Homogeneous Finite-Source Retrial Queues with Search of Customers from the Orbit

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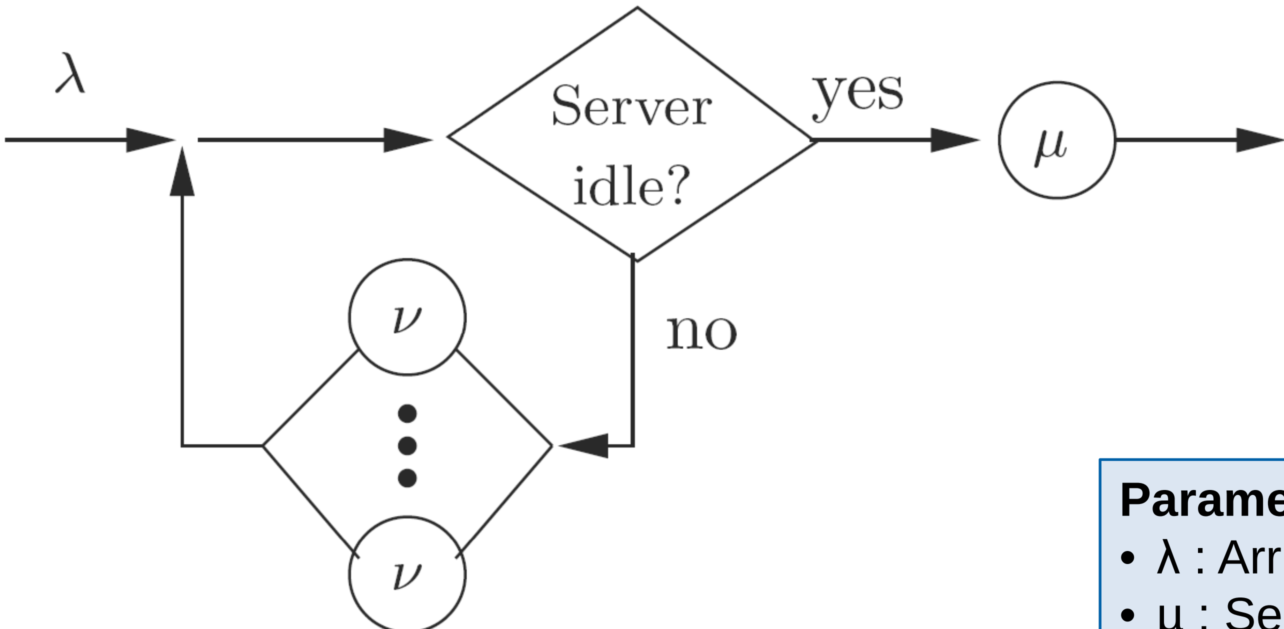
Outline

- Basic Retrial Queueing System
- Extension of Basic Model
- Underlying Markov Chain
- Performance Measures
- Numerical Results
- Conclusion and Future Work

Basic Retrial Queueing System

- Example: M/M/1 Retrial Queueing System



Parameters:
- λ : Arrival Rate
- μ : Service Rate
- ν : Retrial Rate

Model Extension

- Finite Source
  - Arrival rate depends on number of customers in the system
- Orbital Search
  - After service completion, with a certain probability, the server immediately retrieves an orbiting customer for service.
- Application Examples
  - Call centers
  - Telephone networks
  - Network access control
  - P2P file-sharing protocols

