



# A survey of recent results on finite source retrieval queues with collisions

J. Sztrik

University of Debrecen, Hungary

<http://irh.inf.unideb.hu/user/jsztrik>

A. Nazarov, A. Kvach

National Research Tomsk State University, Russia

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# Outline

- 1 Finite source retrial queueing system with collisions
- 2 Performance measures
- 3 Tool supported, algorithmic, simulation and asymptotic approaches
- 4 Comparisons, examples
- 5 Bibliography

# Finite source retrial queueing system with collisions

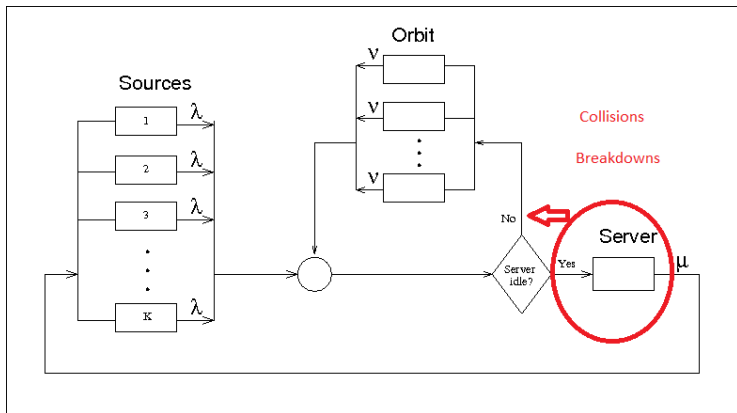


Figure 1: Finite source retrial queueing system with collisions and unreliable server

## Performance measures

- *Distribution of number of requests in the system, including in service and in orbit*
- *Distribution of number of retrials*
- *Distribution of the response/waiting time of a customer*

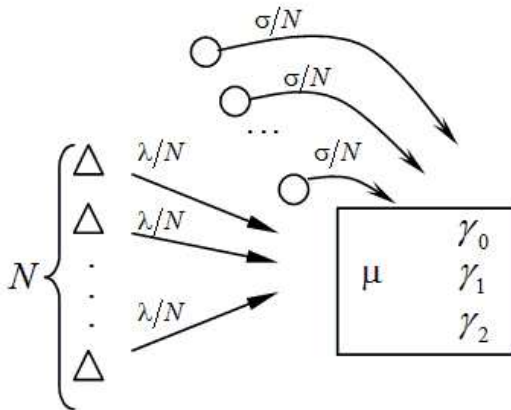
## Tool supported and algorithmic approaches

- *MOSEL (Modeling, Specification and Evaluation Language) solution*
  
- *Algorithmic method*

## Simulation approach

- *The effect of distributions of the involved random variables on the distribution of the number of customers in the system*
- *The effect of distributions of the involved random variables on the mean and variance of the response/waiting time of a request*
- *The effect of distributions of the involved random variables on the mean and variance of the number of retrials*

# Asymptotic method



## *Asymptotic of the first order*

Let  $i(t)$  be number of customers in a closed retrial queueing system  $M/M/1//N$  with the collisions of customers and unreliable server, then

$$\lim_{N \rightarrow \infty} E \exp \left\{ jw \frac{i(t)}{N} \right\} = \exp \{ jw \kappa_1 \}, \quad (1)$$

where value of parameter  $\kappa_1$  is the positive solution of the equation

$$(1 - \kappa_1) \lambda - \mu R_1(\kappa_1) = 0, \quad (2)$$

where the stationary distributions of probabilities  $R_k(\kappa_1)$  of the service state  $k$  are obtained as follows

$$R_0(\kappa_1) = \left\{ \frac{\gamma_0 + \gamma_2}{\gamma_2} + \frac{\gamma_1 + \gamma_2}{\gamma_2} \cdot \frac{a(\kappa_1)}{a(\kappa_1) + \gamma_1 + \mu} \right\}^{-1},$$

$$R_1(\kappa_1) = \frac{a(\kappa_1)}{a(\kappa_1) + \gamma_1 + \mu} \cdot R_0(\kappa_1), \quad (3)$$

$$R_2(\kappa_1) = \frac{1}{\gamma_2} [\gamma_0 R_0(\kappa_1) + \gamma_1 R_1(\kappa_1)],$$

here  $a(\kappa_1)$  is

$$a(\kappa_1) = (1 - \kappa_1) \lambda + \sigma \kappa_1. \quad (4)$$

*Asymptotic of the second order*

$$\lim_{N \rightarrow \infty} E \exp \left\{ jw \frac{i(t) - \kappa_1 N}{\sqrt{N}} \right\} = \exp \left\{ \frac{(jw)^2}{2} \kappa_2 \right\}, \quad (5)$$

where the value of parameter  $\kappa_2$  is defined by expression

$$\kappa_2 = \frac{\gamma_2 \mu (R_1 - b_1) + (1 - \kappa_1) \lambda \{ (\gamma_1 + \gamma_2) b_1 + (1 - \kappa_1) \lambda R_2 \}}{(\lambda + \mu b_2) \gamma_2 - (1 - \kappa_1) \lambda (\gamma_1 + \gamma_2) b_2}, \quad (6)$$

where

$$b_1 = \frac{(1 - \kappa_1) \lambda}{a + \gamma_1 + \mu} R_0, \quad b_2 = \frac{(\sigma - \lambda)(R_0 - R_1)}{a + \gamma_1 + \mu}. \quad (7)$$

From the proved theorem it follows that if  $N \rightarrow \infty$  the limiting distributions for the centered and normalized number of customers in the system has a Gaussian distribution with variance  $\kappa_2$ , defined by the expression (6).

