NEW SZÉCHENYI PLAN

FIRST – Future Internet Research, Services and Technology

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A queueing model to study the effect of service network breakdown in a CogInfoCom system

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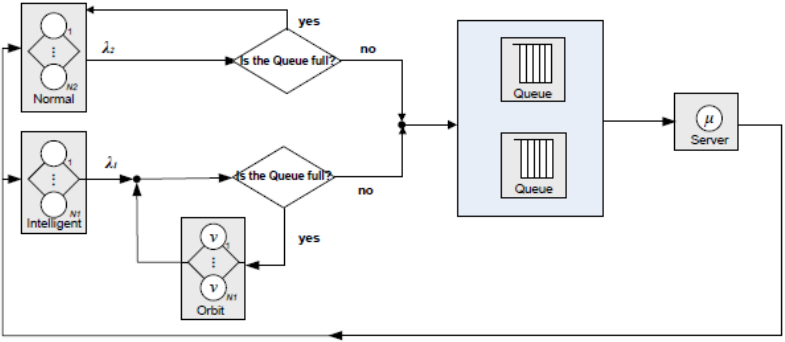
Outline

Finite-source retrial queueing system



Retrial queueing system

Mathematical model

- $y(t)$ is the operation level of the Server. $y(t)$=1 if the Server is in fully operation case, $y(t)$=2 if the Server is in limited operation case, and $y(t)$=3 if the Server is broken at time t,
- $c(t) = 0$ if the server is idle, $c(t) = 1$ if the server is busy with a request coming from the "Intelligent" class, $c(t) = 2$ when the server is busy with a request coming from the "Normal" class,
- $q_1(t)$ denotes the number of requests in the queue for requests from "Intelligent" class at time t,
- $q_2(t)$ denotes the number of requests in the queue for requests from "Normal" class at time t,
- $o(t)$ is the number of waiting jobs in the orbit at time t.

The system state at time t can be described by the following continuous time Markov chain with finite state space

$$X(t) = (y(t); c(t); q_1(t); q_2(t); o(t))$$

Let us denote the steady-state distribution by

$$P(y, c, q_1, q_2, o) = \lim_{t \to \infty} P(y(t) = y; c(t) = c; \\ q_1(t) = q_1; q_2(t) = q_2; o(t) = o)$$

Performance measures

Once we have obtained these limiting probabilities the **main system performance measures** can be derived in the following way.

1. **Utilization of the Server**

$$U_S = \sum_{y=1}^{2} \sum_{c=1}^{2} \sum_{q_1=0}^{B} \sum_{q_2=0}^{B-q_1} \sum_{o=0}^{N1-q_1} P(y, c, q_1, q_2, o)$$

2. **Availability of the Server**

$$A_S = \sum_{y=1}^{2} \sum_{c=0}^{2} \sum_{q_1=0}^{B} \sum_{q_2=0}^{B-q_1} \sum_{o=0}^{N1-q_1} P(y, c, q_1, q_2, o)$$

3. **Average number of requests in the orbit**

$$\overline{O} = E(o(t)) =$$
$$= \sum_{y=1}^{3} \sum_{c=0}^{2} \sum_{q_1=0}^{B} \sum_{q_2=0}^{B-q_1} \sum_{o=0}^{N1-q_1} oP(y, c, q_1, q_2, o)$$

4. **Average number of requests in the queue**

$$\overline{Q} = E(q_1(t) + q_2(t)) =$$
$$= \sum_{y=1}^{3} \sum_{c=0}^{2} \sum_{q_1=0}^{B} \sum_{q_2=0}^{B-q_1} \sum_{o=0}^{N1-q_1} q_1 P(y, c, q_1, q_2, o) +$$
$$+ \sum_{y=1}^{3} \sum_{c=0}^{2} \sum_{q_1=0}^{B} \sum_{q_2=0}^{B-q_1} \sum_{o=0}^{N1-q_1} q_2 P(y, c, q_1, q_2, o)$$

5. **Average number of requests in the network**

$$\overline{M} = \overline{O} + \overline{Q} + \\ + \sum_{y=1}^{3} \sum_{c=1}^{2} \sum_{q_1=0}^{B} \sum_{q_2=0}^{B-q_1} \sum_{o=0}^{N1-q_1} P(y, c, q_1, q_2, o)$$

6. **Average number of Intelligent entities in the communication network**

$$\overline{N1} = E(q_1(t)) + E(o(t)) + \sum_{y=1}^{3}\sum_{q_1=0}^{B}\sum_{q_2=0}^{B-q_1}\sum_{o=0}^{N1-q_1} P(y,1,q_1,q_2,o)$$

