NEW SZÉCHENYI PLAN

FIRST – Future Internet Research, Services and Technology

TÁMOP-4.2.2.C-11/1/KONV-2012-0001

Tool supported analysis of queueing systems with Future Internet applications

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HUNGARY'S RENEWAL

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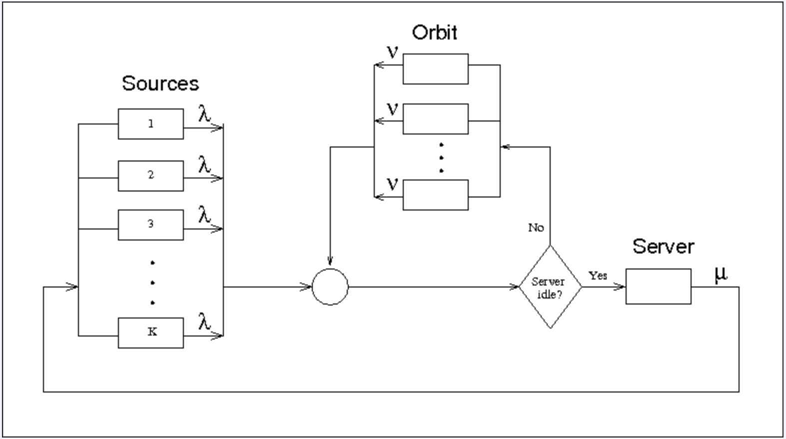
Outline

Modelling tools

- University of Dortmund: *HIT, HiQPN, APNN*
  http://ls4-www.infromatik.uni-dortmund.de/tools.html/
- University of Illinois at Urbana-Champaign: *MÖBIUS*
  http://www.mobius.uiuc.edu/
- University of Erlangen: *PEPSY, MOSEL*
  http://www4.informatik.uni-erlangen.de/Projects/MOSEL/
- University of Oxford: *PRISM*
  http://www.prismmodelchecker.org/

Finite-source retrial queueing system



Retrial queueing system

Mathematical model

The system state at time t can be described with the process

$$X(t) = (Y(t); C(t); N(t))$$

where $Y(t) = 0$ if the server is up, $Y(t) = 1$ if the server is failed,

$C(t) = 0$ if the server is idle, $C(t) = 1$ if the server is busy,

$N(t)$ is the number of sources of repeated calls at time t.

We define the stationary probabilities:

$$P(q;r;j) = \lim_{t\to\infty} P(Y(t) = q, C(t) = r, N(t) = j)$$

$$q = 0,1, \;\; r = 0,1, \;\; j = 0,...,K^*,$$

$$\text{where } K^* = \begin{cases} K-1 & \text{for blocked case,} \\ K-r & \text{for unblocked case.} \end{cases}$$

Once we have obtained these limiting probabilities the **main system performance measures** can be derived in the following way.

1. **Utilization of the server**

$$U_S = \sum_{j=0}^{K-1} P(0,1,j)$$

2. **Utilization of the repairman**

$$U_R = \sum_{q=0}^{1} \sum_{j=0}^{K^*} P(1,q,j)$$

3. **Availability of the server**

$$A_S = \sum_{q=0}^{1} \sum_{j=0}^{K^*} P(0,q,j) = 1 - U_R$$

4. **The mean number of calls staying in the orbit or in service**

$$M = E[N(\infty)+C(\infty)] = \sum_{q=0}^{1}\sum_{r=0}^{1}\sum_{j=0}^{K^*} jP(q,r,j) + \sum_{q=0}^{1}\sum_{j=0}^{K-1} P(q,1,j).$$

5. **Utilization of the sources**

$$U_{SO} = \begin{cases} \frac{E[K-C(\infty)-N(\infty);Y(\infty)=0]}{K} & \text{for blocked case,} \\ \frac{K-M}{K} & \text{for unblocked case.} \end{cases}$$