### Corollary

*As a consequence the distribution of the number of customers in the system is Gaussian with mean  $N\kappa_1$  and variance  $N\kappa_2$ , respectively.*

$$\lim_{N \rightarrow \infty} \mathbb{E} \exp \left\{ j\omega \frac{T}{N} \right\} = q + (1 - q) \frac{\sigma q}{\sigma q - j\omega}, \quad (8)$$

where value of parameter  $q$  is defined by expression

$$q = \frac{(1 - \kappa_1)\lambda}{(1 - \kappa_1)\lambda + \sigma\kappa_1}. \quad (9)$$

### Corollary

*Characteristic function of the sojourn time of the customer in the system in a prelimiting situation of finite  $N$  can be approximated by a function of the form*

$$\mathbb{E} e^{juT} = q + (1 - q) \frac{\sigma q}{\sigma q - juN}, \quad (10)$$

Let  $\nu$  be the number of transitions of the tagged customer into the orbit, then

$$\lim_{N \rightarrow \infty} \mathbf{E} z^\nu = \frac{q}{1 - (1 - q)z}, \quad (11)$$

where value of parameter  $q$  is

$$q = \frac{(1 - \kappa_1)\lambda}{a}. \quad (12)$$

### Corollary

*The probability distribution  $P\{\nu = n\}$ ,  $n = \overline{0, \infty}$  of the number of transitions of the tagged customer into the orbit is geometric and has the form*

$$P\{\nu = n\} = q(1 - q)^n, \quad n = \overline{0, \infty}. \quad (13)$$

## Comparisons, examples

$$N = 100, \quad \lambda = 1, \quad \mu = 1, \quad \sigma = 5, \quad \gamma_0 = \gamma_1 = 0.1, \quad \gamma_2 = 1.$$

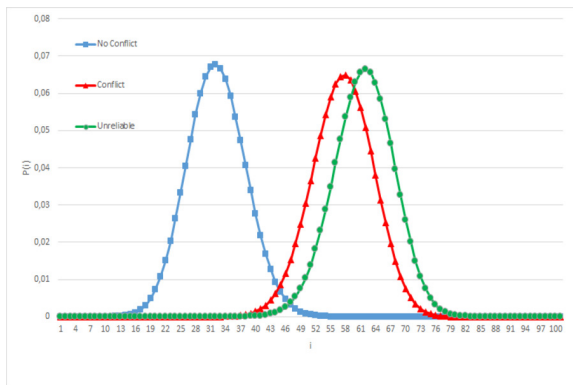


Figure 2: Reliable without conflict, reliable with conflict, and unreliable with conflict

$$N = 10, \quad \mu = 1, \quad \gamma_0 = 0, \quad \gamma_1 = 0.1, \quad \gamma_2 = 1$$

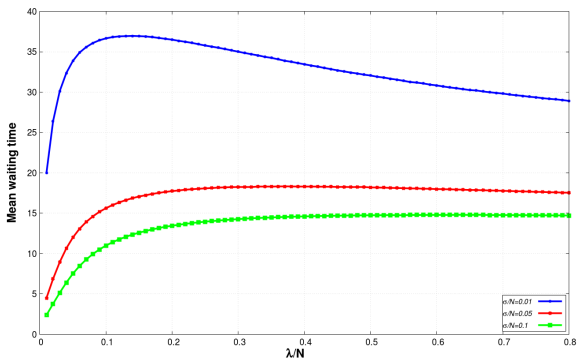


Figure 3: Mean waiting time in the orbit without collisions

$$N = 10, \quad \mu = 1, \quad \gamma_0 = 0, \quad \gamma_1 = 0.1, \quad \gamma_2 = 1$$

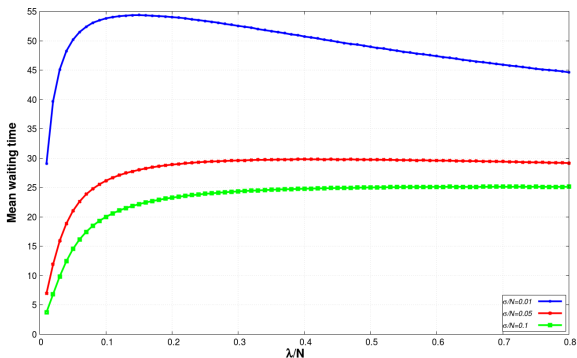


Figure 4: Mean waiting time in the orbit with collisions

$$\lambda = 0.5, \quad \mu = 1, \quad \sigma = 5, \quad \gamma_0 = 0.1, \quad \gamma_1 = 0.2, \quad \gamma_2 = 1.$$

