M/M/1 retrial queue with working vacations and negative customer arrivals

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Abstract: The M/M/1 retrial queue with working vacations and negative customers is introduced. The arrival processes of positive customers and negative customers are Poisson. Upon the arrival of a positive customer, if the server is busy the customer would enter an orbit of infinite size and the orbital customers send their requests for service with a constant retrial rate. The single server takes an exponential working vacation once customers being served depart from the system and no customers are in the orbit. Arriving negative customers kill a batch of the positive customers waiting in the orbit randomly. Efficient methodology to compute the stationary distribution for this new queue is developed and presented.

Keywords: retrial queue, working vacations, negative customers

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1 Introduction

Vacation queues and retrial queues have been applied to evaluate the performance of various systems [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. Recently, the M/M/1 retrial queue with working vacations was introduced [11] and analyzed. Then onwards, several works [12, 13, 14, 15] have appeared that analyze the single server retrial queue with working vacations in the discrete time domain and the continuous time domain.

In this paper, we provide a useful further extension. The result is the M/M/1 retrial queue with working vacations, and with negative customer arrivals. The notation and the concept of negative customers in queueing systems were introduced by Gelenbe [16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26]. G-queues with negative arrivals in the GI/GI context were first published in [16]. Note that stability issues in G-network models were first discussed in [17, 18, 19, 20, 21, 22]. Discrete-time queues were analyzed in [27, 28, 29]. Queues with negative customers have used extensively to model breakdowns, packet losses, task terminations in speculative parallelism, faulty components in manufacturing systems, server breakdowns and a reaction network of interacting molecules [30, 31, 32, 33, 34, 35], Optical Burst/Packet (OBS) Switching networks [36], wireless networks [33, 37, 38], failures in manufacturing cells [39]. The bibliography on G-networks and negative customers can be found in [40].

The M/M/1 retrial queue with working vacations that is conceived in this paper has negative customer arrivals in addition to positive customers. Positive customers are also referred as customers. The arrival processes of both these customer types are Poisson. Upon the arrival of a positive customer, if the server is busy the customer would wait in an orbit of infinite size. If the server is not occupied, then the customer would obtain its service started immediately and the server gets occupied. Note that the blocked positive customer enter the orbit according to FCFS discipline. Customers waiting in the orbit send request for service from the server with a constant retrial rate. The single server takes a working vacation at times when requests being served depart from the system and no customers are in the orbit. Each vacation lasts for a duration that is exponentially distributed. These vacation periods are working vacations during which customers are indeed served, but with a rate smaller than the normal service rate. At the end of each working vacation, the server takes another new vacation if there is neither a new request nor any retrial request from the orbit. Arriving negative customers kill positive customers waiting in the orbit in a pre-defined manner to model an impatience of positive customers in practice. It is worth mentioning that a queue in which server goes becomes unavailable for a random time after each service period was considered in [1]. However, this kind of vacations is different from a vacation model studied in this paper.

The rest of the paper is organized as follows. In Section 2, we conceive and analyze the M/M/1 retrial queue with working vacations and negative customers. We also derive and present a closed form expression for the steady state probabilities in Section 3. Numerical results are presented in Section 4. Finally, the paper is concluded in Section 5.

2 System Descriptions and Modeling

The working of the M/M/1 retrial queue with working vacations and negative customers is explained as follows. Positive customers and negative customers arrive according Poisson processes with rates λ^+ and λ^-, respectively. Upon the arrival of a customer (positive customer), if the server is busy, it would join the orbit of infinite size. Each customer in the orbit retries for service with a retrial rate of α (the time between two consecutive retrials from a customer is thus exponentially distributed).

The service rate is μ_b when the server is not on vacation. The single server takes a working vacation at times when a request being served departs from the system and no requests are in the orbit. The duration of a working vacation is exponentially distributed with parameter θ. During the working vacation periods, arriving customers or requests are served with a rate $\mu_v < \mu_b$. At the end of each vacation, the server takes another vacation if there is no any customer in the orbit.

