A Survey of Recent Results in Finite-Source Retrial Queues with Collisions and Impatient Customers in the Orbit

J. Sztrik, Á. Tóth

University of Debrecen, Faculty of Informatics

http://irh.inf.unideb.hu/user/jsztrik

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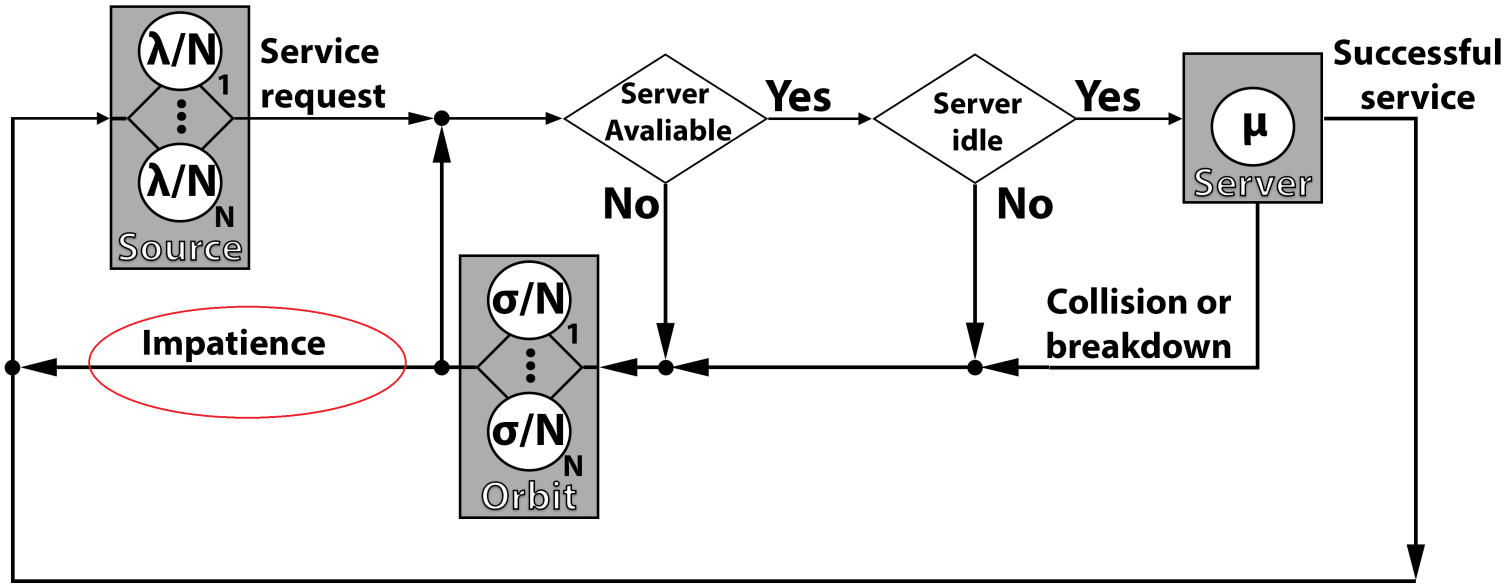
Outline

Finite-source retrial queueing system with collisions and impatience



Description of the systems

Performance measures

- *Distribution of number of requests in the system, including in service and in the orbit*
- *Mean waiting/response time of an arbitrary, successful and impatient customer*
- *Mean total service time of an arbitrary, successful and impatient customer*
- *Distribution of number of retrials of an arbitrary, successful and impatient customer*
- *Distribution of waiting/response time of an arbitrary, successful and impatient customer*

Tool supported approach

- *MOSEL (MOdeling, Specification and Evaluation Language) solution*
- *PRISM (PRobabIliStic Model checking) solution*

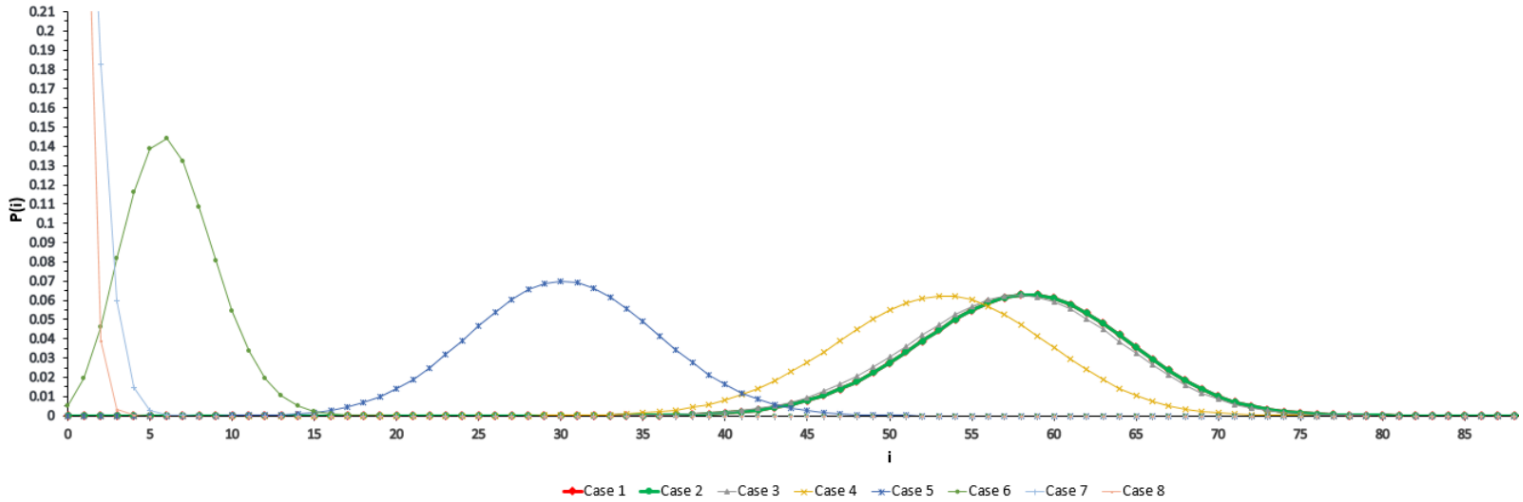
Simulation approach

- *The effect of distributions of the involved random variables on the distribution of the number of customers in the system*
- *The effect of distributions of the involved random variables on the mean and variance of the waiting/response time of a request*
- *The effect of distributions of the involved random variables on the mean and variance of the number of retrials*

Exponentially distributed impatience time

	N	λ/N	σ/N	γ0	γ1	γ2	μ	τ
Case 1	100	0.01	0.1	0.1	0.1	1	1	1E-10
Case 2	100	0.01	0.1	0.1	0.1	1	1	0.000001
Case 3	100	0.01	0.1	0.1	0.1	1	1	0.0001
Case 4	100	0.01	0.1	0.1	0.1	1	1	0.001
Case 5	100	0.01	0.1	0.1	0.1	1	1	0.01
Case 6	100	0.01	0.1	0.1	0.1	1	1	0.1
Case 7	100	0.01	0.1	0.1	0.1	1	1	1
Case 8	100	0.01	0.1	0.1	0.1	1	1	5