6. **Overall utilization**

$$U_O = U_S + KU_{SO} + U_R.$$

7. **The mean rate of generation of primary calls**

$$\overline{\lambda} = \begin{cases} \lambda E[K - C(\infty) - N(\infty); Y(\infty) = 0] & \text{for blocked case,} \\ \lambda E[K - C(\infty) - N(\infty)] & \text{for unblocked case.} \end{cases}$$

8. **The mean response time**

$$E[T] = M/\overline{\lambda}$$

9. **The blocking probability of a primary call**

$$B = \begin{cases} \frac{\lambda E[K - C(\infty) - N(\infty); Y(\infty) = 0; C(\infty) = 1]}{\overline{\lambda}} & \text{for blocked case,} \\ \\ \frac{\lambda E[K - C(\infty) - N(\infty); C(\infty) = 1]}{\overline{\lambda}} & \text{for unblocked case.} \end{cases}$$

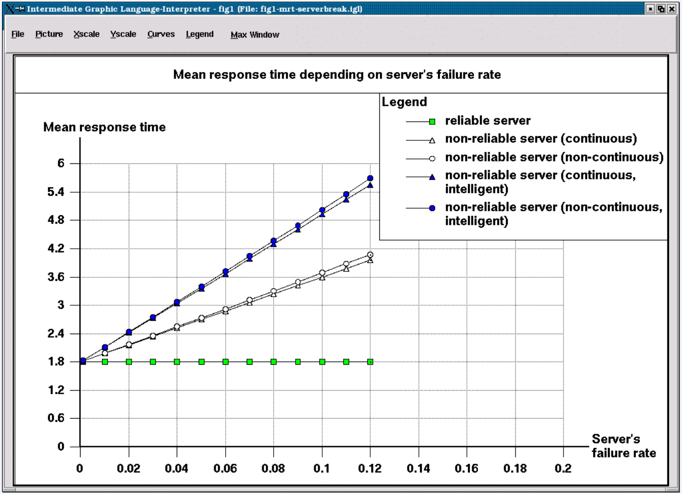
Evaluation Tool MOSEL

MOSEL (MOdeling, Specification and Evaluation Language) developed at the University of Erlangen, Germany, is used to formulate and solved the problem.

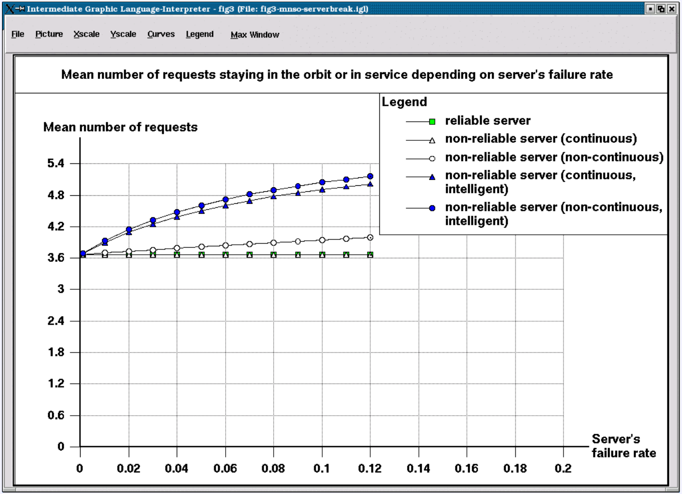
Case studies

	K	λ	μ	ν	δ, γ	τ
Figure 1	6	0.8	4	0.5	x axis	0.1
Figure 2	6	0.1	0.5	0.05	x axis	0.1
Figure 3	6	0.8	4	0.5	0.05	x axis
Figure 4	6	0.1	0.5	0.05	0.05	x axis