An arriving negative customer kills a number of positive customers waiting in the orbit, according to a rule that is explained as follows. When a negative customers arrives, an integer variable $m \geq 1$ is selected randomly, with given probability mass function p_m. If orbiting customers are greater than the realized m, then m orbiting customers are killed. If the number of waiting customers in the orbit is less than or equal to m, then the orbit becomes empty. Let $H(z)$ be the probability generating function of the batch size of the killings

$$H(z) = \sum_{m=1}^{\infty} z^m p_m. \tag{1}$$

Let $I(t)$ denote the state of the server and $J(t)$ be the number of customers in the orbit, at time t.. The single server can be in one of the following mutually exclusive and exhaustive states:

(1) the server is on the working vacation and free at time t. Let this state be numbered as, $I(t) = 0$.

(2) the server is on a working vacation and busy, at time t. This state is denoted by, $I(t) = 1$.

(3) the server is not on a working vacation and not occupied at time t. This state is, $I(t) = 2$.

(4) the server is not on a working vacation and it is busy at time t. This state is, $I(t) = 3$.

The system can now be modeled by a continuous time Markov process (CTMP) $Y = \{I(t), J(t)\}$ on the state space $S = \{(i, j) : 0 \leq i \leq 3,\ \ j \geq 0\}$.

For $J(t) = j > 0$, the possible events in the system can be enumerated as follows:

(1A) the arrival of a new customer occurs,

(1A.1) if the server is free, then the server changes to the busy state (i.e.: $I(t)$ changes either from 0 to 1, or from 2 to 3). In this case, the transition either from state $(0, j)$ to $(1, j)$ or from $(3, j)$ to $(4, j)$ occurs.

(1A.2) if the server is occupied ($I(t) = 1$ or $I(t) = 3$), then the customer goes into the orbit. That is, the transition from state (i, j) to $(i, j + 1)$ happens.

(2A) the departure of a customer from the system takes place after the completion of its service, then the server becomes free. Then, $I(t)$ changes either from 1 to 0 or from 3 to 2 and $J(t)$ remains unchanged.

(3A) the end of the server vacation occurs, then $I(t)$ changes either from 0 to 2 or from 1 to 3. The system changes its state either from $(0, j)$ to $(2, j)$ or from $(1, j)$ to $(3, j)$.

(4A) the successful service request of a customer from the orbit, then $I(t)$ changes either from 0 to 1 or from 2 to 3. The system changes its state either from $(0, j)$ to $(1, j - 1)$ or from $(2, j)$ to $(3, j - 1)$.

(5A) the arrival of a negative customer deletes k positive customers from the orbit, the system changes its state either from (i, j) to $(i, j - k)$ or from (i, j) to $(i, 0)$. k is a random variable here.

At time t, if no customer is in the orbit ($J(t) = 0$), the following events are possible in the system.

(1B) upon the arrival of a new positive customer at time t,

(1B.1) if the server is free, then the server changes to the busy state. The system changes from state $(0, 0)$ to $(1, 0)$.

(1B.2) if the server is occupied ($I(t) = 1$ or $I(t) = 3$), then the customer goes into the orbit. The system changes either from state $(1, 0)$ to $(1, 1)$ or from $(3, 0)$ to $(3, 1)$.

(2B) the departure of a request after the completion of its service occurs, then the server becomes free. This would bring a transition either from state $(1, 0)$ to $(0, 0)$ or from $(3, 0)$ to $(0, 0)$.

(3B) the status change of the server (i.e.: the end of the vacation), then $I(t)$ changes from 1 to 3. The system changes from state $(1, 0)$ to $(3, 0)$.

As a consequence, the following types of possible transitions between the states of CTMP Y can be identified:

(a) a purely phase transition, from state (i, j) to state (k, j) ($\forall$ $(i, j) \in S$ and $(k, j) \in S$), the transition rate is denoted by $A_j(i, k)$;

(b) an one–step upward transition from state (i, j) to state $(k, j + 1)$ ($\forall$ $(i, j) \in S$ and $(k, j + 1) \in S$), the transition rate is represented by $B_j(i, k)$;

(c) an $m-$step downward transition from state (i, j) to state $(k, j - m)$ ($\forall$ $(i, j) \in S$ and $(k, j - m) \in S$) for $m \geq 1$, the transition rate is is $C_{j,m}(i, k)$.

Let A_j, B_j and $C_{j,m}$ be matrices of size 4×4 with elements $A_j(i, k)$, $B_j(i, k)$ and $C_{j,m}(i, k)$, respectively.

