Structured Markov Chains Arising from Finite-Source Retrial Queues with Orbital Search

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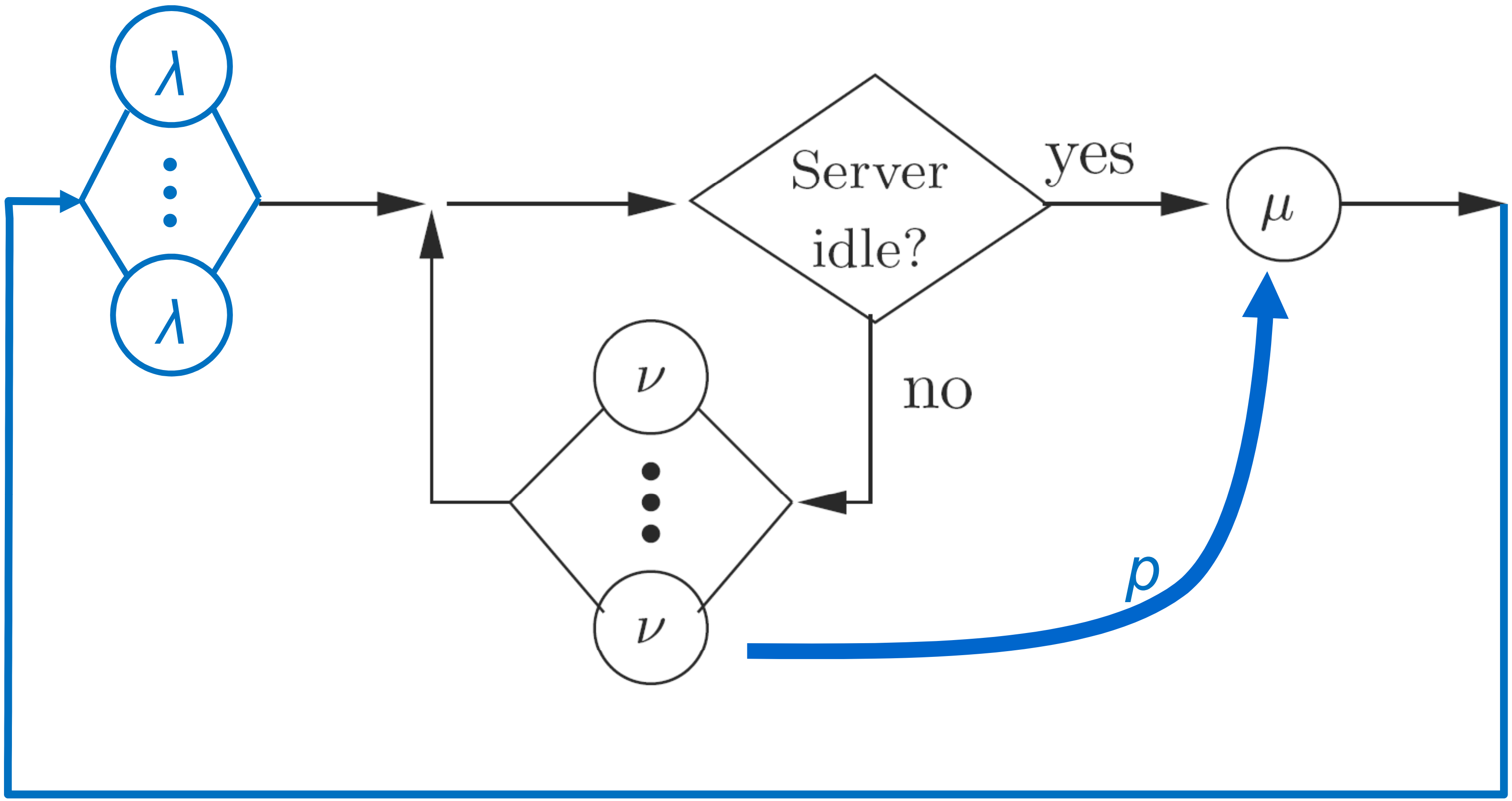
Overview

