac{(\alpha-1)^2}{\alpha(\alpha-2)}-1,\quad \alpha>2.$$

The below interrelation can be used to get $\alpha$ and k, supposing that the mean and the variance are known:

$$\alpha=1+\frac{\sqrt{1+C_X^2}}{\sqrt{C_X^2}},\qquad k=\frac{\alpha-1}{\alpha}\times\overline{X}$$

### C. Hypo-exponential distribution

Supposing, $X_i \in Exp(\mu_i)(i = 1, ..., n)$ are exponentially distributed random variables. If all the parameters are different the the density function of a Hypo-exponential distribution is:

$$f_{Y_n}(x) = \begin{cases} 0 & \text{if } x < 0 \\ (-1)^{n-1}\left[\prod_{i=1}^{n} \mu_i\right] \sum_{j=1}^{n} \frac{e^{-\mu_j x}}{\prod_{k=1,k\neq j}^{n}(\mu_j - \mu_k)} & \text{if } x \geq 0. \end{cases}$$

The Mean value, Variation and the squared coefficient of variation are:

$$\overline{Y_n} = \sum_{i=1}^{n} \frac{1}{\mu_i}, \qquad Var(Y_n) = \sum_{i=1}^{n} \frac{1}{\mu_i^2}$$

$$C_{Y_n}^2 = \frac{\sum_{i=1}^{n}\left(\frac{1}{\mu_i}\right)^2}{\left(\sum_{i=1}^{n}\frac{1}{\mu_i}\right)^2}.$$

The below equations allow us to get the parameters of a 2-phase distrbution if the mean and the variance are predefined:

$$\overline{X} = \frac{1}{\mu_1} + \frac{1}{\mu_2}, \qquad Var(X) = \frac{1}{\mu_1^2} + \frac{1}{\mu_2^2}$$

### D. Hyper-exponential distribution

Let, $X_1, X_2, \cdots, X_n$ are independent and exponentially distributed random variables, where the rate parameter of $X_i$ is $\lambda_i$. Every random variable $X_i$ has the probability $p_i$ where $p_1 + \cdots + p_n = 1$. A random variable $X$ is said to follow a Hyper-exponential distribution if its density function is the following:

$$f_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ \sum_{i=1}^{n} p_i \lambda_i e^{-\lambda_i x} & \text{if } x \geq 0. \end{cases}$$

If $X$ is a random variable, in which $C_X^2 > 1$, the next fit which is called two-moment, can be used:

$$f_Y(t) = p\lambda_1 e^{-\lambda_1 t} + (1-p)\lambda_2 e^{-\lambda_2 t}.$$

$Y$ is a 2-phase Hyper-exponentially distributed random variable. The most commonly used procedure is the balanced mean method, that is

$$\frac{p}{\lambda_1} = \frac{1-p}{\lambda_2}.$$

The below formulas allow us to get the three parameters of the Hyper-exponential distribution :

$$p = \frac{1}{2}\left(\sqrt{\frac{C_X^2 - 1}{C_X^2 + 1}}\right), \qquad \lambda_1 = \frac{2p}{\overline{X}}, \qquad \lambda_2 = \frac{2(1-p)}{\overline{X}}.$$

## III. System Model

As shown in Figure 1 our system model is a finite source queuing system with retrials which contains two sub-systems for the PUs and SUs, knowing that, these two subsystems are connected to each other. The model's first subsystem is dedicated for PU requests, with a finite number of $N1$ sources. Each inter-arrival time is exponentially distributed with parameter $\lambda_1$ for a primary request which is sent to a preemptive priority queue (FIFO). If the queue is empty, the service starts instantly and its duration is exponentially distributed with parameter $\mu_1$. Otherwise, the newly generated request will have to wait in the queue. The requests of the SUs are generated in an exponentially distributed inter-request time with parameter $\lambda_2$ and is served during an exponentially distributed time with parameter $\mu_2$, supposing that this second subsystem has a finite number of sources $N2$.

It should be noted that, if a high priority requester joins the primary server and finds it busy with an unlicensed (secondary) request, the latter request is interrupted and sent back either to the SSU (Secondary Service Unit) or to the orbit depending on the accessibility of the secondary channel. However, if the primary server is processing a licensed request, the new customer will have to wait in the queue.

When secondary users arrive, they can process their services immediately if the dedicated channel is free, if not, they will check the availability of the primary channel hoping to start the service. Furthermore, if the primary channel is busy too this request will be forwarded to the orbit. These postponed tasks try to get served after an interval of time which is exponentially distributed with parameter $\nu$.

As mentioned before, the server at both subsystems are subject to breakdowns and repairs, the failures of the service units can occur both in busy and idle status.