Therefore, we can write

$$A_0 = \begin{bmatrix} 0 & \lambda^+ & 0 & 0 \\ \mu_v & 0 & 0 & \theta \\ 0 & 0 & 0 & 0 \\ \mu_b & 0 & 0 & 0 \end{bmatrix} ; \; A_j = A = \begin{bmatrix} 0 & \lambda^+ & \theta & 0 \\ \mu_v & 0 & 0 & \theta \\ 0 & 0 & 0 & \lambda^+ \\ 0 & 0 & \mu_b & 0 \end{bmatrix} , j \geq 1;$$

$$B_j = B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \lambda^+ & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda^+ \end{bmatrix} , j \geq 0;$$

$$C_{1,1} = C = \begin{bmatrix} \lambda^- & \alpha & 0 & 0 \\ 0 & \lambda^- & 0 & 0 \\ 0 & 0 & \lambda^- & \alpha \\ 0 & 0 & 0 & \lambda^- \end{bmatrix} ; \; C_{j,1} = C_1 = \begin{bmatrix} p_1\lambda^- & \alpha & 0 & 0 \\ 0 & p_1\lambda^- & 0 & 0 \\ 0 & 0 & p_1\lambda^- & \alpha \\ 0 & 0 & 0 & p_1\lambda^- \end{bmatrix} \; \forall j \geq 2;$$

$$C_{j,m} = C_m = \begin{bmatrix} p_m\lambda^- & 0 & 0 & 0 \\ 0 & p_m\lambda^- & 0 & 0 \\ 0 & 0 & p_m\lambda^- & 0 \\ 0 & 0 & 0 & p_m\lambda^- \end{bmatrix},\ j > m > 1\ ;$$

$$C_{j,j} = \begin{bmatrix} \sum_{m=j}^{\infty} p_m\lambda^- & 0 & 0 & 0 \\ 0 & \sum_{m=j}^{\infty} p_m\lambda^- & 0 & 0 \\ 0 & 0 & \sum_{m=j}^{\infty} p_m\lambda^- & 0 \\ 0 & 0 & 0 & \sum_{m=j}^{\infty} p_m\lambda^- \end{bmatrix} \quad \forall j \geq 2.$$

One can observe that the following equation holds

$$C = C_{j,j} + \sum_{i=1}^{j-1} C_i. \tag{2}$$

3 The Steady State Probabilities

CTMP Y is on a two-dimensional lattice, finite in the phase $I(t)$ and infinite in the level $J(t)$ of the process. Due to the transitions caused by negative customers, the process Y is of GI/M/1-type, skip-free to the right ([1, 41]). Therefore, the infinitesimal generator matrix of Y is

$$Q = \begin{bmatrix} A_0^* & B_0 & 0 & 0 & 0 & 0 & \dots \\ C_{1,1} & A_1^* & B_1 & 0 & 0 & 0 & \dots \\ C_{2,2} & C_{2,1} & A_2^* & B_2 & 0 & 0 & \dots \\ C_{3,3} & C_{3,2} & C_{3,1} & A_3^* & B_3 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix} = \tag{3}$$

$$\begin{bmatrix} A_0^* & B & 0 & 0 & 0 & 0 & \dots \\ C_{1,1} & A^* & B & 0 & 0 & 0 & \dots \\ C_{2,2} & C_1 & A^* & B & 0 & 0 & \dots \\ C_{3,3} & C_2 & C_1 & A^* & B & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix}, \tag{4}$$

where $A_0^* = A_0 - D^{A_0} - D^{B_0}$, $A_j^* = A_j - D^{A_j} - D^{B_j} - D^C$, $j \geq 1$, and D^Z $(Z = A_j, B_j, C_j)$ is a diagonal matrix whose diagonal element is the sum of all elements in the corresponding row of Z. It can be observed that $A_j^* = A_j - D^{A_j} - D^{B_j} - D^C$ does not depend on j, so let $A_j^* = A^*$.

We denote the steady state probabilities as,

$$\pi_{i,j} = \lim_{t\to\infty} P(I(t) = i, J(t) = j),$$

and let the row-vector, $\mathbf{v}_j = [\pi_{0,j}, \pi_{1,j}, \pi_{2,j}, \pi_{3,j}]$. From the explained operation of the queue, it can be seen that the probability there is no waiting customer in the orbit, no customer occuping the server and the server is not on a working vacation, is zero. This means, $\pi_{2,0} = 0$ holds.

