



# A Contribution to Modeling Two-Way Communication with Retrial Queueing Systems

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# Outline

- 1 Finite source retrial queueing system
- 2 Performance measures
- 3 Mathematical modeling by the help of MOSEL
- 4 Comparisons, examples
- 5 Bibliography

## Finite source retrial queueing system

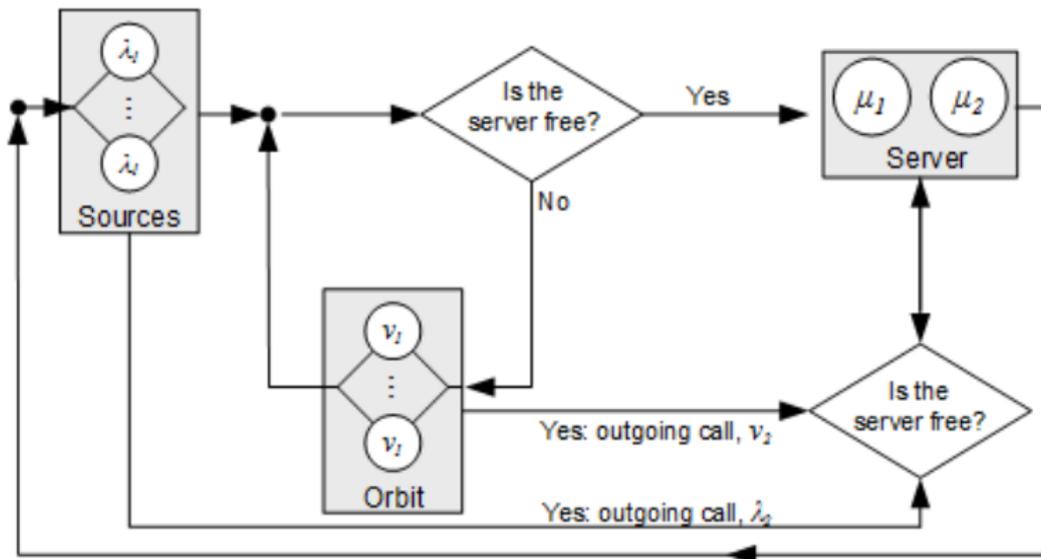


Figure 1: Finite source retrial queueing system with components

- **Case 1** The call came from the orbit, which means this call has an unserved incoming request. After the outgoing call is served, this incoming request remains unserved. This call is sent back to the orbit after the outgoing service is finished, thus the call will be able to retry its incoming request,
- **Case 2** As in the previous case, after the outgoing call is processed, it has an unserved incoming call. In this case the server is able to serve the incoming request immediately after the outgoing job was finished. That means a two-phase service. First the outgoing call is served, after the incoming one. When both of the service phases have been finished, the call is sent back to the source.

## Performance measures

- *Utilization of the server*
- *Mean number of customers in the system/orbit*
- *Mean response/waiting time of an incoming customer*

## 2-dimensional Markov chain

Let us define the state of the server at time  $t$  by  $S(t)$ , that is

$$S(t) = \begin{cases} 0, & \text{when the server is idle} \\ 1, & \text{when an incoming request is in service} \\ 2, & \text{when an outgoing request from source is in service} \\ 3, & \text{when an outgoing request from orbit is in service} \end{cases}$$

$O(t)$  = the number of customers in the orbit at time  $t$

$$p_{i,j} = \lim_{t \rightarrow \infty} P(S(t) = i, O(t) = j), i = 0, 1, 2, 3 \text{ and } j = 0, 1, \dots, N.$$