Failures appear randomly for primary and secondary servers according to Hyper-Exponentially, Hypo-Exponentially and Gamma-distributed time with intensities $\gamma_1$ and $\gamma_2$, respectively. The repair times are distributed according to Hyper-Exponentially, Hypo-Exponentially and Gamma-distributed as well, with intensities $\sigma_1$ and $\sigma_2$, respectively.

Using the stochastic model, we can clarify our system through the below formulas:

- $k1(t)$: represents the number of licensed sources at given time $t$;
- $k2(t)$: refers to the number of unlicensed at time given $t$;
- $q(t)$: is the number of primary requests in the queue at time $t$;
- $o(t)$: denotes the number of tasks in the orbit at time $t$;
- $y(t) = 0$, if the primary channel is free, $Y(t) = 1$, if the primary channel is busy a high-priority request and $Y(t) = 2$, if the primary service unit is busy with a low-priority request at time $t$;
- $c(t) = 0$, if the secondary service unit is idle(free) and

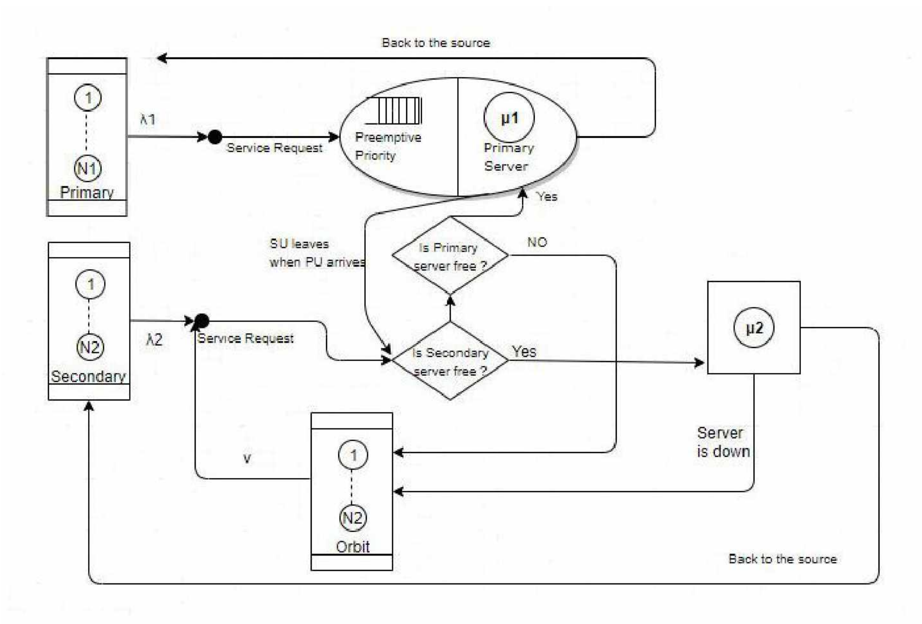


Fig. 1: Cognitive radio network with finite source retrial queueing systems

TABLE I: Parameters of the simulation

| Parameters | Value at moment $t$ | Maximum Value |
|---|---|---|
| Primary sources | $k1(t)$ | N1 |
| Secondary Sources | $k2(t)$ | N2 |
| Primary arrival rate | $\lambda_1$ | |
| Secondary arrival rate | $\lambda_2$ | |
| Number of requests at the queue (FIFO) | $q(t)$ | N1-1 |
| Number of requests at the orbit | $o(t)$ | N2-1 |
| Primary service rate | $\mu_1$ | |
| Secondary service rate | $\mu_2$ | |
| Failure rate of the primary server | $\gamma_1$ | |
| Failure rate of the secondary server | $\gamma_2$ | |
| Repair rate of the primary server | $\sigma_1$ | |
| Repair rate of the secondary server | $\sigma_2$ | |

$c(t) = 1$, if the secondary service unit is busy at time $t$.

In the present work we add Gamma-distribution to the Hypo-Exponential and Hyper-Exponential for the operation and repair times in order to investigate and display the impact of the distributions and their parameters on the behavior of the system.

The set of parameters used in the simulation are shown in Table I .

Table I presents all the values needed for the simulation we can see that the primary number of sources is $k1$ at moment t, however the Maximum number for this values is $N1$, similarly for the second server has $k2$ a number of sources and $N2$ is the max number of secondary sources .

As the Maximum number of primary sources in the system is $N1$, the Maximum of the requests in the queue will be $N1 - 1$ since the server deals with one user in the same time, similarly for the orbit the Maximum number of requests at the orbit will be $N2 - 1$.