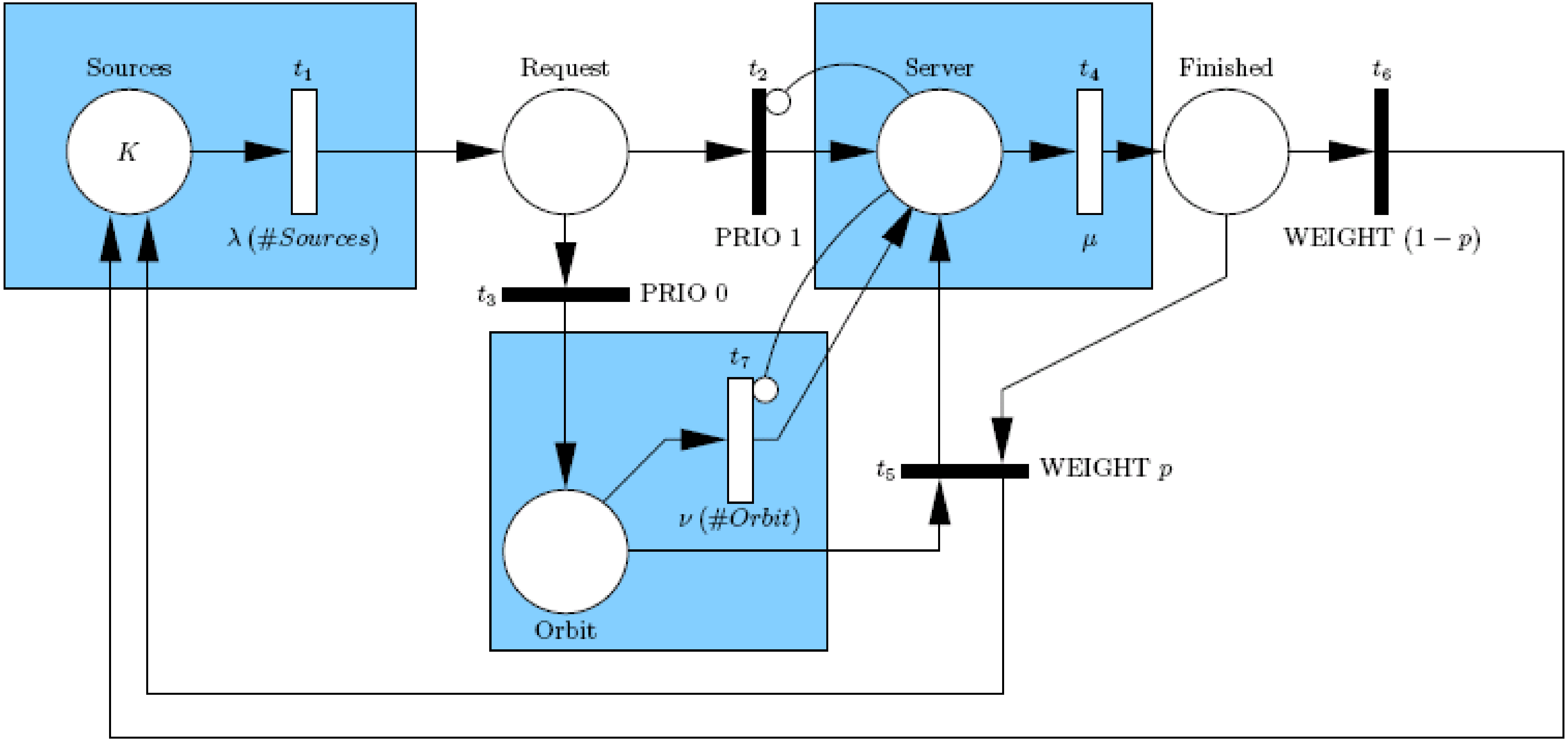
1. Basic Retrial Queueing System
2. Model Variation
3. Preliminary Numerical Results
4. Underlying Structured Markov Chain
5. Discussion of Future Steps

1. Retrial Queueing Systems



2. Model Variation

- Finite source & orbital search
- Stochastic Petri net model:



3. Preliminary Numerical Results

- Example parameters:

Parameter	Symbol	Value/Range
Number of sources	K	3
Service rate	μ	1
Generation rate	λ	$0.0001 \ldots 1$
Retrial rate	ν	0.001, 0.005
Search probability	p	1E-8...0.99