MOSEL ( MOdeling, Specification and Evaluation Language )  
solution

- *Utilization with respect to incoming requests*

$$U_1 = \sum_{o=0}^{N-1} P(1, o)$$

- *Utilization with respect to outgoing requests*

$$U_2 = \sum_{s=2}^3 \sum_{o=0}^{N-1} P(s, o)$$

- *Average number of jobs in the orbit*

$$\bar{O} = \sum_{s=0}^3 \sum_{o=0}^N oP(s, o)$$

- *Average number of active primary incoming customers*

$$\bar{M} = N - \bar{O} - U_1 - U_2$$

- *Average generation rate of primary incoming customers*

$$\bar{\lambda}_1 = \lambda_1 \bar{M}$$

- *Mean waiting time in the orbit of a primary incoming call by using [Little-formula](#)*

$$\bar{W} = \frac{\bar{O}}{\bar{\lambda}_1}$$

## Comparisons, examples

Table 1: Numerical values of model parameters

Examples											
No.	$N$	$\lambda_1$	$\lambda_2$	$\nu_1$	$\nu_2$	$\mu_1$	$\mu_2$	$C_W$	$C_1$	$C_2$	$C_P$
Fig. 2	10	0.1..5.1	0.2	0.1	0.2	1	1				
Fig. 3	10	0.1..5.1	0.2	0.1	0.2	1	1				
Fig. 4	10	0.1..5.1	0.2	0.1	0.2	1	1				
Fig. 5	10	0.1..5.1	0.2	0.1	0.2	1	1				
Fig. 6	10	0.1..5.1	0.2	0.1	0.2	1	1	100	10	10	1
Fig. 7	10	0.1..5.1	0.2	0.1	0.2	1	1	100	10	10	1