TABLE II: Numerical values of model parameters

| Figure No. | $N1$ | $N2$ | $\lambda_1$ | $\lambda_2$ | $\mu_1$ | $\mu_2$ | $\sigma_1$ | $\sigma_2$ | $\gamma_1$ | $\gamma_2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Figure 2,3 | 6 | 10 | 0.7 | 0.2 | 2 | 1.5 | x-axis | 0.5 | 6 | 5 |
| Figure 4,5 | 6 | 10 | 0.7 | 0.2 | 2 | 1.5 | 0.5 | 0.5 | 6 | x-axis |

TABLE III: Values of the distribution parameters

| Distribution | | Hyper | Hypo | Gamma |
|---|---|---|---|---|
| Figure 2,4 | Mean | N/A | 0.2 | 0.2 |
| | Variance | N/A | 0.03 | 0.03 |
| | Parameters | N/A | $\lambda_1 = 0.0292$<br>$\lambda_2 = 0.1707$ | $\alpha = 1.333$<br>$\beta = 6.667$ |
| Figure 3,5 | Mean | 0.2 | N/A | 0.2 |
| | Variance | 0.4 | N/A | 0.4 |
| | Parameters | $\lambda_1 = 0.2$<br>$\lambda_2 = 0.632$ | N/A | $\alpha = 0.1$<br>$\beta = 0.5$ |

## IV. Results of the Simulation

Using our simulation program we could display many figures for several cases combinations, whilst in this paper we focus on the effect of the failure and repair times on the mean response time of secondary users using different distributions.

It should be noted that we generated all the figures under the assumption that both servers are non-reliable.

Both Figure 2 and 3 show the mean response time of secondary users in function of the primary repair intensity using different distributions (Hypo, Hyper and Gamma) for the primary operating time, supposing that the Exponential distribution was used for the rest of the inter-event times (arrival, service, retrial, secondary operation). As expected, the mean response time of the secondary users decreases with the increment of the repair intensity as shown in Figures 2 and 3.

If we check closely the Figure 2, where the the squared


978-1-7281-1401-9/19/$31.00 ©2019 IEEE

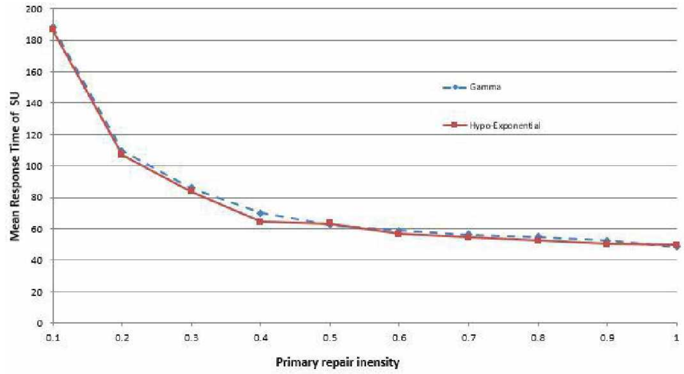


Fig. 2: The effect of the primary repair intensity on the mean response time of the Secondary Users.

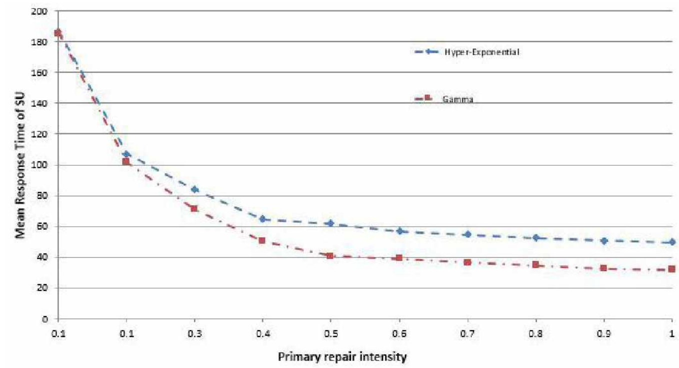


Fig. 3: The effect of the primary repair intensity on the mean response time of the Secondary Users.

coefficient of variation was less than one we can't see any big difference between the distributions except the fourth value of the primary repair intensity, in which Gamma was a bit higher then Hypo. However, the difference between the distributions was significant in Figure 3, where the squared coefficient of variation is greater than one, more differences between the distributions can be observed.

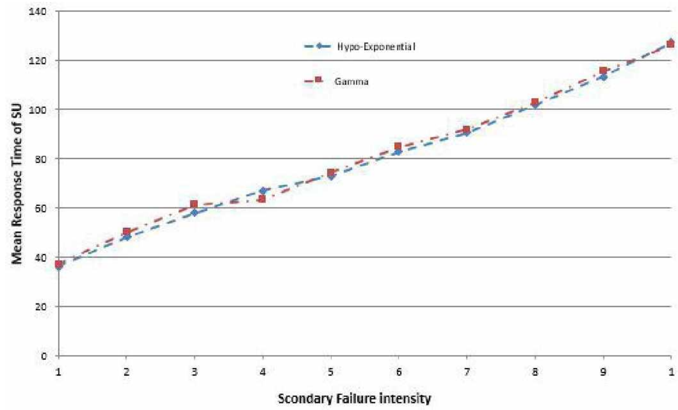


Fig. 4: The effect of the secondary failure intensity on the mean response time of the Secondary Users.