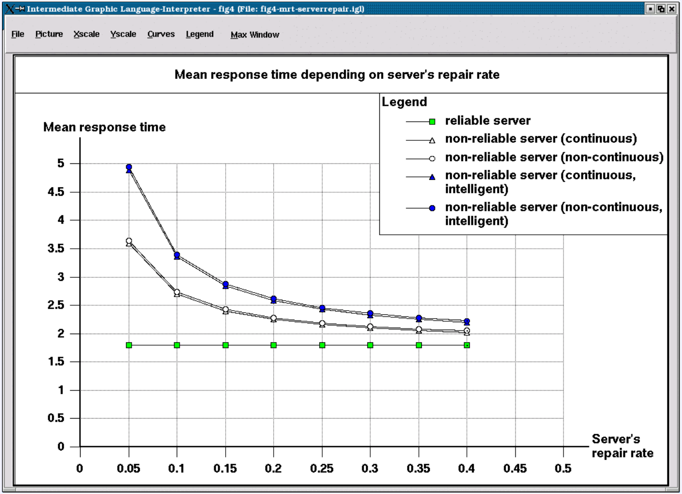
Input system parameters



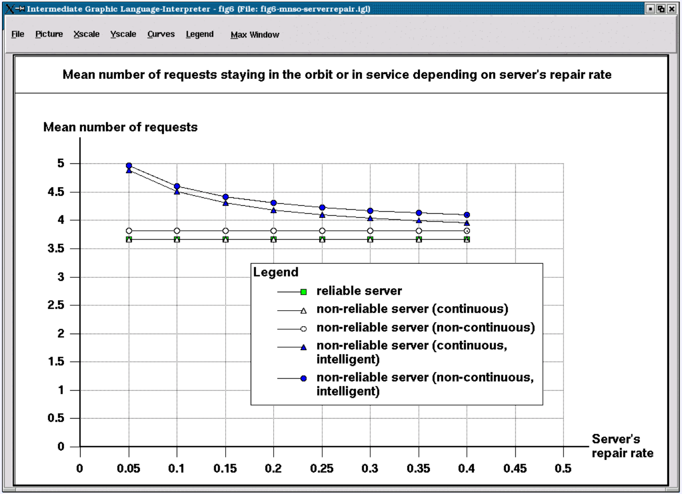
$E[T]$ versus server's failure rate



M versus server's failure rate



$E[T]$ versus server's repair rate

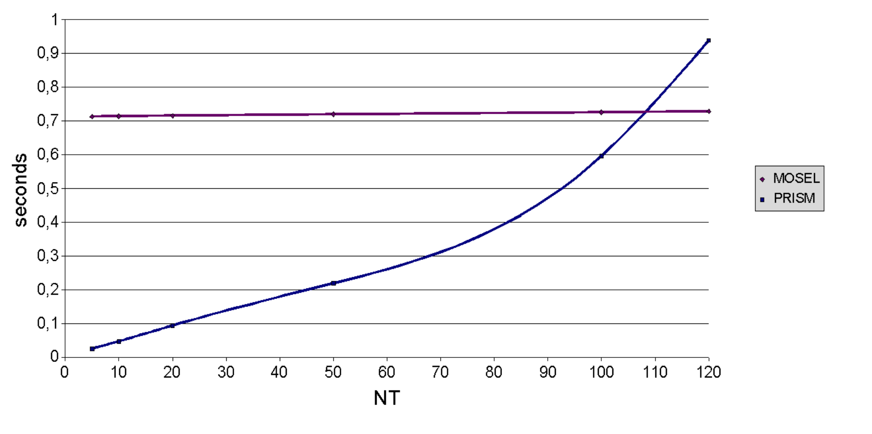


M versus server's repair rate

Benchmarks

MOSEL (MOdeling, Specification and Evaluation Language) developed at University of Erlangen, Germany,

PRISM (PRobabIliStic Model Checker) developed at University of Oxford, England.



Execution Time

NT	MOSEL	PRISM
5	0.712	0.025
10	0.713	0.047
20	0.715	0.094
50	0.719	0.219
100	0.725	0.596
120	0.728	0.938
150	-	1.550
200	-	2.377

Total execution times of MOSEL and PRISM in seconds

NT	Model const.	Model checking	Total
5	0.015	0.010	0.025
10	0.031	0.016	0.047
20	0.047	0.047	0.094
50	0.141	0.078	0.219
100	0.391	0.205	0.596
120	0.594	0.344	0.938
150	1.071	0.479	1.550
200	1.609	0.768	2.377

Execution times of PRISM in seconds

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Thank You for Your Attention