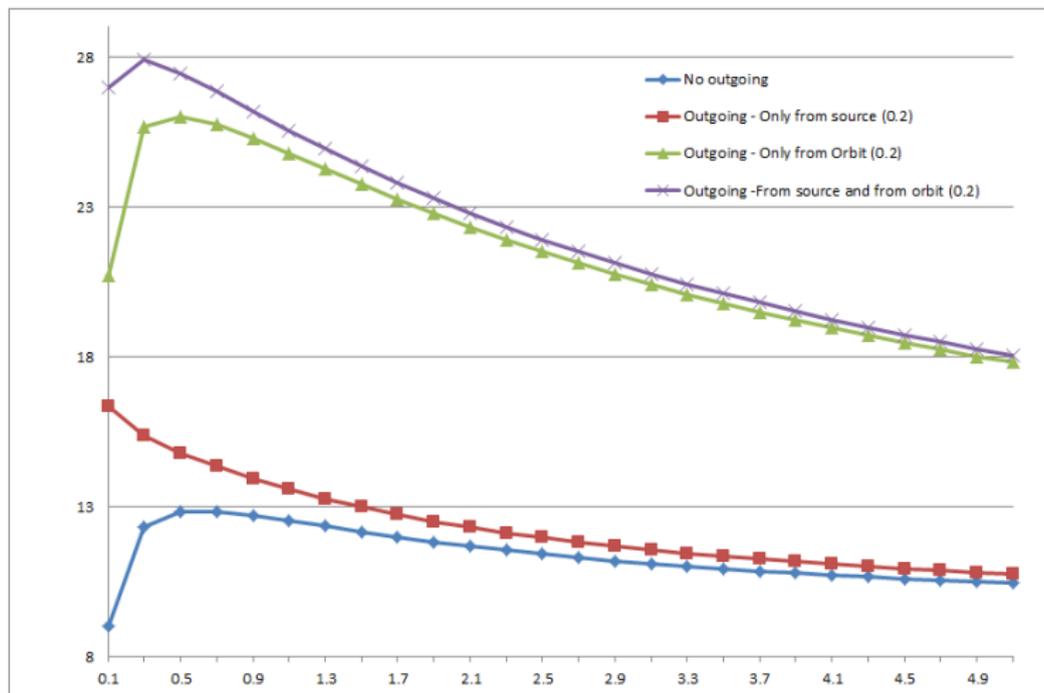


Figure 2: Mean waiting time (Case 1) vs.  $\lambda_1$

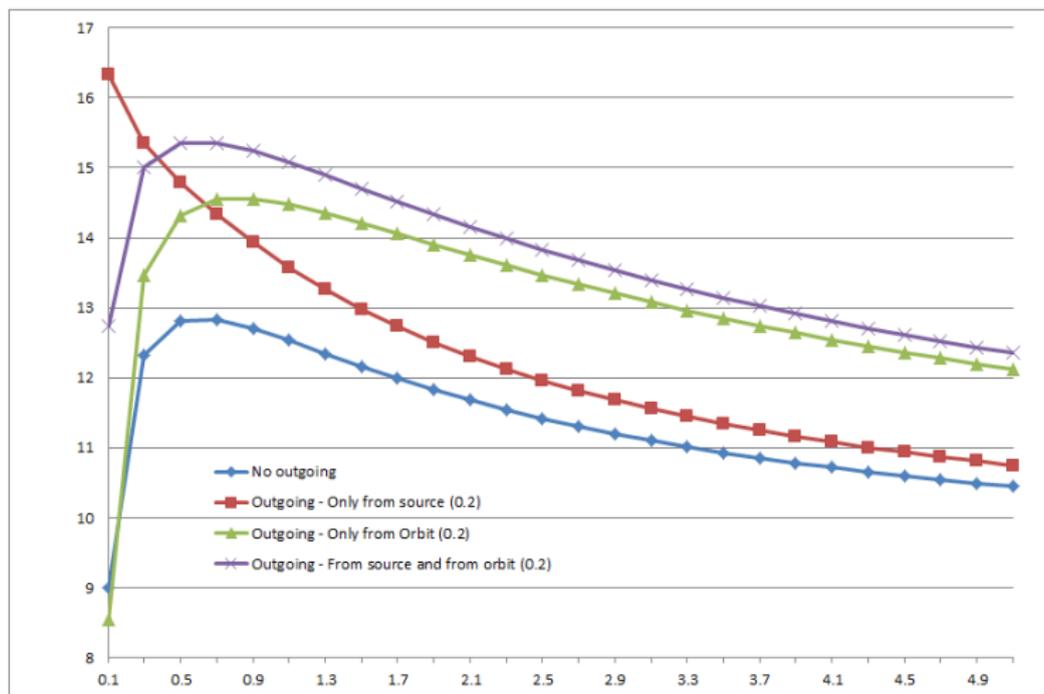


Figure 3: Mean waiting time (Case 2) vs.  $\lambda_1$

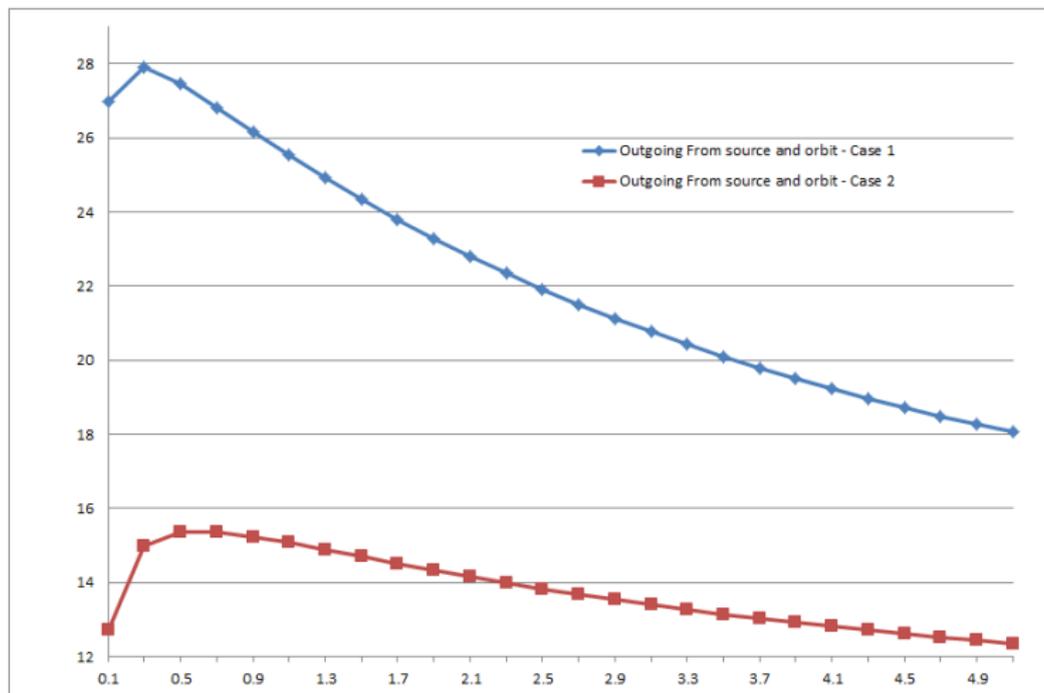


Figure 4: Mean waiting time vs.  $\lambda_1$