Input parameters of the model



Distribution of number of customers

E(NS): mean number of customers in the system
E(T): mean sojourn time of an arbitrary customer
E(TS): mean sojourn time of a successful customer
E(TA): mean sojourn time of a reneging customer
E(O): mean number of customers in the orbit
E(W): mean waiting time of an arbitrary customer
E(WS): mean waiting time of the successfully served customers
E(WA): mean waiting time of the customers who leave the system without service
E(ST): mean total service time of an arbitrary customer
E(STS): mean total service time of the successfully served customers
E(STA): mean total service time of a reneging customer
Pa: probability of abandonment
Us: server utilization

Notations for the estimations

	E(NS)	E(T)	Little	E(TS)	E(TA)	E(O)	E(W)	E(WS)	E(WA)	E(ST)	E(STS)	E(STA)	Pa	Us
Case 1	57.989	138.003	138.030	138.003	0.000	57.568	137.003	137.003	0.000	1.000	1.000	0.000	0.000	0.420
Case 2	57.985	137.961	138.009	137.960	148.596	57.565	136.962	136.960	147.692	1.000	1.000	0.904	0.000	0.420
Case 3	57.444	134.959	134.987	134.823	144.989	57.025	133.972	133.835	144.081	0.987	0.988	0.907	0.013	0.420
Case 4	53.003	112.727	112.778	111.576	121.875	52.585	111.839	110.678	121.060	0.888	0.898	0.815	0.112	0.418
Case 5	30.243	43.345	43.355	39.391	48.634	29.844	42.773	38.767	48.131	0.572	0.624	0.503	0.428	0.399
Case 6	6.319	6.744	6.745	3.671	8.488	5.980	6.383	3.168	8.205	0.362	0.502	0.282	0.638	0.339
Case 7	0.990	1.000	1.000	0.551	1.197	0.687	0.694	0.063	0.972	0.306	0.488	0.226	0.694	0.303
Case 8	0.436	0.438	0.438	0.485	0.419	0.140	0.140	0.003	0.199	0.298	0.482	0.220	0.702	0.297
		138.003					137.003			1.000				
		137.961					136.962			1.000				
		134.959					133.972			0.987				
		112.727					111.839			0.888				
		43.345					42.773			0.572				
		6.744					6.383			0.362				
		1.000					0.694			0.306				
		0.438					0.140			0.298				

Estimations

Var(NS): variance of customers in the system
Var(T): variance of sojourn time of an arbitrary customer
Var(TS): variance of sojourn time of a successful customer
Var(TA): variance of sojourn time of a reneging customer
Var(W): variance of waiting time of an arbitrary customer
Var(WS): variance of waiting time of a successful customer
Var(WA): variance of waiting time of a reneging customer
Var(ST): variance of service time of an arbitrary customer
Var(STS): variance of service time of a successful customer
Var(STA): variance of waiting time of an arbitrary customer
Pao: probability that an abandoned customer never leaves the orbit
E(WAO): mean waiting time of an abandoned customer who never leaves the orbit
E(STSI): mean total interrupted service time of a successfully served customer
E(STSUI): mean total uninterrupted service time of a customer

Notations for the estimations

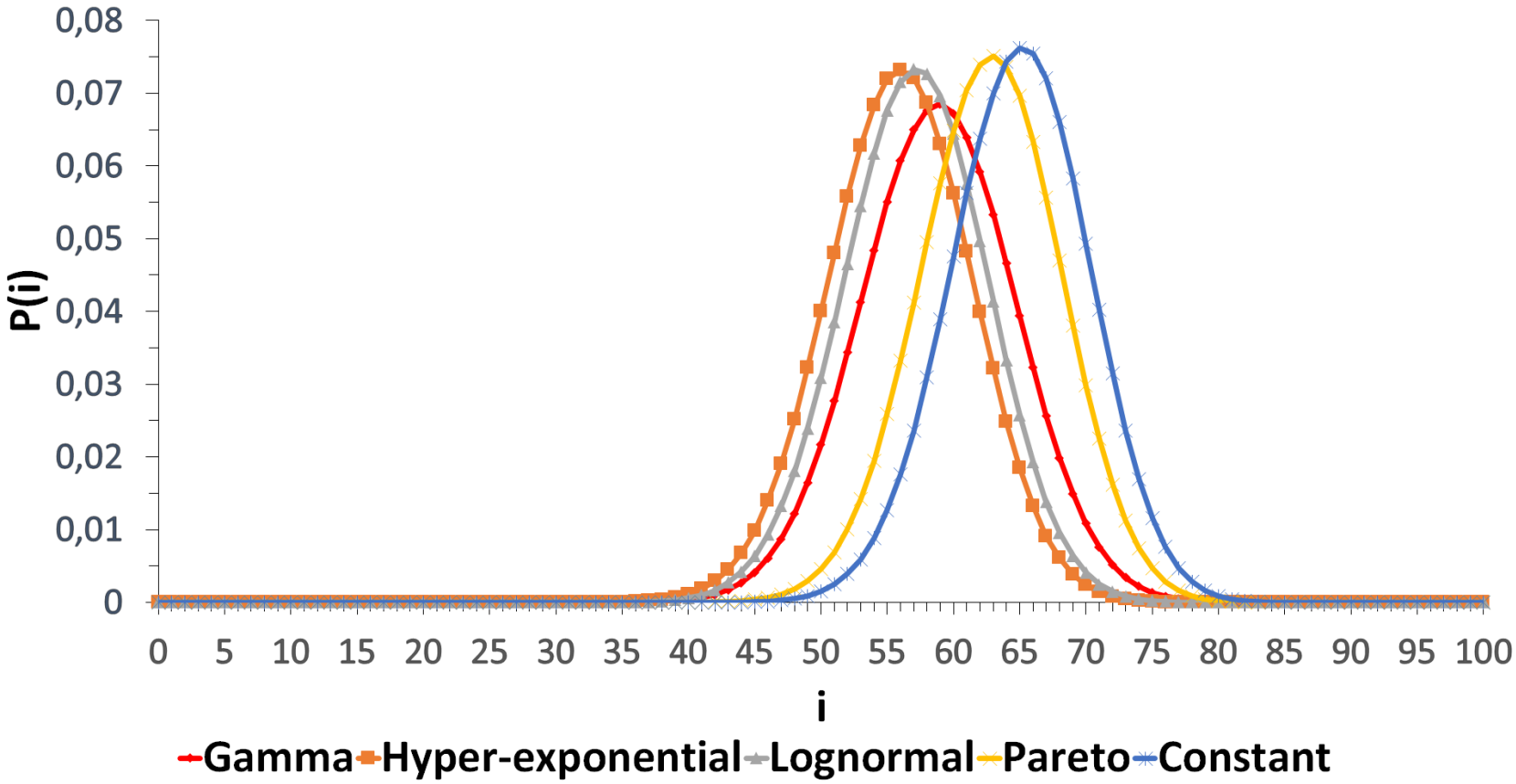
Var(NS)	Var(T)	Var(TS)	Var(TA)	Var(W)	Var(WS)	Var(WA)	Var(ST)	Var(STS)	Var(STA)	Pao	E(WAO)	E(STSI)	E(STSUI)
40.007	21911.908	21911.908	0.000	21657.862	21657.862	0.000	1.000	1.000	0.000	0.000	0.000	0.861	0.139
40.124	21881.572	21881.345	23376.692	21627.907	21627.682	23112.600	0.999	0.999	1.003	0.074	20.174	0.860	0.139
40.251	20963.887	20960.814	21088.319	20719.418	20716.321	20844.001	0.975	0.975	0.965	0.078	20.334	0.847	0.140
40.608	14752.973	14728.385	14854.131	14571.818	14547.085	14672.623	0.805	0.806	0.798	0.091	19.931	0.749	0.149
32.106	2326.767	2264.987	2360.505	2287.902	2225.639	2321.008	0.386	0.390	0.373	0.187	16.398	0.410	0.214
7.570	65.502	44.016	69.292	63.994	41.980	67.298	0.233	0.257	0.202	0.427	6.376	0.112	0.391
0.980	0.999	0.387	1.141	0.867	0.118	0.945	0.205	0.240	0.168	0.532	0.943	0.016	0.472
0.344	0.215	0.235	0.204	0.036	0.001	0.039	0.199	0.233	0.164	0.543	0.197	0.004	0.478