7. **Average number of active Intelligent entities in the source**

$$\overline{\Lambda_1} = N1 - \overline{N1}$$

8. **Average number of Normal entities in the communication network**

$$\overline{N2} = E(q_2(t)) + \sum_{y=1}^{3} \sum_{q_1=0}^{B} \sum_{q_2=0}^{B-q_1} \sum_{o=0}^{N1-q_1} P(y,2,q_1,q_2,o)$$

9. **Average number of active Normal entities in source**

$$\overline{\Lambda_2} = N2 - \overline{N2}$$

10. **Average generation rate of Intelligent entities**

$$\overline{\lambda_1} = \lambda_1 \overline{\Lambda_1}$$

11. **Mean response time for Intelligent entities**

$$\overline{T_1} = \frac{\overline{N1}}{\overline{\lambda_1}}$$

12. **Mean waiting time for Intelligent entities**

$$\overline{W_1} = \frac{\overline{O} + E(q_1(t))}{\overline{\lambda_1}}$$

13. **Mean wasted time for Intelligent entities**

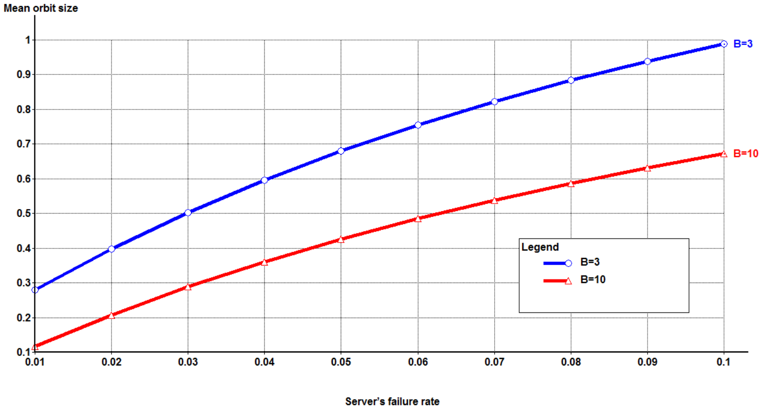
$$\overline{Wasted_1} = \overline{T_1} - \overline{W_1} - \frac{1}{\mu_1}$$

Numerical examples by the help of MOSEL

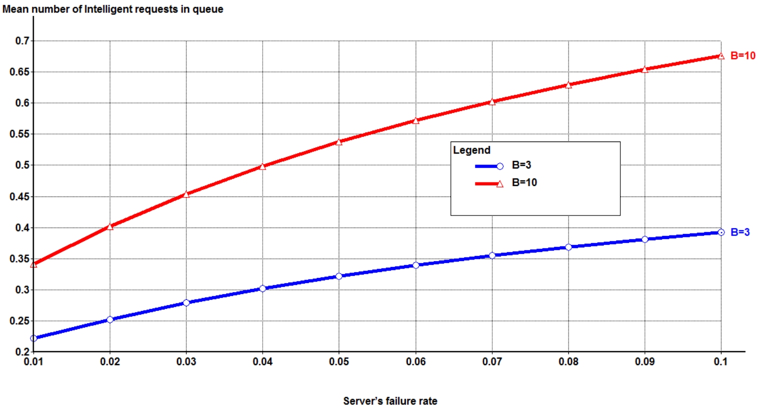
MOSEL (MOdeling, Specification and Evaluation Language) developed at the University of Erlangen, Germany, is used to formulate and solved the problem.

Input parameters

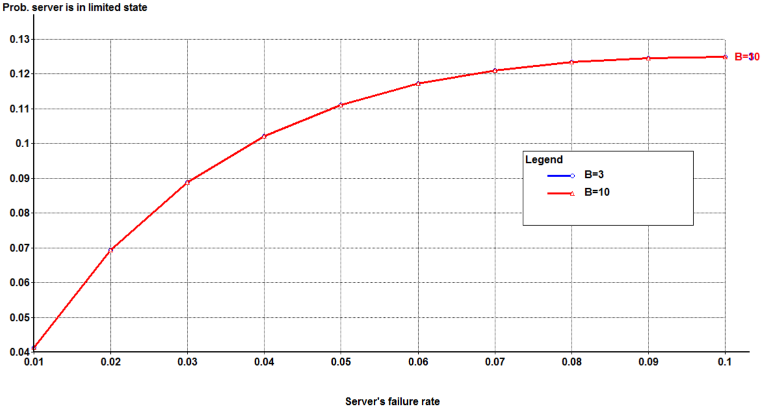
Parameter	Symbol	Value
Overall generation rate	λ	1.5
Intelligent generation rate	$\lambda_1 = \frac{2}{10}\lambda$	0.3
Normal generation rate	$\lambda_2 = \frac{8}{10}\lambda$	1.2
Number of Intelligent entities	$N1$	3
Number of Normal entities	$N2$	50
Retrial rate	ν	4
Service rate	μ_1	20
Service rate in limited state	μ_2	5
Server's failure rate	δ	[0.01..0.1]
Server's repair rate	$\beta_2 = \beta_3$	0.1
Prob. server‘s state changes from level 1 to 2	p	0.5
Buffer size	B	3; 10