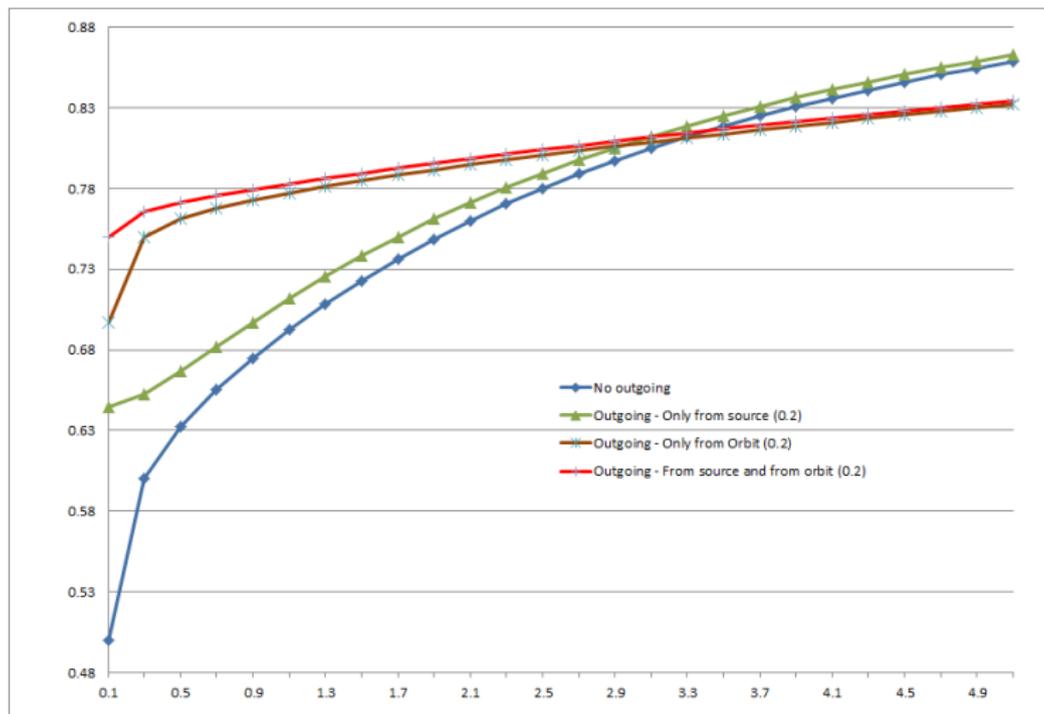


Figure 5: Utilization of the server in Case 1 vs.  $\lambda_1$

Fair treatment of the problem by introducing the average loss per long run unit time

$$E(L) = C_w(1 - U_1 - U_2) + C_1\mu_1U_1 + C_2\mu_2U_2 + C_P(E(O) + U_1 + U_2)$$