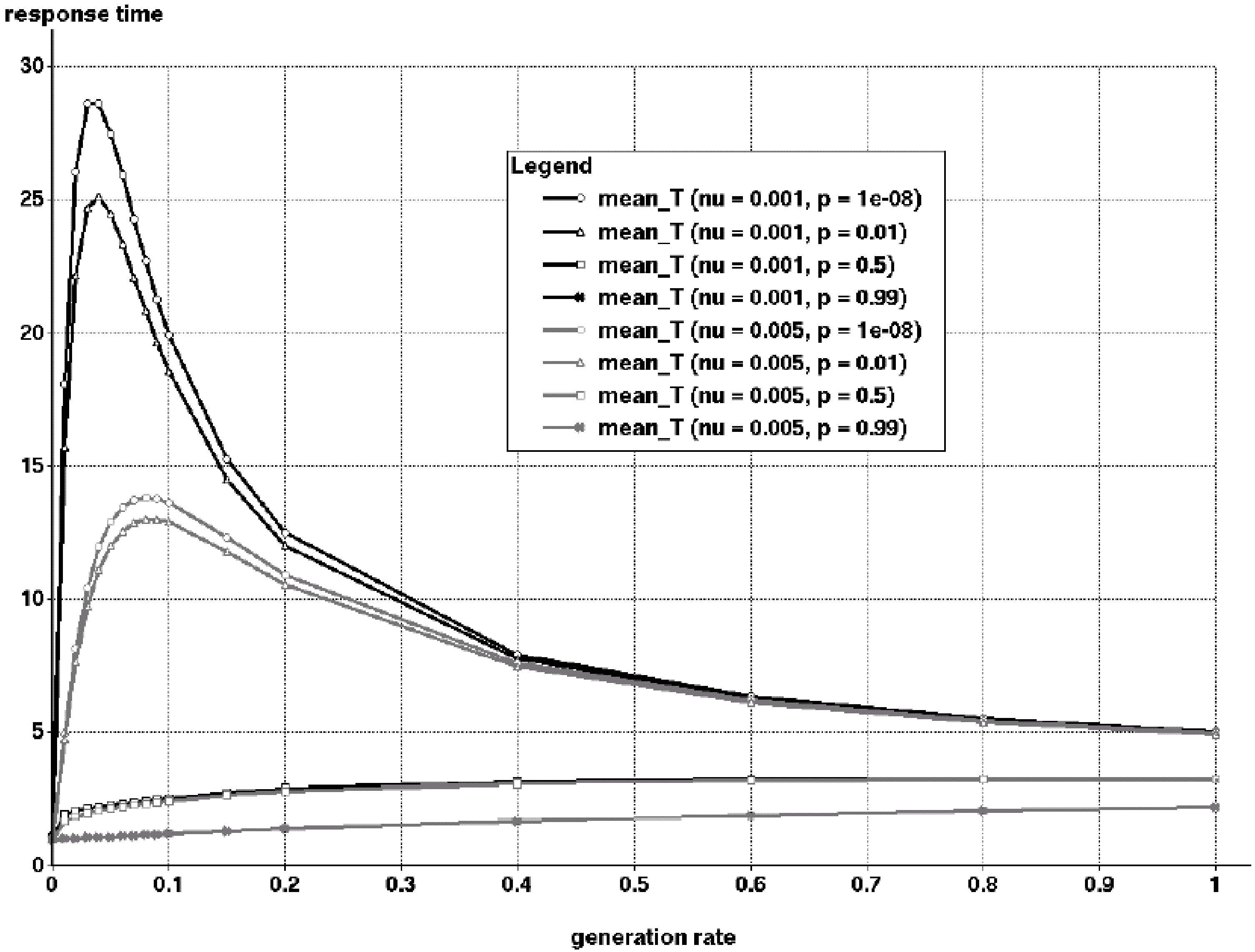
3. Preliminary Numerical Results

- Model specification and evaluation using MOSEL-2

```
/*** CONSTANTS AND PARAMETERS ****************************************************/
CONST     K      := 3;                                          // population size
CONST     mu     := 1;                                          // service rate
PARAMETER lambda := 0.0001, 0.001, 0.01 .. 0.1 STEP 0.01;       // request gen. rate
PARAMETER nu     := 0.001, 0.0025, 0.005;                       // retrial rate
PARAMETER p      := 0.00000001, 0.00001, 0.01;                  // search probability

/*** NODES ***********************************************************************/
NODE Sources[K]   := K;                                         // sources
NODE Request[1]   := 0;                                         // generated call
NODE Server[1]    := 0;                                         // the server
NODE Orbit[K]     := 0;                                         // retrying customers
NODE Finished[1]  := 0;                                         // serviced customer

/*** RULES ***********************************************************************/
FROM Sources   TO Request  RATE Sources*lambda;                 // primary requests
FROM Request   TO Server   PRIO 1;                              // to server if idle
FROM Request   TO Orbit    PRIO 0;                              // to orbit if busy
FROM Orbit     TO Server   RATE Orbit*nu;                       // retrials
FROM Server    TO Finished RATE mu;                             // service
FROM Finished  TO Sources  WEIGHT 1-p;                          // without polling
FROM Finished, Orbit TO Server, Sources WEIGHT p;               // with polling

/*** RESULTS *********************************************************************/
PRINT util_server  := UTIL(Server);                             // server utilization
PRINT mean_lambda  := MEAN(Sources)*lambda;                     // mean arrival rate
PRINT mean_T       := (K-MEAN(Sources))/mean_lambda;            // mean response time
PRINT mean_orbit   := MEAN(Orbit);                              // mean cust. in orbit
```