Estimations

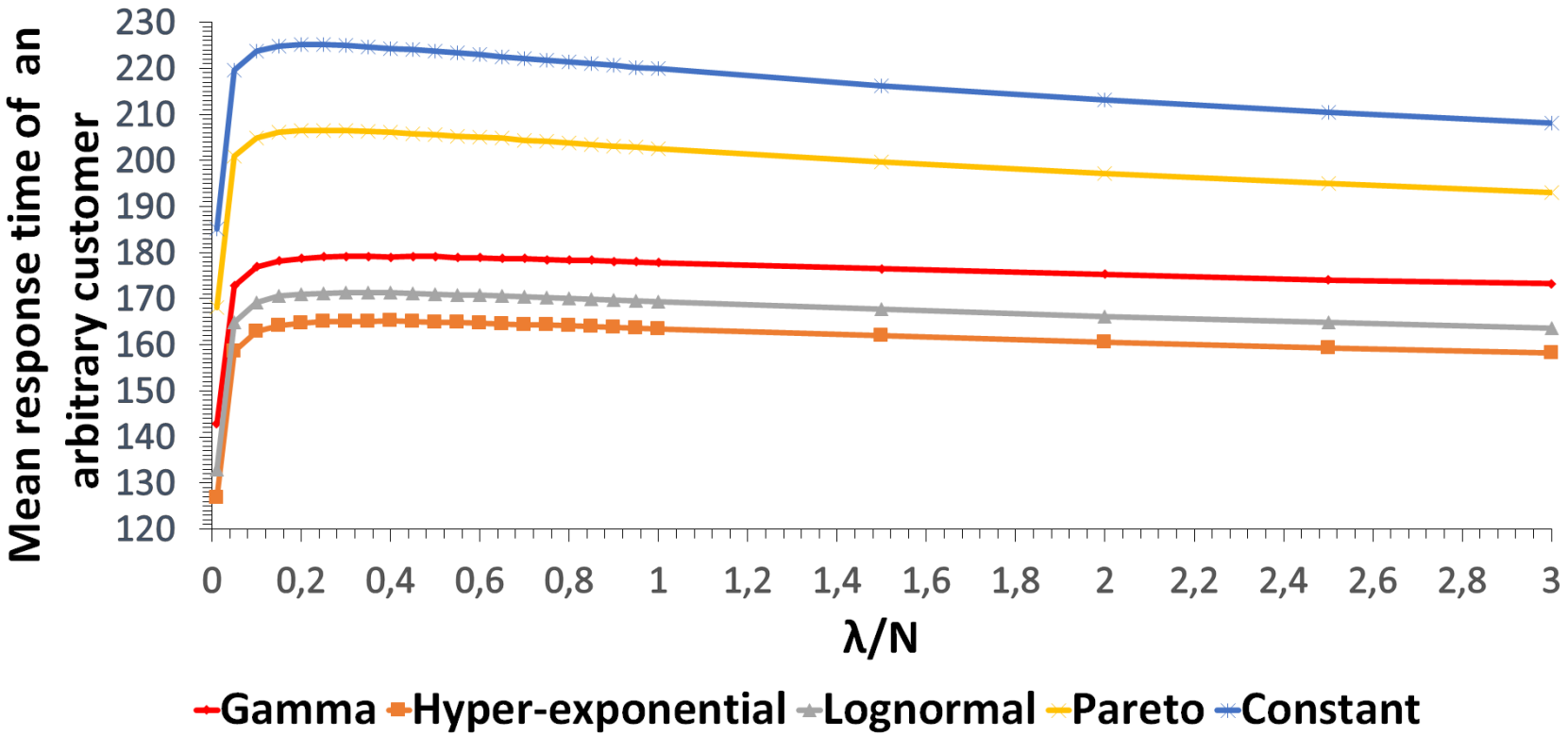
The effect of distribution of the impatience time
Squared coefficient of variation is more than one

Parameters of the distribution of impatient calls

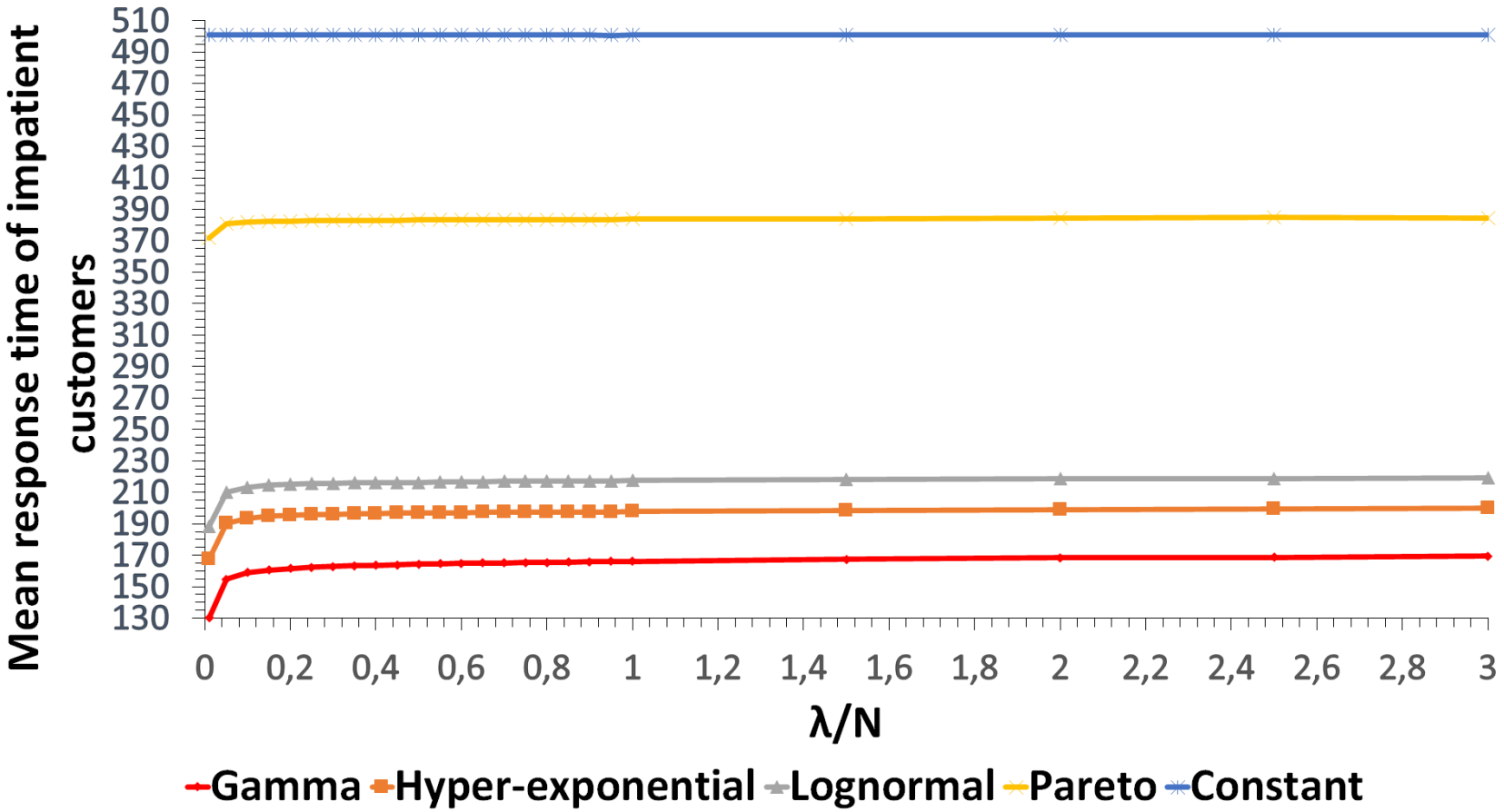
Distribution	**Gamma**	**Hyper-exponential**	**Pareto**	**Lognormal**
Parameters	$\alpha = 0.390625$ $\beta = 0.0007813$	$p = 0.33098$ $\lambda_1 = 0.00132$ $\lambda_2 = 0.00268$	$\alpha = 2.1792$ $k = 270.56302$	$m = 5.57973$ $\sigma = 1.12684$
Mean	500			
Variance	640000			
Squared coefficient of variation	2.56			