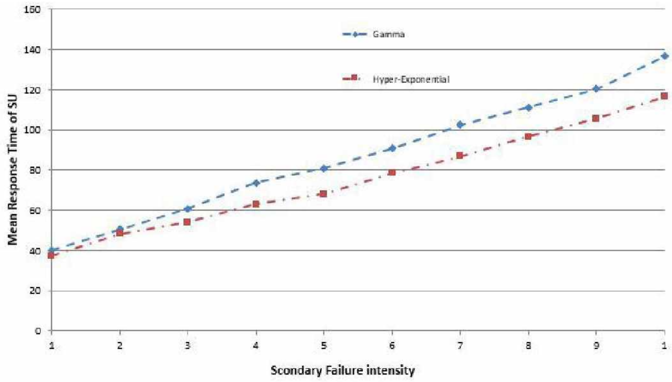


Fig. 5: The effect of the secondary failure intensity on the mean response time of the Secondary Users.

The last two results are related to the effect of the failure intensity of the secondary server versus the mean response time of secondary users. The repair times of the secondary users are Hypo, Hyper and Gamma distributed while all the remaining random variables are exponential.

In Figure 4 where the squared coefficient of variation is less than one, we can see how the the mean response time of the secondary users increases with the increment of the failure intensity of the secondary users and no essential effect can be seen, regardless of the different distributions with the same mean and variance. Furthermore, all the values of the mean response time were nearly similar.

Figure 5 shows the mean response time of SU in function with the secondary failure intensity, supposing that the means and variances of the two distributions are equal and their squared coefficient of variation is greater than one as shown in Table III.

Even though the mean and the variance of the distributions are equal, an important difference can be seen between the values of mean response time of the SUs. The effect of the used distributions can be obviously observed in this Figure.

## V. Conclusion

In this paper a finite-source retrial queuing system is presented with a nonreliable servers. We showed the effect of several distributions on the mean response time of secondary users. A significant effect of these distributions was seen when the squared coefficient of variation is greater than one having the same mean and variance, however the impact was almost negligible when it is less than one. Lastly, as future works, we will deal with more distributions, in order to investigate further their influence on the cognitive radio networks

Finally, as future works, we will deal with new distributions as lognormal and Pareto to investigate the effect of the distributions on a such networks.

## Acknowledgment

The work of Dr. János Sztrik is supported by the EFOP-3.6.1-16-2016-00022 project. The project is co-financed by the European Union and the European Social Fund.

The work of Mohamed Hedi Zaghouani is supported by the Stipendium Hungaricum Scholarship.


## References


[1] F. Paluncic, A. S. Alfa, B. T. Mahraj and H. M. Simba, Queueing Models for Cognitive Radio Networks: A survey. *IEE Access* **6**, 10–1109 (2018)

[2] E. W. Wong, C. H. Foch and F. Adachi, Analysis of cognitive radio spectrum decision for cognitive radio networks. *IEEE J. Select. Areas Common.* **29**, 757–769, (2011).

[3] N. Devroye, M. Vu and V. Tarock, Cognitive radio networks. *IEEE Signal Process. Mag.* **25**, 12–23, (2008).

[4] S. Gunawardena and W. Zhuang, Modeling and Analysis of Voice and Data in Cognitive Radio Networks. *Springer, Berlin* 2191–5768, (2014).

[5] T. Van Do, Nam H. Do, A. Horvath and W. Jinting, Modelling opportunistic spectrum renting in mobile cellular networks. *Journal of Network and Computer Applications, Elsevier* **52**, 129–138 (2015)

[6] Y. Zhao and L. Bai, Performance Analysis and Optimization for Cognitive Radio Networks with Classified Secondary Users and Impatient Packets. *Mobile Information Systems, Hindawi* **2017**, Article ID 3613496 (2017)

[7] J. Sztrik, T. Bérczes, B. Almási, A. Kuki, and J. Wang, Performance modeling of finite-source cognitive radio networks. *J. Acta Cybern.* **22** No3, 617–631 (2016).

[8] J. Sztrik, T. Bérczes, H. Nemouchi, and A. Z. Melikov, Performance modeling of finite-source cognitive radio networks using simulation. *Com. Comput. Inform. Sci.* **678** , 64–73 (2016).

[9] H. Nemouchi and J. Sztrik, Performance simulation of finite source cognitive radio with servers subjects to breakdowns and repairs. *Journal of Mathematical Sciences* **37**, 1072–3374 (2019).




978-1-7281-1401-9/19/$31.00 ©2019 IEEE