The balance equations, which equate the probability fluxes from and to the states of CTMC Y, and the normalization equation pertaining to CTMC Y can be written as follows:

- for $J(t) = 0$,

$$\mathbf{v}_0 A_0^* + \sum_{k=1}^{\infty} \mathbf{v}_k C_{k,k} = 0. \tag{5}$$

- for row j for $j \geq 1$,

$$\mathbf{v}_{j-1} B + \mathbf{v}_j A^* + \sum_{m=1}^{\infty} \mathbf{v}_{j+m} C_m = 0. \tag{6}$$

The normalization equation is

$$\sum_{j=0}^{\infty} \mathbf{v}_j \boldsymbol{e} = 1, \tag{7}$$

where $\boldsymbol{e}$ is the column vector of size 4 with each element equals to unity.

Theorem 1 *The determinant of the characteristic matrix polynomial* $Q(x) = B + A^*x + \sum_{m=1}^{\infty} C_m x^{m+1}$ *associated with equation (6) can be expressed as follows*

$$Det[Q(x)] = (Q_{00}(x)Q_{11}(x) - (\lambda^+ + \alpha x)\mu_v x^2)(Q_{22}(x)Q_{33}(x) - (\lambda^+ + \alpha x)\mu_b x^2), \tag{8}$$

where

$$\begin{aligned} Q_{00}(x) &= -(\alpha + \lambda^+ + \theta + \lambda^-)x + \lambda^- x H(x), \\ Q_{11}(x) &= \lambda^+ - (\lambda^+ + \theta + \mu_v + \lambda^-)x + \lambda^- x H(x), \\ Q_{22}(x) &= -(\lambda^+ + \alpha + \lambda^-)x + \lambda^- x H(x), \\ Q_{33}(x) &= \lambda^+ - (\lambda^+ + \mu_b + \lambda^-)x + \lambda^- x H(x). \end{aligned}$$

Proof: Using the expressions of B, A^*, C_m and equation (1), we can obtain after some algebra

$$Q(x) = \begin{bmatrix} Q_{00}(x) & \lambda^+ x + \alpha x^2 & \theta x & 0 \\ \mu_v x & Q_{11}(x) & 0 & \theta x \\ 0 & 0 & Q_{22}(x) & \lambda^+ x + \alpha x^2 \\ 0 & 0 & \mu_b x & Q_{33}(x) \end{bmatrix}.$$

Therefore, $Det[Q(x)] = (Q_{00}(x)Q_{11}(x) - (\lambda^+ + \alpha x)\mu_v x^2)(Q_{22}(x)Q_{33}(x) - (\lambda^+ + \alpha x)\mu_b x^2)$ holds. □

Based on Theorem 1, the roots of $Det[Q(x)]$ can be determined from $Q_{00}(x)Q_{11}(x) - (\lambda^+ + \alpha x)\mu_v x^2 = 0$ and $Q_{22}(x)Q_{33}(x) - (\lambda^+ + \alpha x)\mu_b x^2 = 0$. Based on [42], the necessary and sufficient condition for the ergodicity of CTMC Y is that the number of eigenvalues of $Q(x)$ inside the unit disk is 4. Let us denote these eigenvalues as x_1, x_2, x_3 and x_4, then $|x_i| < 1$.

Note that $Q_{00}(x)Q_{11}(x) - (\lambda^+ + \alpha x)\mu_v x^2$ has two roots that are inside the unit circle: $0 < x_1 < 1$ and $x_2 = 0$. Similarly, $Q_{22}(x)Q_{33}(x) - (\lambda^+ + \alpha x)\mu_b x^2$ has two roots that are inside the unit circle: $x_3 = 0$ and $0 < x_4 < 1$.

Following [42], the steady state probabilities can be expressed as a linear sum of factors $x_i^j \Psi_i$ (where $|x_i| < 1$):

$$\mathbf{v}_j = \sum_{i=1}^{4} a_i x_i^j \Psi_i \ \ (j \geq 0), \tag{9}$$

where a_i $(i = 1, \ldots, 4)$ are the coefficients to be determined and Ψ_is are the left-hand-side (LHS) eigenvectors of $Q(x)$ for eigenvalue x_i, $i = 1, 2, 3, 4$.