Comparison of steady-state distributions



Mean response time vs. arrival intensity



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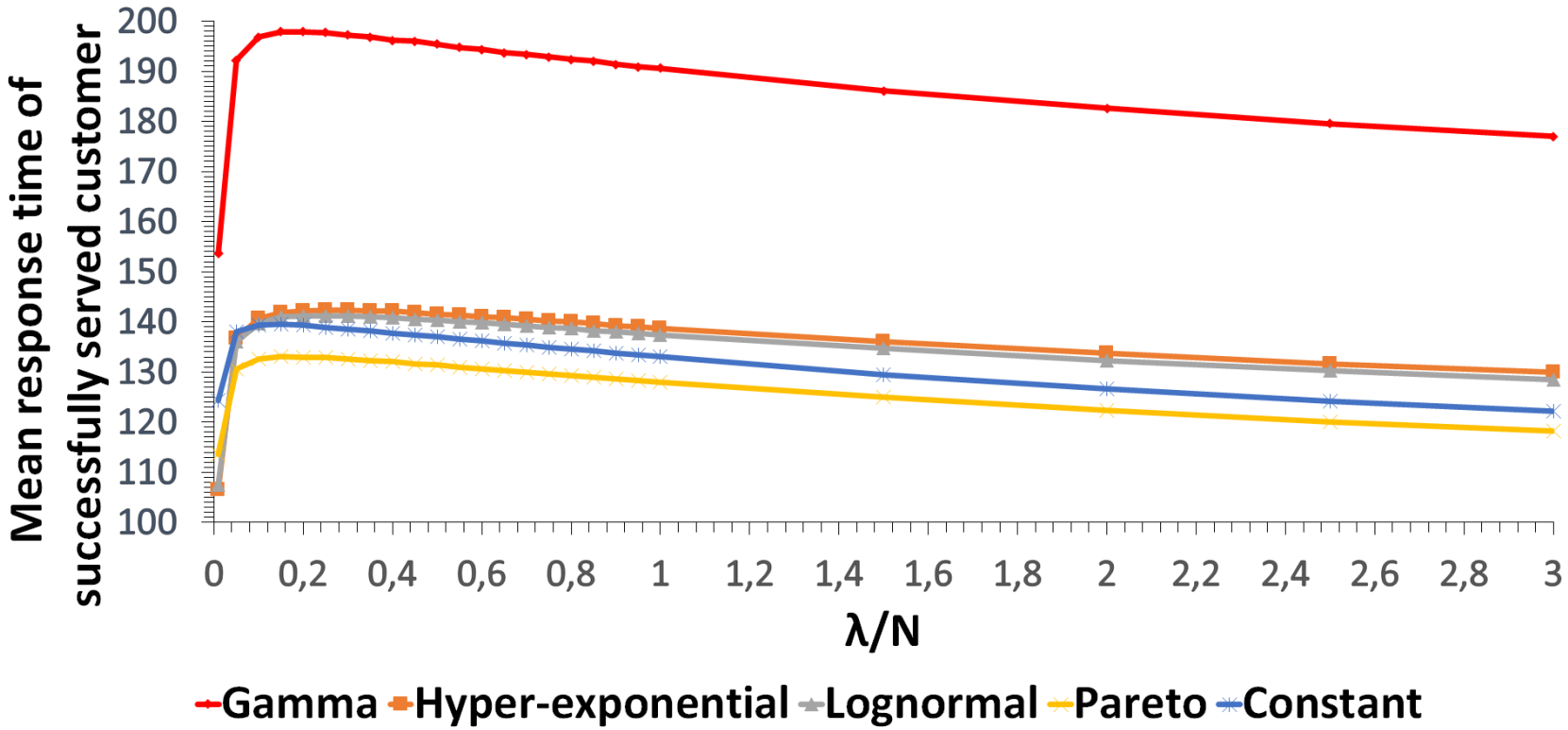