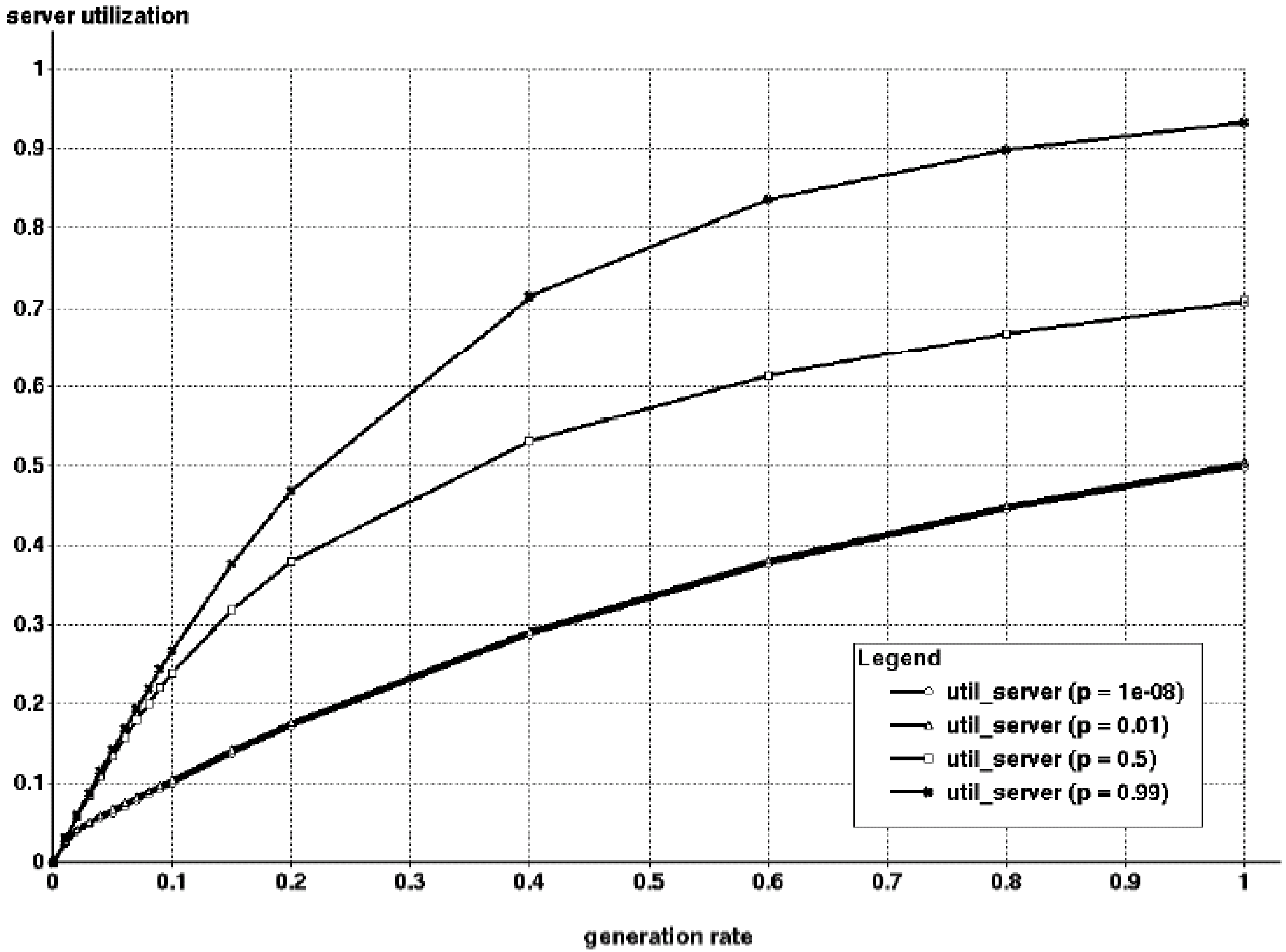
3. Preliminary Numerical Results

- Mean response time:



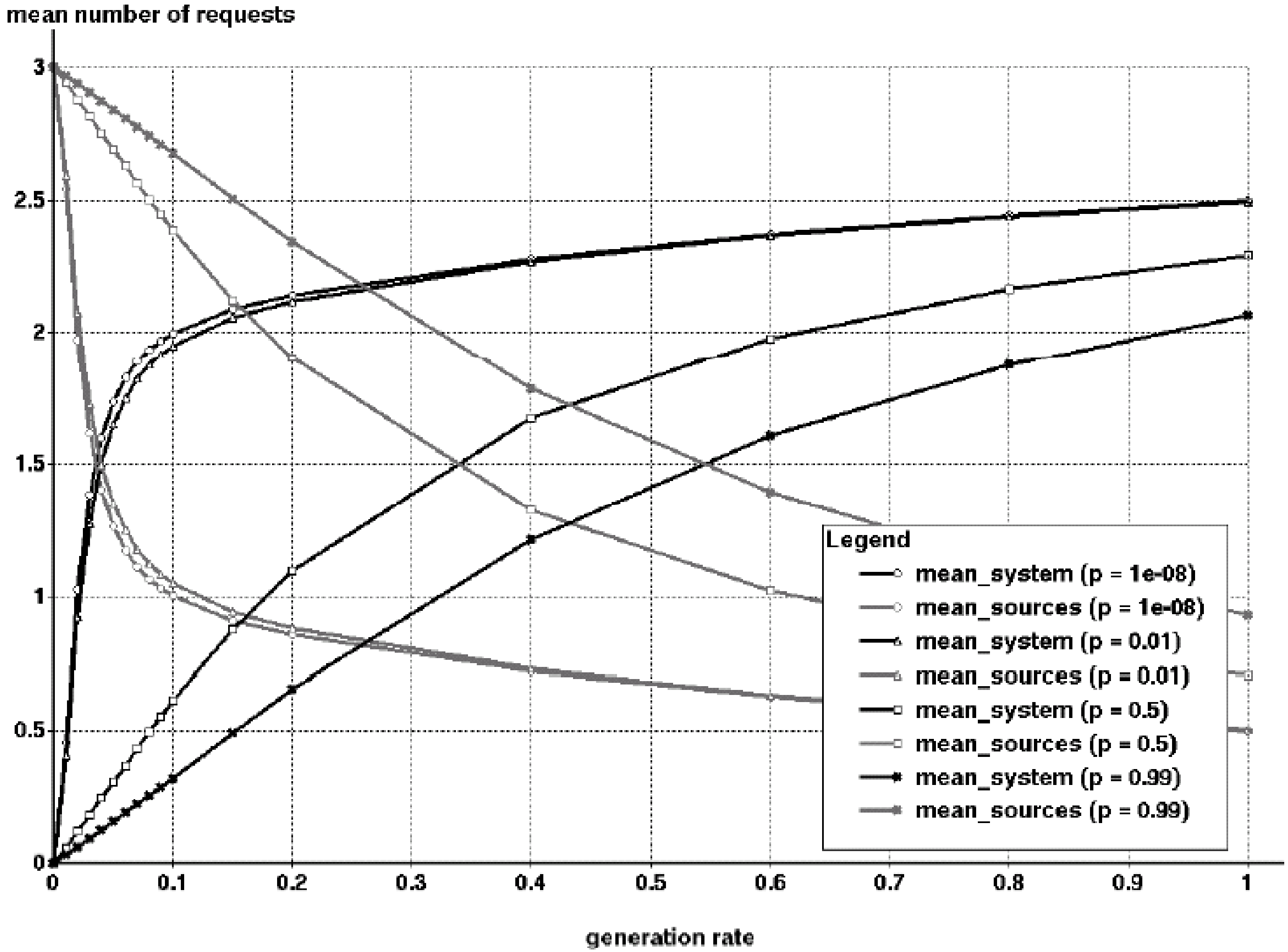
3. Preliminary Numerical Results

- Utilization:



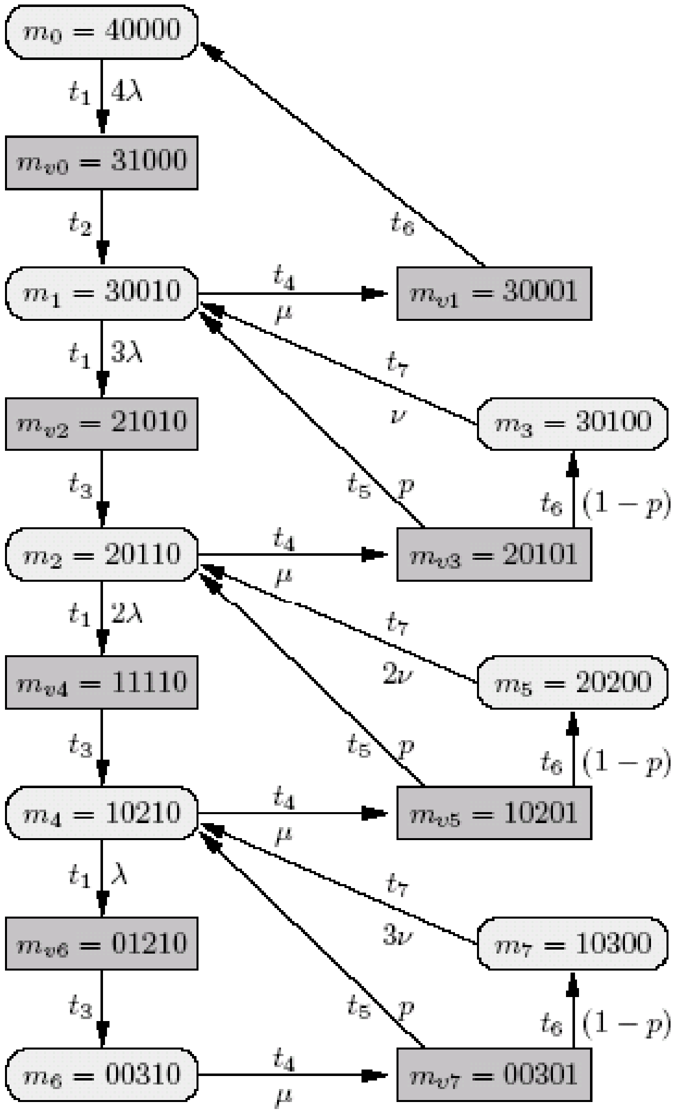
3. Preliminary Numerical Results

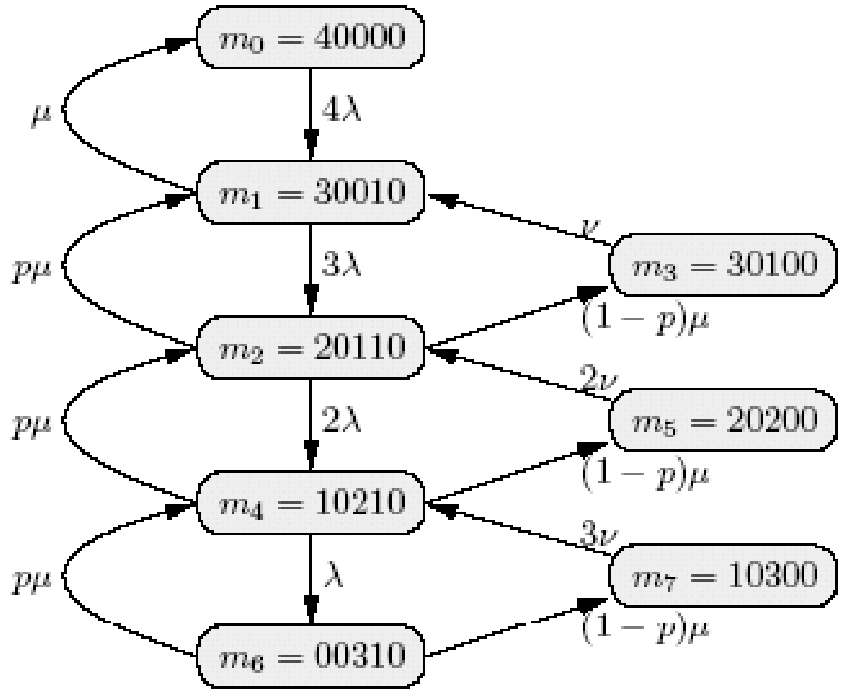
- Active sources and requests in system:



4. Underlying Struct. Markov Chain

- Extended and reduced reachability graph of GSPN (K=4):