To compute the coefficients a_i $(i = 1, \ldots, 4)$ we will proceed as follows. Using $\Psi_i Q(x_i) = 0$, $i = 1, 2, 3, 4$, we can obtain the expression for the LHS eigenvectors of $Q(x)$:

1) $\Psi_1 = [1, -Q_{00}(x_1)/(\mu_v x_1), y_1, y_2]$ is the LHS eigenvector of $Q(x)$ for eigenvalue x_1, where y_1 and y_2 can be determined as the solution of the following linear equations:

$$\begin{aligned} \theta x_1 + Q_{22}(x_1)y_1 + \mu_b x_1 y_2 = 0, \\ -Q_{00}(x_1)\theta/\mu_v + (\lambda^+ x_1 + \alpha x_1^2)y_1 + Q_{33}(x_1)y_2 = 0. \end{aligned}$$

   Therefore, we get

$$\begin{aligned} y_1 &= -\frac{\mu_b Q_{00}(x_1)\theta x_1 + \mu_v Q_{33}(x_1)\theta x_1}{\mu_v (Q_{22}(x_1)Q_{33}(x_1) - \lambda^+ \mu_b x_1^2 - \alpha \mu_b x_1^3)}, \\ y_2 &= \frac{Q_{00}(x_1)Q_{22}(x_1)\theta + \mu_v \lambda^+ \theta x_1^2 + \mu_v \alpha \theta x_1^3}{\mu_v (Q_{22}(x_1)Q_{33}(x_1) - \lambda^+ \mu_b x_1^2 - \alpha \mu_b x_1^3)}. \end{aligned}$$

2) $\Psi_2 = [1, 0, 0, 0]$ is the corresponding LHS eigenvector of zero eigenvalue x_2 of $Q(x)$.

3) $\Psi_3 = [0, 0, 1, 0]$ is the corresponding LHS eigenvector of zero eigenvalue x_3 of $Q(x)$.

4) $\Psi_4 = [0, 0, 1, -Q_{22}(x_4)/(\mu_b x_4)]$ is the LHS eigenvector of $Q(x)$ for eigenvalue x_4.

Theorem 2 *The balance equation (5) can be written in the following form*

$$\mathbf{v}_0 A_0^* + \sum_{i=1}^{4} \frac{a_i x_i}{1 - x_i} \Psi_i C + \sum_{i=1}^{4} \frac{a_i}{1 - x_i} \Psi_i \left(B + A^* x_i\right) = 0 \tag{10}$$

Proof: Substituting equation (2) into (5), we get

$$\mathbf{v}_0 A_0^* + \sum_{k=1}^{\infty} \mathbf{v}_k (C - \sum_{m=1}^{k-1} C_m) = 0. \tag{11}$$

Since

$$\sum_{k=1}^{\infty} \mathbf{v}_k \sum_{m=1}^{k-1} C_m = \sum_{m=1}^{\infty} \sum_{k=m+1}^{\infty} \mathbf{v}_k C_m = \sum_{i=1}^{4} \sum_{m=1}^{\infty} \sum_{k=m+1}^{\infty} a_i x_i^k \Psi_i C_m$$
$$= \sum_{i=1}^{4} \frac{a_i}{1-x_i} \Psi_i \sum_{m=1}^{\infty} C_m x_i^{m+1} = \sum_{i=1}^{4} \frac{a_i}{1-x_i} \Psi_i (Q(x_i) - B - A^* x_i) \tag{12}$$

and

$$\sum_{k=1}^{\infty} \mathbf{v}_k C = \sum_{i=1}^{4} \sum_{k=1}^{\infty} a_i x_i^k \Psi_i C = \sum_{i=1}^{4} \frac{a_i x_i}{1-x_i} \Psi_i C, \tag{13}$$

equation (11) can be expressed as

$$\mathbf{v}_0 A_0^* + \sum_{i=1}^{4} \frac{a_i x_i}{1-x_i} \Psi_i C - \sum_{i=1}^{4} \frac{a_i}{1-x_i} \Psi_i \left(Q(x_i) - B - A^* x_i\right) = 0. \tag{14}$$

Substituting $\Psi_i Q(x_i) = 0$, $i = 1, 2, 3, 4$ into equation (14), we yield (10). □