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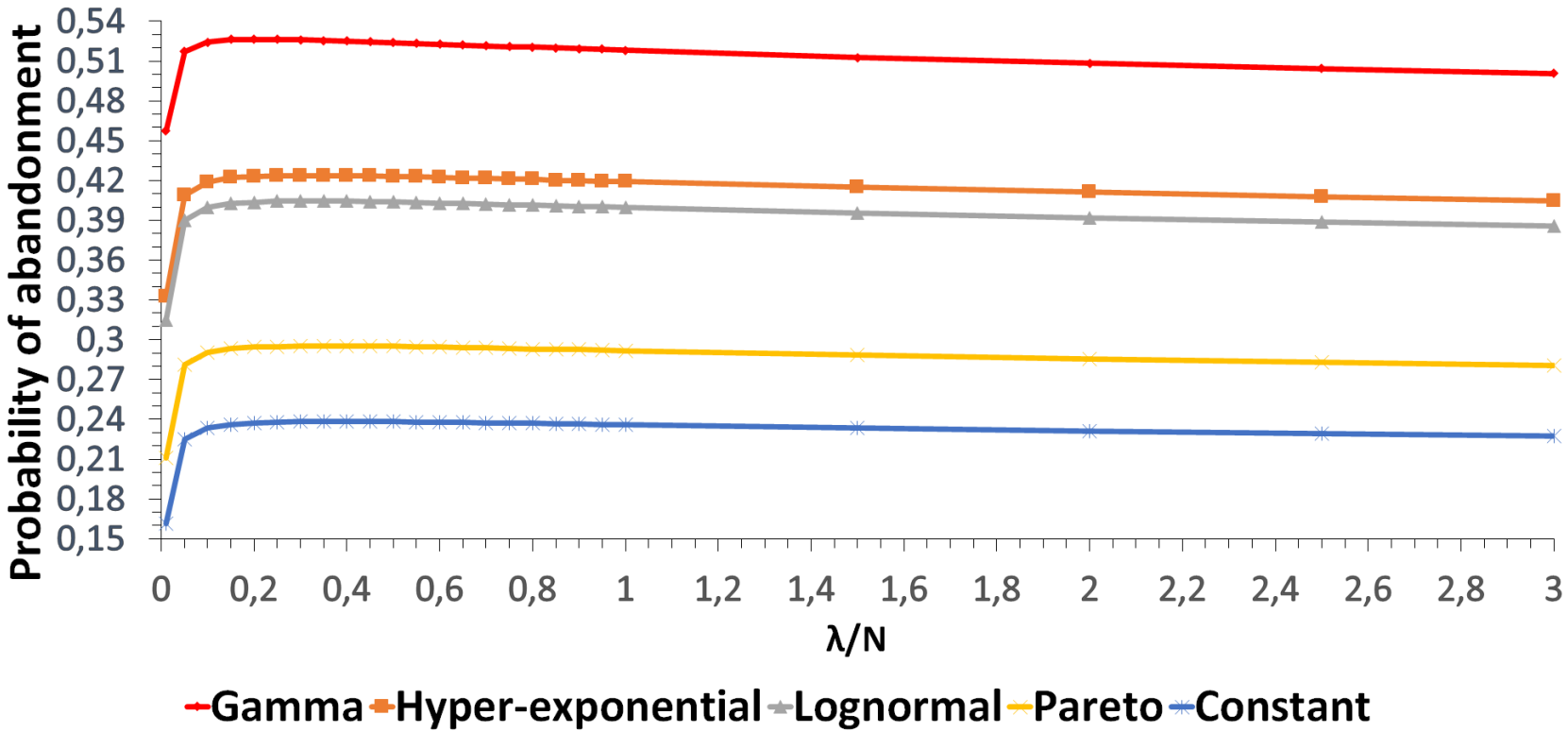
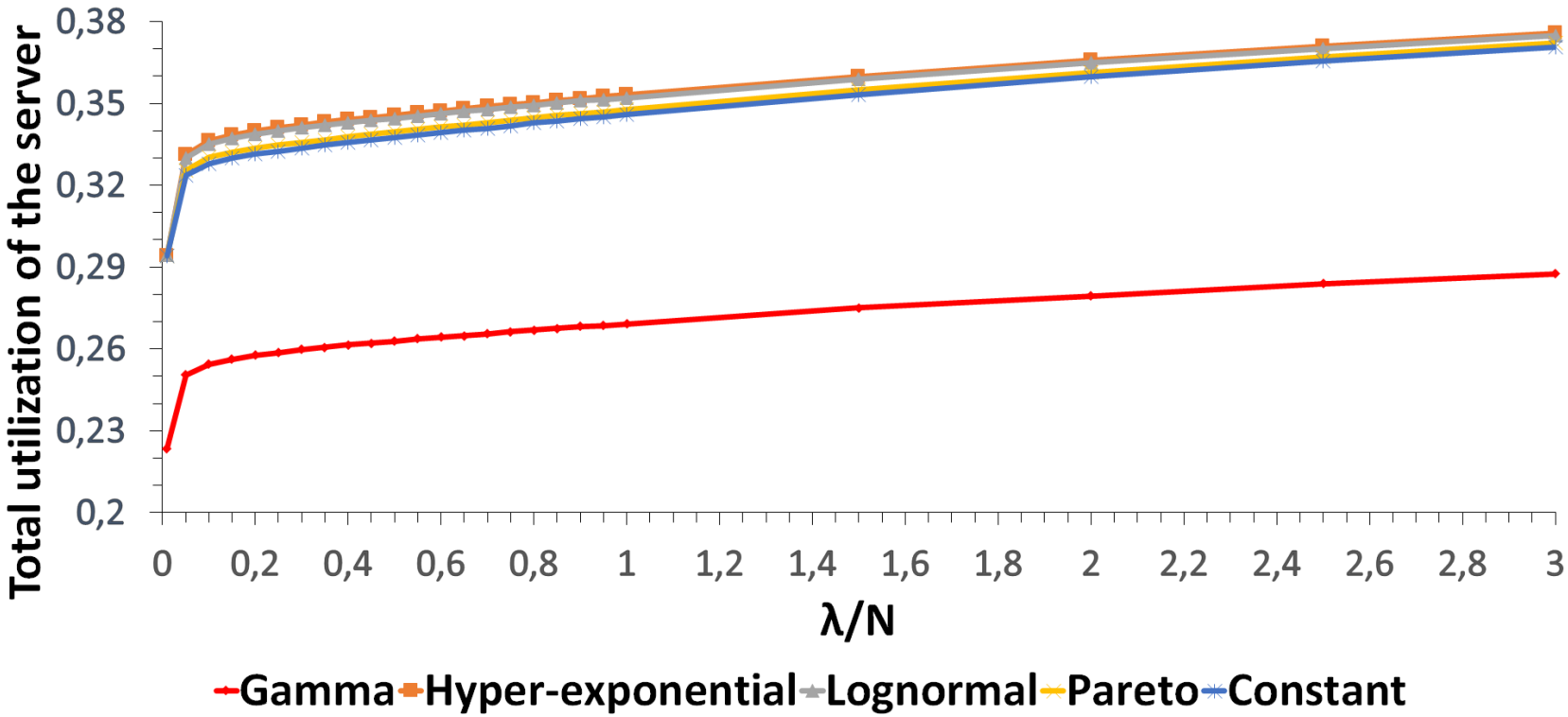