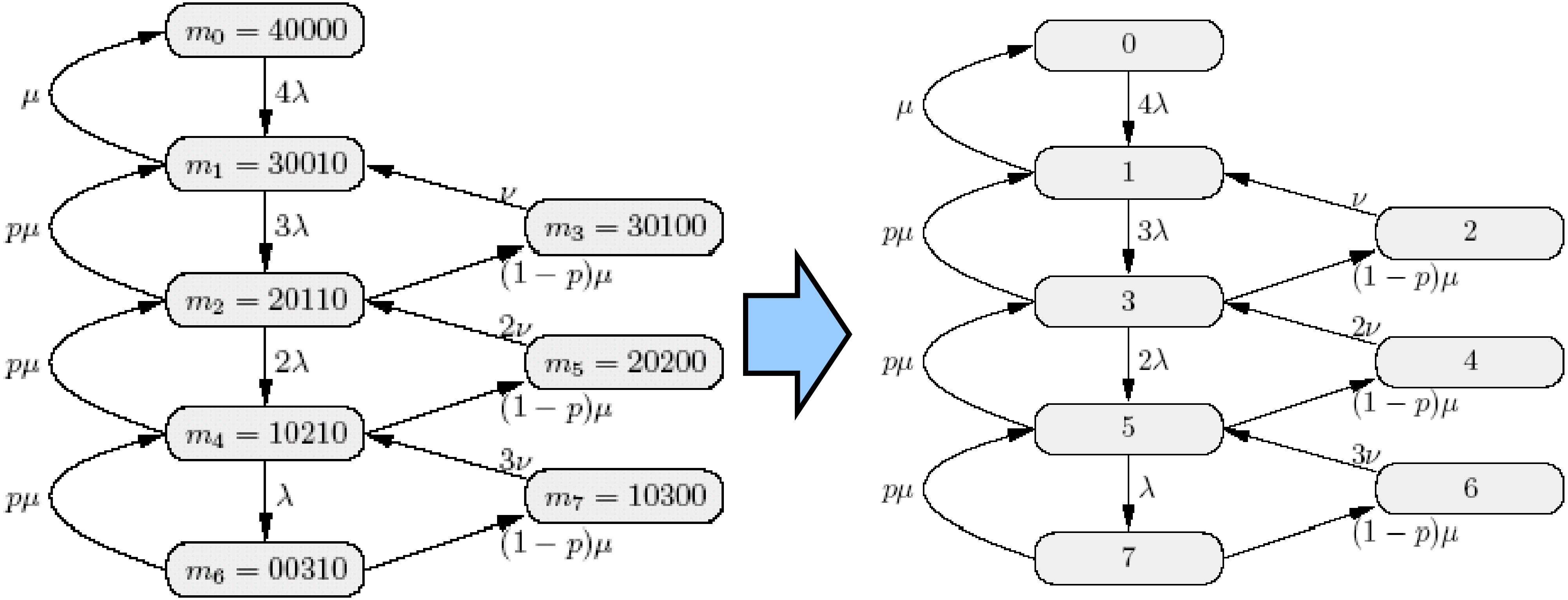




4. Underlying Struct. Markov Chain

- Underlying continuous-time Markov chain:

$$X(t) = (C(t), N(t))$$



(Label of state *s*: *ID(s)* = *2N(s)+C(s)*)

4. Underlying Struct. Markov Chain

- CTMC's infinitesimal generator matrix:

$$Q = \begin{pmatrix} -4\lambda & 4\lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ \mu & -\mu-3\lambda & 0 & 3\lambda & 0 & 0 & 0 & 0 \\ 0 & \nu & -\nu & 0 & 0 & 0 & 0 & 0 \\ 0 & p\mu & (1-p)\mu & -\mu-2\lambda & 0 & 2\lambda & 0 & 0 \\ 0 & 0 & 0 & 2\nu & -2\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & p\mu & (1-p)\mu & -\mu-\lambda & 0 & \lambda \\ 0 & 0 & 0 & 0 & 0 & 3\nu & -3\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & p\mu & (1-p)\mu & -\mu \end{pmatrix}$$

$$\mathbf{Q} = \begin{pmatrix} \mathbf{A}_{0,0} & \mathbf{A}_{0,1} & \mathbf{0} & \mathbf{0} \\ \mathbf{A}_{1,0} & \mathbf{A}_{1,1} & \mathbf{A}_{1,2} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{2,1} & \mathbf{A}_{2,2} & \mathbf{A}_{2,3} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_{3,2} & \mathbf{A}_{3,3} \end{pmatrix}$$

4. Underlying Struct. Markov Chain

- CTMC's infinitesimal generator matrix:

$$Q = \begin{pmatrix} \mathbf{A}_{0,0} & \mathbf{A}_{0,1} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ \mathbf{A}_{1,0} & \mathbf{A}_{1,1} & \mathbf{A}_{1,2} & 0 & \cdots & 0 & 0 & 0 & 0 \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & \cdots & 0 & \mathbf{A}_{i,i-1} & \mathbf{A}_{i,i} & \mathbf{A}_{i,i+1} & 0 & \cdots & 0 \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & 0 & \cdots & 0 & \mathbf{A}_{K-2,K-3} & \mathbf{A}_{K-2,K-2} & \mathbf{A}_{K-2,K-1} \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \mathbf{A}_{K-1,K-2} & \mathbf{A}_{K-1,K-1} \end{pmatrix}$$

$$\mathbf{A}_{0,0} = \begin{pmatrix} -K\lambda & K\lambda \\ \mu & -(K-1)\lambda \end{pmatrix}, \quad \mathbf{A}_{0,1} = \begin{pmatrix} 0 & 0 \\ 0 & (K-1)\lambda \end{pmatrix},$$

$$\mathbf{A}_{i,i-1} = \begin{pmatrix} 0 & i\nu \\ 0 & p\mu \end{pmatrix}, \quad \text{for } 0 < i < K,$$

$$\mathbf{A}_{i,i} = \begin{pmatrix} -i\nu & 0 \\ (1-p)\mu & -(\mu + (K-1-i)\lambda) \end{pmatrix}, \quad \text{for } 0 < i < K,$$

$$\mathbf{A}_{i,i+1} = \begin{pmatrix} 0 & 0 \\ 0 & (K-1-i)\lambda \end{pmatrix}, \quad \text{for } 0 < i < K-1.$$

5. Discussion of Future Steps

$$\boldsymbol{\pi}\mathbf{Q} = \mathbf{0}, \quad \boldsymbol{\pi}\mathbf{1} = 1 .$$

How can I get the vector of steady-state probabilities explicitly in a (preferably compact and possibly approximate) closed form?!

I'm grateful for any useful pointer!