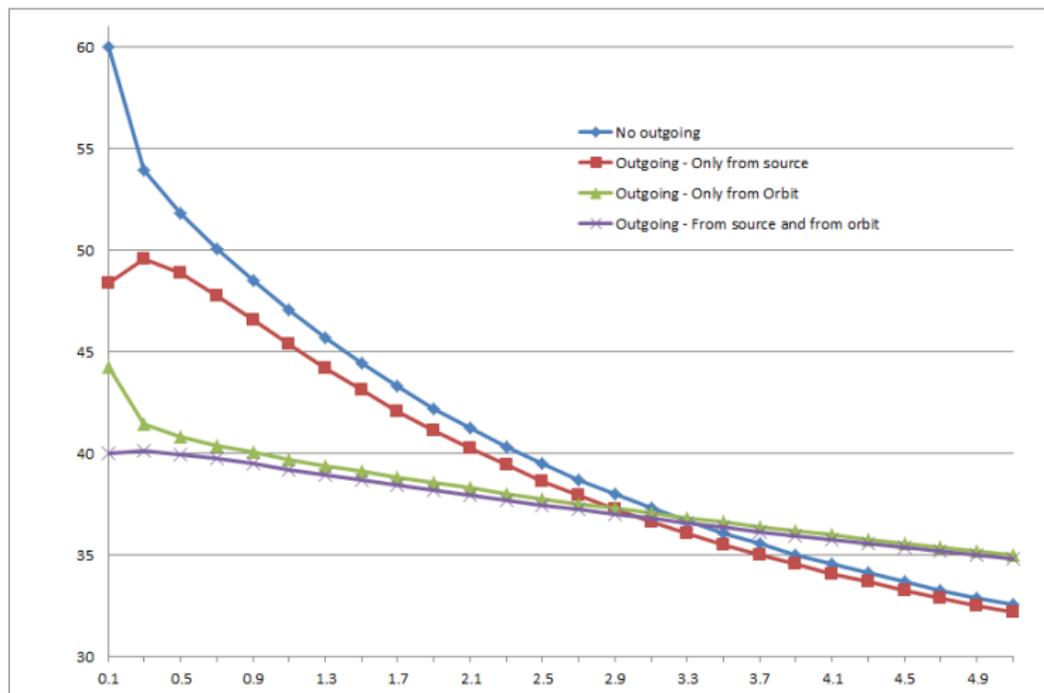


Figure 6: The loss function in Case 1 vs.  $\lambda_1$

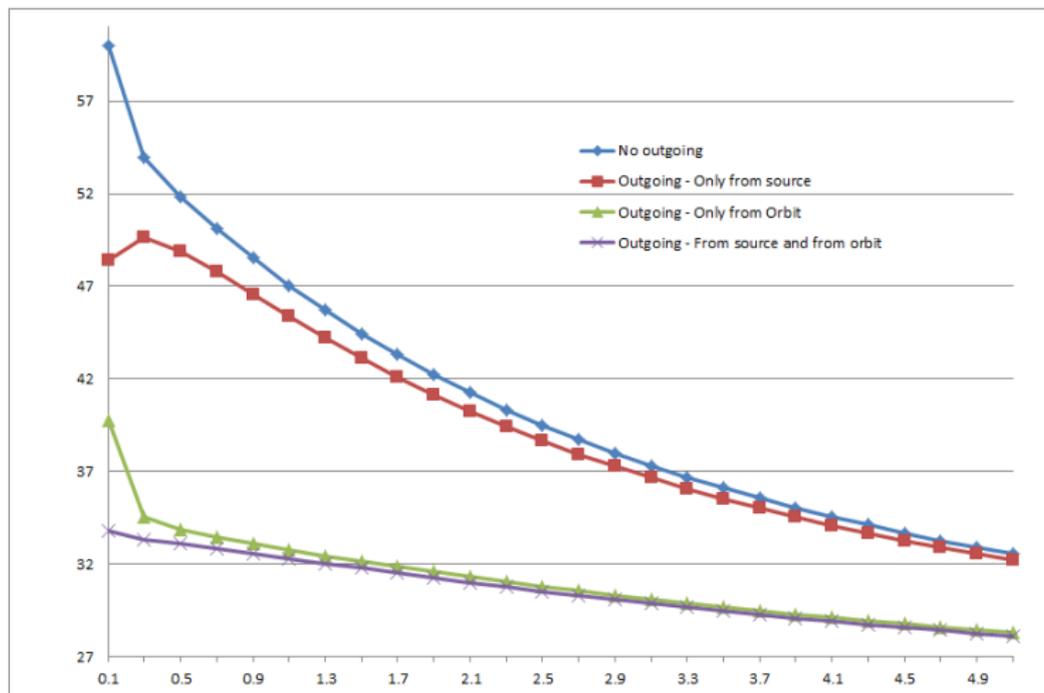


Figure 7: The Loss function in Case 2 vs.  $\lambda_1$

## Conclusions

- 1 Finite source retrial queueing system for two way communication
- 2 Tool supported solution by MOSEL
- 3 Graphical illustrations, comparisons, examples

## Bibliography

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*Thank You  
for Your  
Attention*