Probability of abandonment of a customer vs. arrival intensity

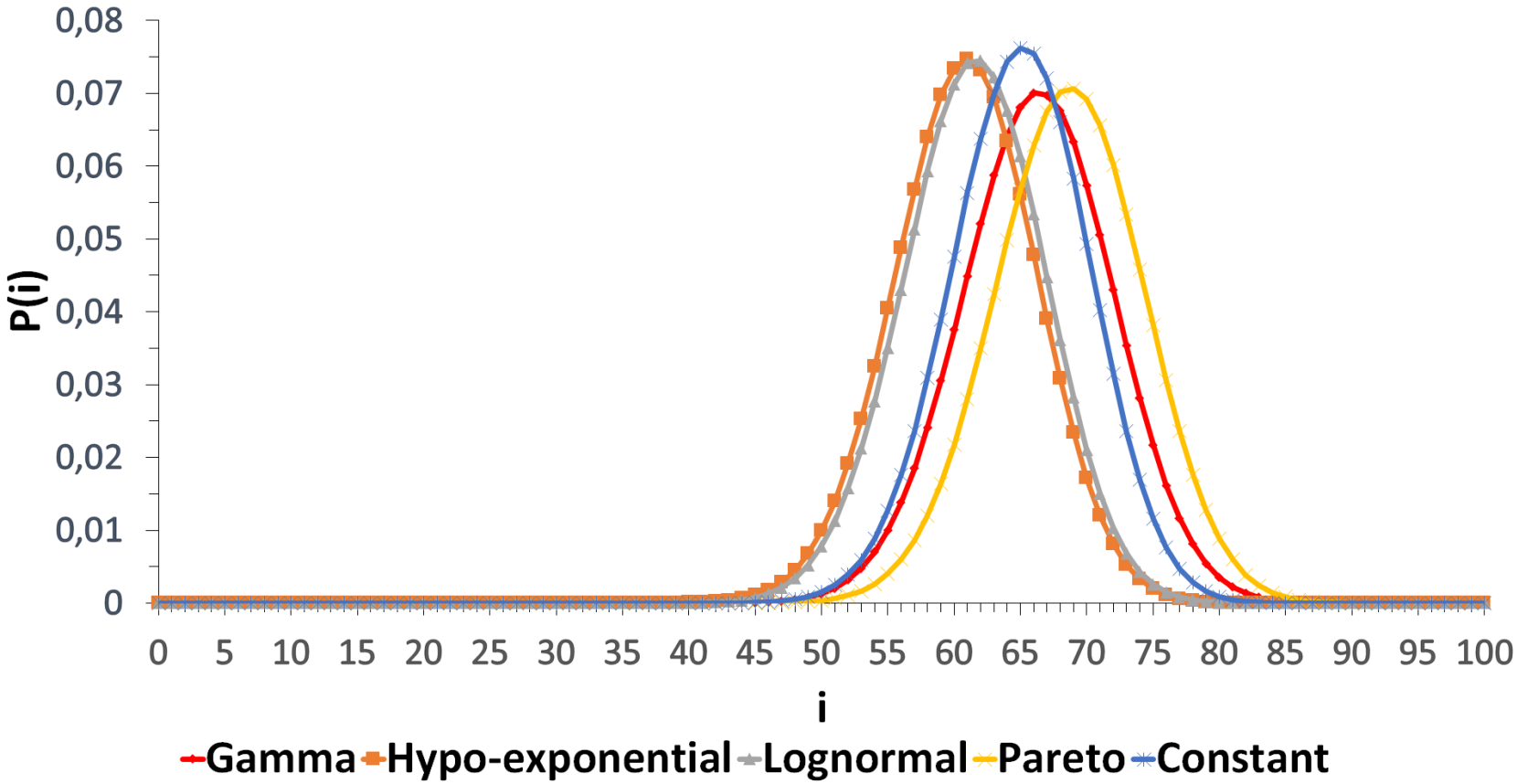


Total utilization of the server vs. arrival intensity

The effect of distribution of the impatience time
Squared coefficient of variation is less than one

Parameters of impatient calls

Distribution	**Gamma**	**Hypo-exponential**	**Pareto**	**Lognormal**
Parameters	$\alpha = 1.47059$ $\beta = 0.002941$	$\mu_1 = 0.01$ $\mu_2 = 0.0025$	$\alpha = 2.5718$ $k = 305.5844$	$m = 5.9552$ $\sigma = 0.72027$
Mean	500			
Variance	170000			
Squared coefficient of variation	0.68			



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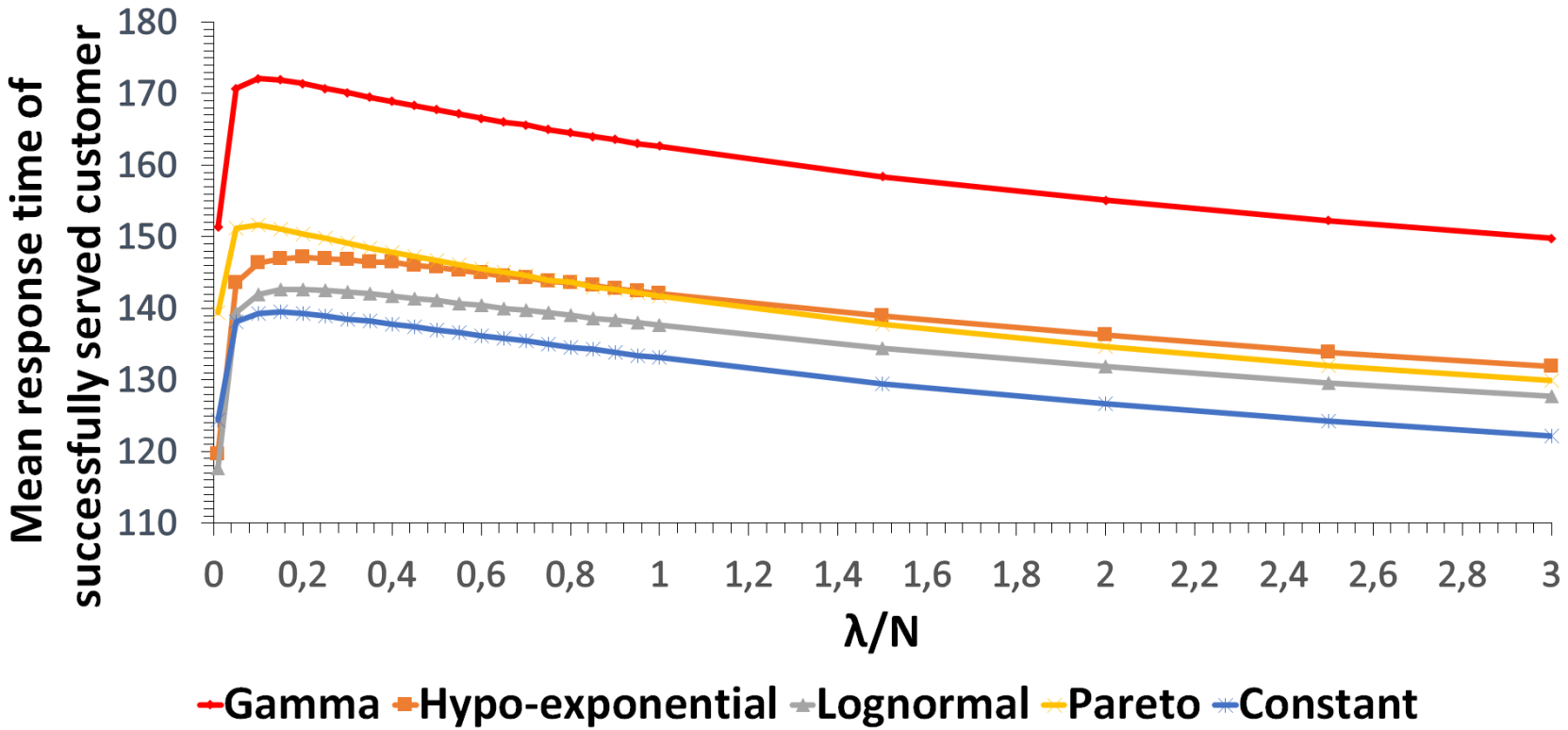
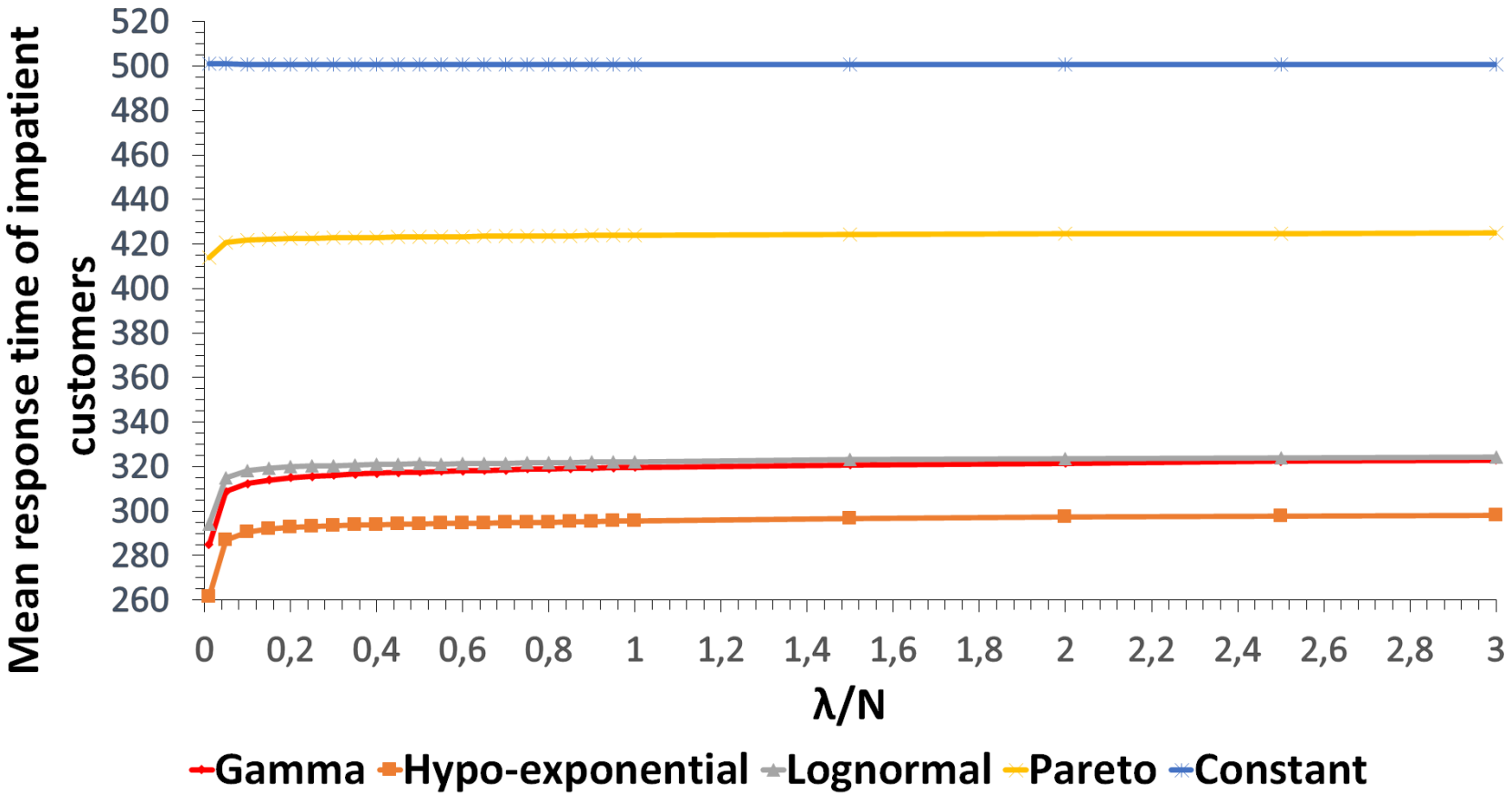