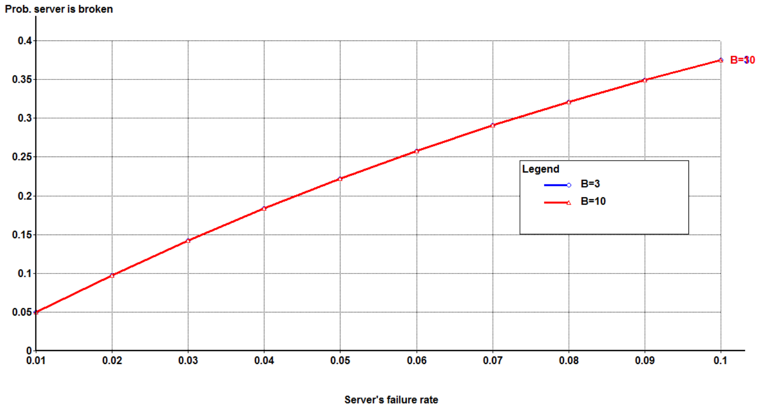
Mean orbit size vs Server's failure rate



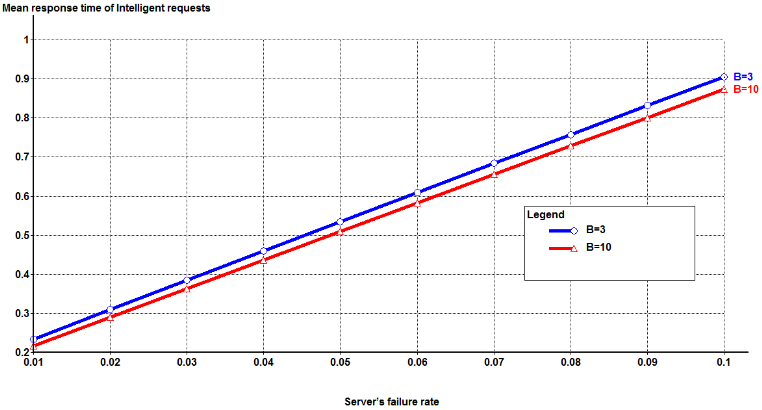
Mean queue length for Intelligent entities vs Server's failure rate



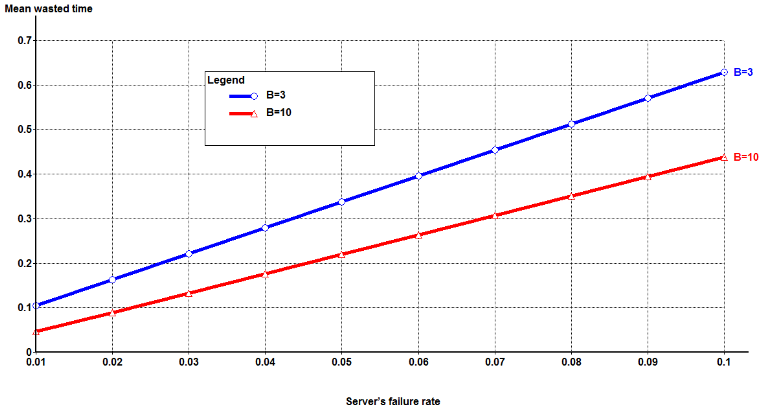
Probability that the server is in limited state vs Server's failure rate



Probability that the server is broken vs Server's failure rate



Mean response time of Intelligent entities vs Server's failure rate



Mean wasted time of Intelligent entities vs Server's failure rate

Conclusions

1. Finite-source retrial queueing system with two types of requests
2. Markovian model via MOSEL
3. Effect of failure rate of the server on performance measures in steady-state
4. Graphical illustrations, case studies

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Thank You for Your Attention