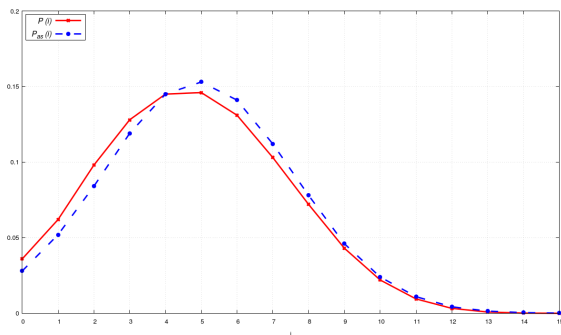


Figure 5: Comparison of the asymptotic and numerical results in the case  $N = 15$

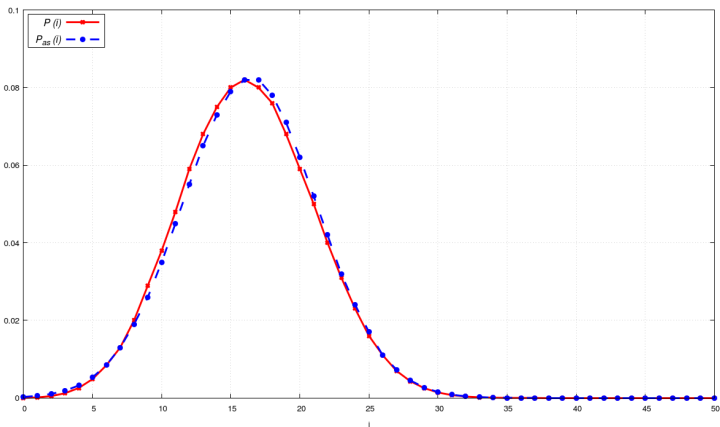


Figure 6: Comparison of the asymptotic and numerical results in the case  $N = 50$

$$N = 100, \quad \mu = 1, \quad \gamma_0 = 0, \quad \gamma_1 = 0.1, \quad \gamma_2 = 1$$

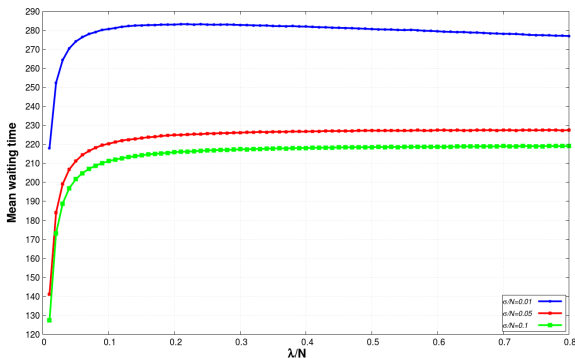


Figure 7: Asymptotic mean waiting time in the orbit

Kolmogorov distance  $\Delta$

$$\Delta = \max_{0 \leq i < \infty} \left| \sum_{n=0}^i (P_{as}(\nu = n) - P_s(\nu = n)) \right| .$$

Realizing the simulation program for

$$\lambda = 1, \quad \mu = 1, \quad \sigma = 4, \quad \gamma_2 = 1$$

and applying the approximation (13), we will provide the Kolmogorov distance  $\Delta$  for various values  $N$  and  $\gamma = \gamma_0 = \gamma_1$  in the Table 1.

**Table 1:** Kolmogorov distance between distribution  $P_s(i)$  and approximation of the geometric distribution  $P_{as}(i)$  for various values of the parameters  $N$  and  $\gamma$

	$N = 20$	$N = 30$	$N = 50$	$N = 100$	$N = 200$
$\gamma = 0.05$	0.026	0.016	0.009	0.005	0.003
$\gamma = 0.1$	0.024	0.015	0.009	0.004	0.002
$\gamma = 0.5$	0.017	0.011	0.006	0.004	0.001

**Table 2:** Mean number of retrials in prelimiting and limiting situations for various values of the parameters  $N$  and  $\gamma$

	$N = 20$	$N = 50$	$N = 100$	$N = 200$	limiting
$\gamma = 0.05$	5.512	5.842	5.900	5.930	5.977
$\gamma = 0.1$	6.090	6.334	6.415	6.442	6.494
$\gamma = 0.5$	10.336	10.640	10.715	10.777	10.821

# Conclusions

- 1 Finite source retrial queueing system with collisions
- 2 Different solution approaches
- 3 Recent results on systems with an unreliable server
- 4 Graphical illustrations, comparisons, examples

# Bibliography

## Tool Supported and Numerical Methods



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## Simulation Methods



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## Asymptotic Methods



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*Thank You  
for Your  
Attention*