Mean response time vs. arrival intensity



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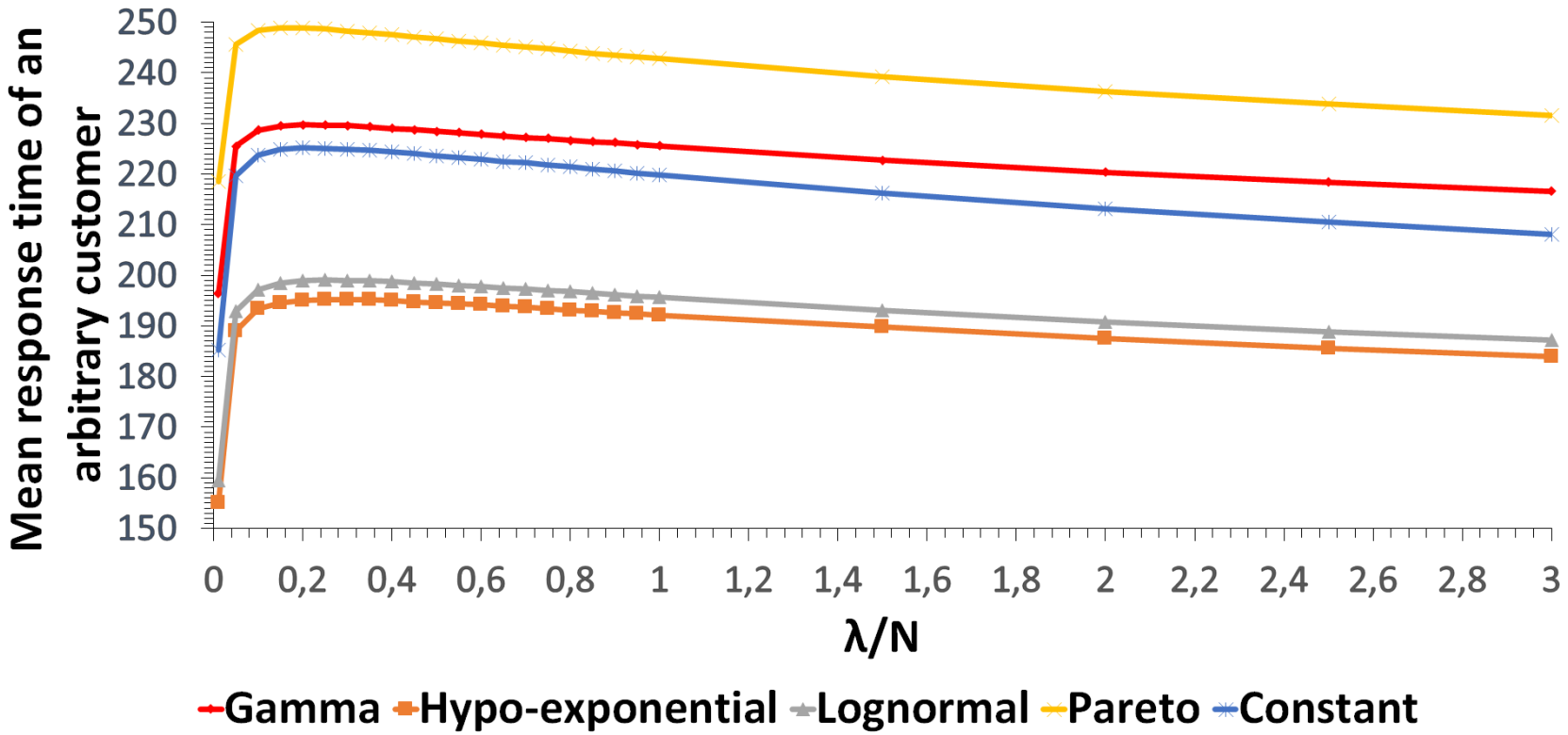