The normalization equation $\sum_{j=0}^{\infty} \mathbf{v}_j \boldsymbol{e} = 1$ can be rewritten as

$$\sum_{j=0}^{\infty} \mathbf{v}_j \boldsymbol{e} = \sum_{j=0}^{\infty} \sum_{i=1}^{4} a_i x_i^j \Psi_i \boldsymbol{e} = \sum_{i=1}^{4} a_i \frac{1}{1-x_i} \Psi_i \boldsymbol{e} = 1, . \tag{15}$$

The computation of the coefficients can be performed using (10) and (15) which needs only small computational requirements, and does not involve the computation of the infinite number of terms. The performance measures can be obtained as follows:

- the average number of customers in the orbit

$$E(J) = \sum_{j=1}^{\infty} j \mathbf{v}_j e = \sum_{j=1}^{\infty} j \sum_{i=1}^{4} a_i x_i^j \Psi_i e = \sum_{i=1}^{4} a_i \frac{x_i}{(1-x_i)^2} \Psi_i e, \tag{16}$$

- the probability that the server is on vacation

$$P_{vac} = \sum_{j=0}^{\infty} (\pi_{0,j} + \pi_{1,j}) = \frac{a_1(1 - Q_{00}(x_1)/(\mu_v x_1))}{(1-x_1)} + a_2. \tag{17}$$

Figure 1 $E(J)$ vs λ^-

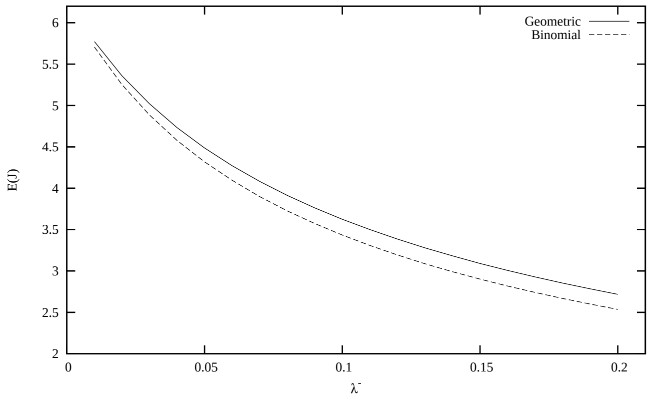
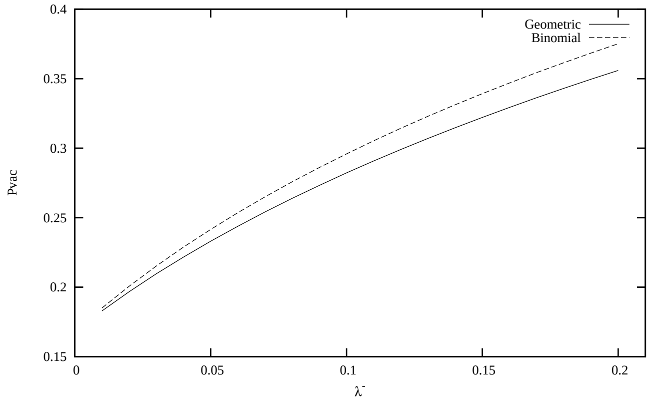


Figure 2 P_{vac} vs λ^-



4 Numerical Results

In this section, we present some numerical results concerning the average number of customers in the orbit and the probability that the server is on vacation. In order to do so, we implemented the method presented in Section 3. For $\lambda^+ = 2.1$, $\alpha = 10.$, $\theta = 1.0$, $\mu_v = 0.8$, $\mu_b = 3.0$, we plot the average number of customers in the orbit vs λ^- and the probability that the server is on vacation vs λ^- in Figures 1 and 2, respectively. It is observed that the binomial distribution of the batch size of the killings has a more severe impact on the number of customers waiting in the orbit than the geometric distribution of the batch size of the killings (note that we keep $\sum_{m=1}^{\infty} p_m m = 5.0$). Therefore, the probability that the server is on vacation is less with the geometric distribution than the binomial distribution of the batch size of the killings.

5 Conclusions

We have introduced the new M/M/1 retrial queue with working vacations and negative customer arrivals. It is remarkable that our solution does not involve the summation of the infinite number of terms, if we have the closed-form expression for the probability generating function of the batch size of the killings. Extensions of this work in several directions (e.g., to model public transportation situation) are possible.

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