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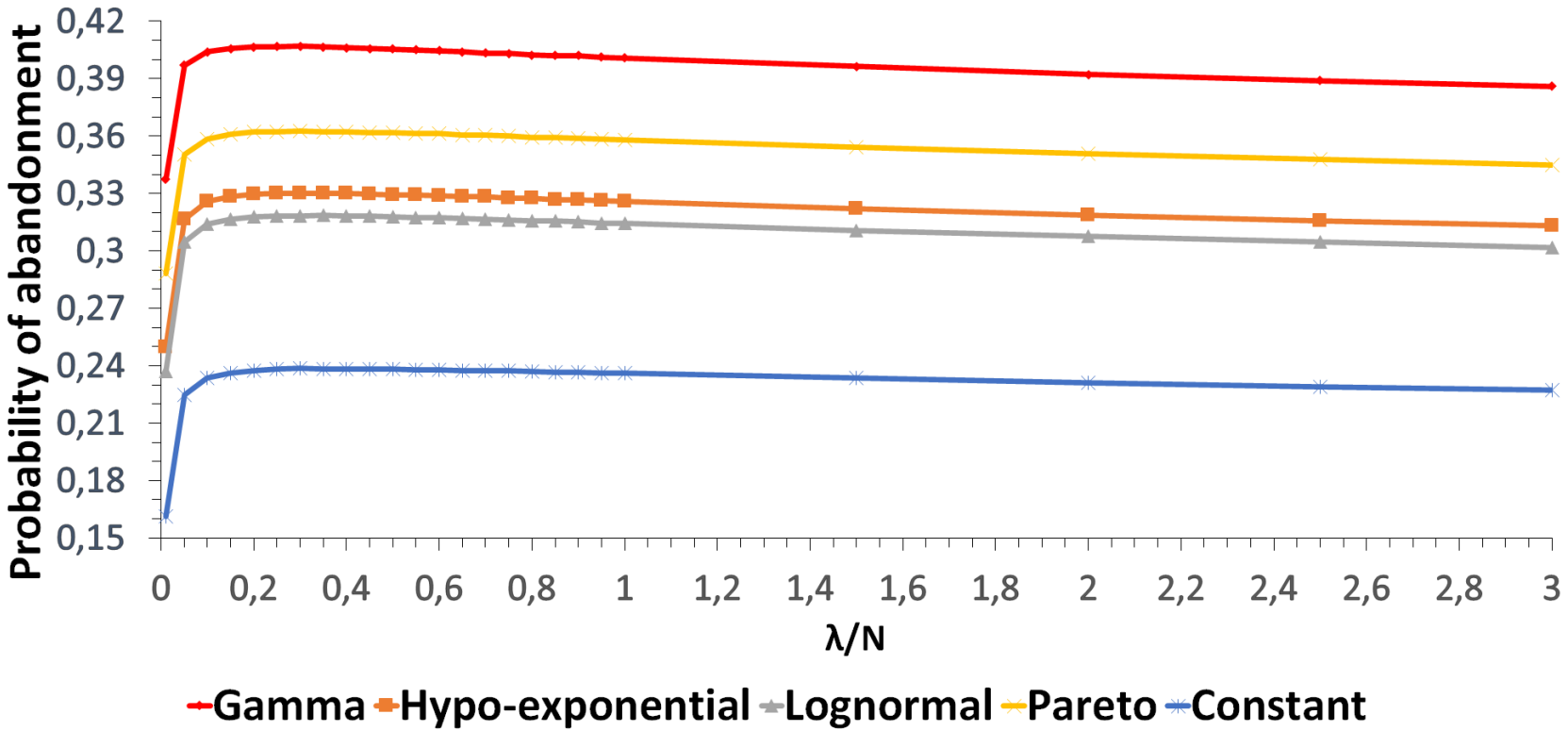
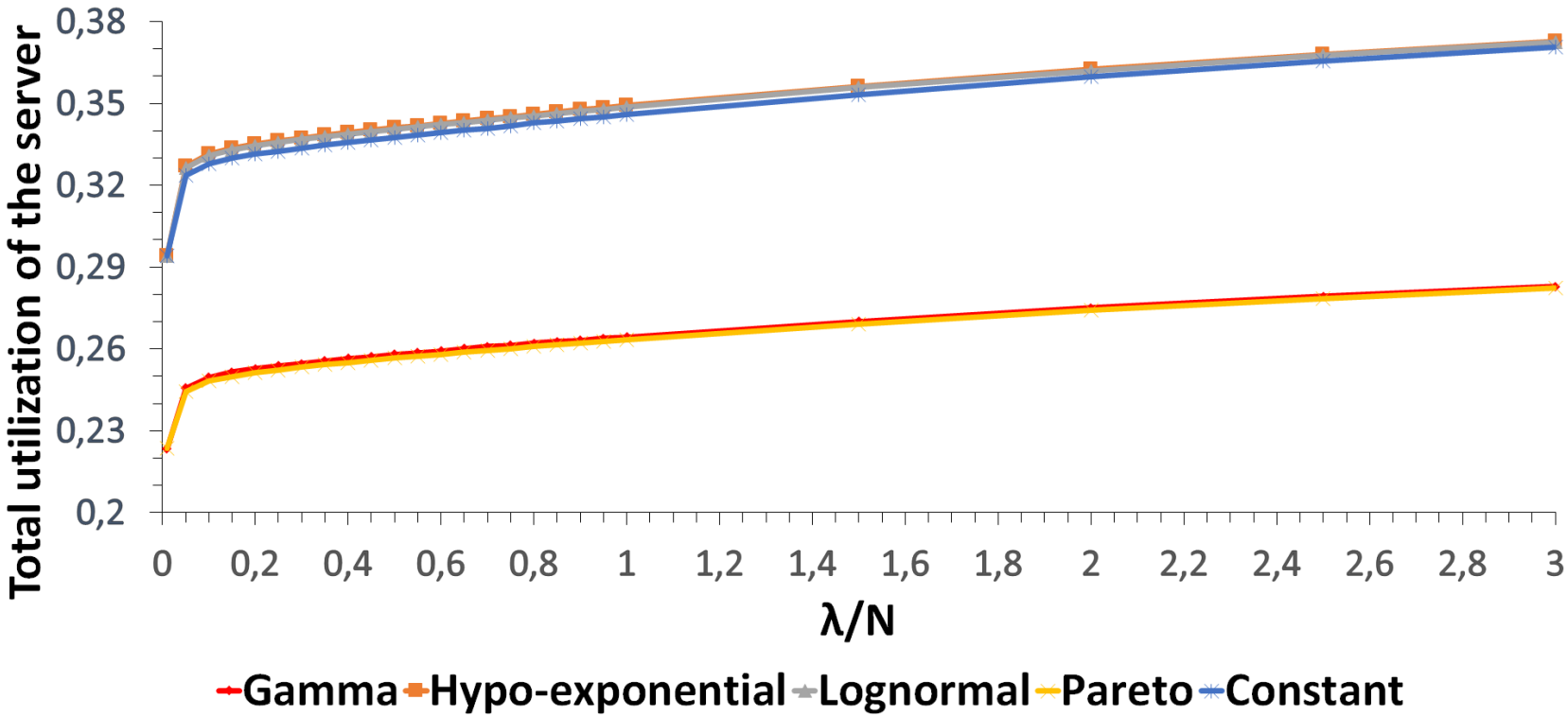


Probability of abandonment of a customer vs. arrival intensity



Total utilization of the server vs. arrival intensity

Conclusions

1. Finite-source retrial queueing system with collisions and impatience
2. Different solution approaches
3. Graphical illustrations, comparisons
4. Future plans

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