Model Extension - Illustration

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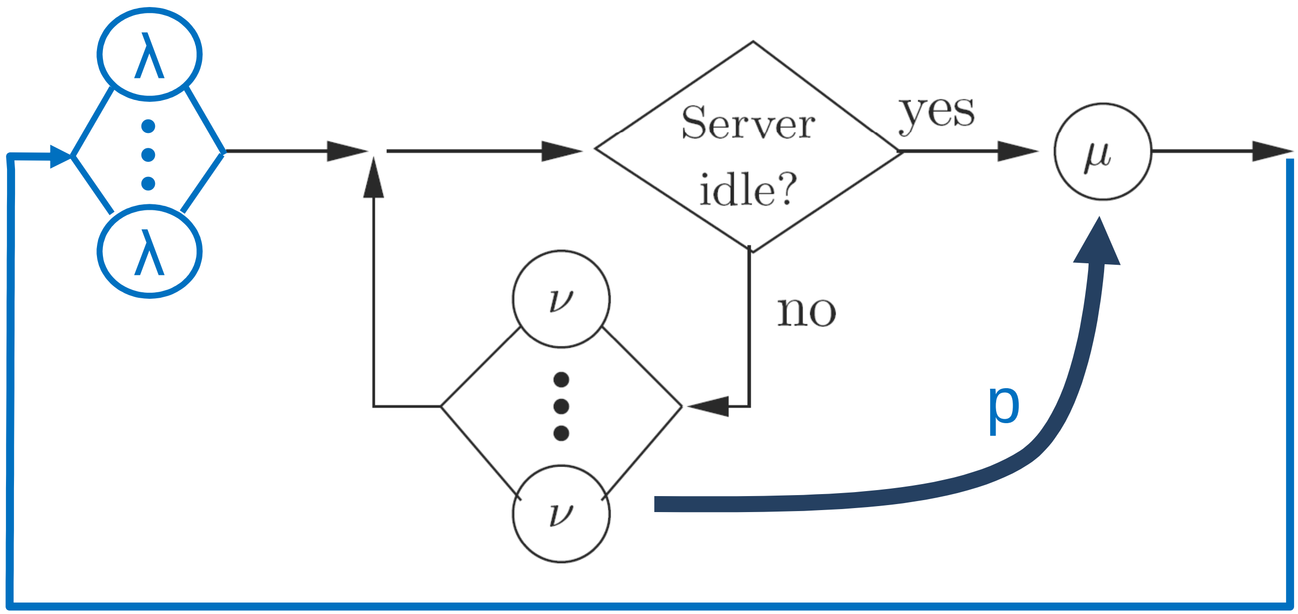
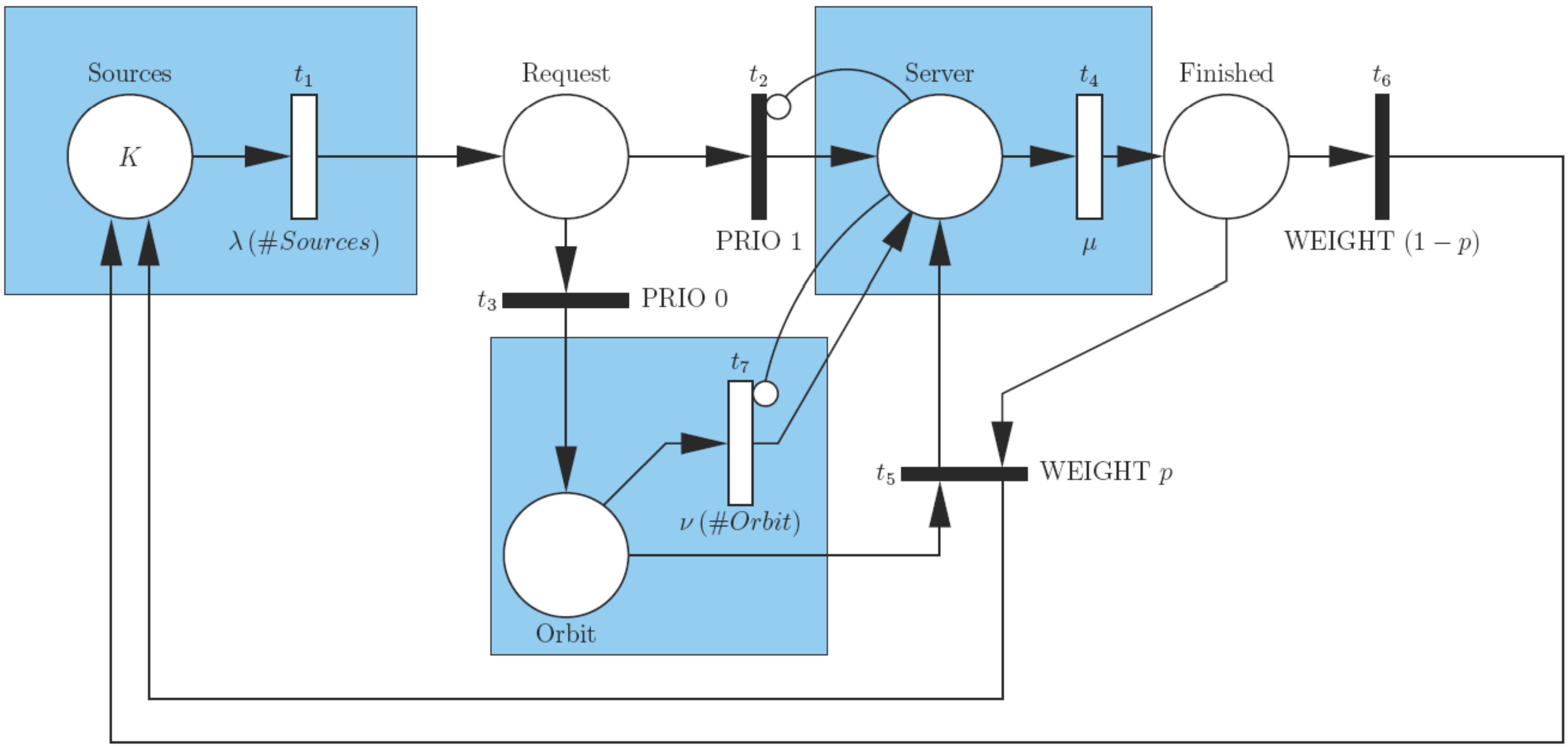
- Finite Source Retrial Queue with Orbital Search



Parameters:
- μ : Service Rate
- K : Number of Sources
- ν : Retrial Rate
- λ : Arrival Rate
- p : Search Probability

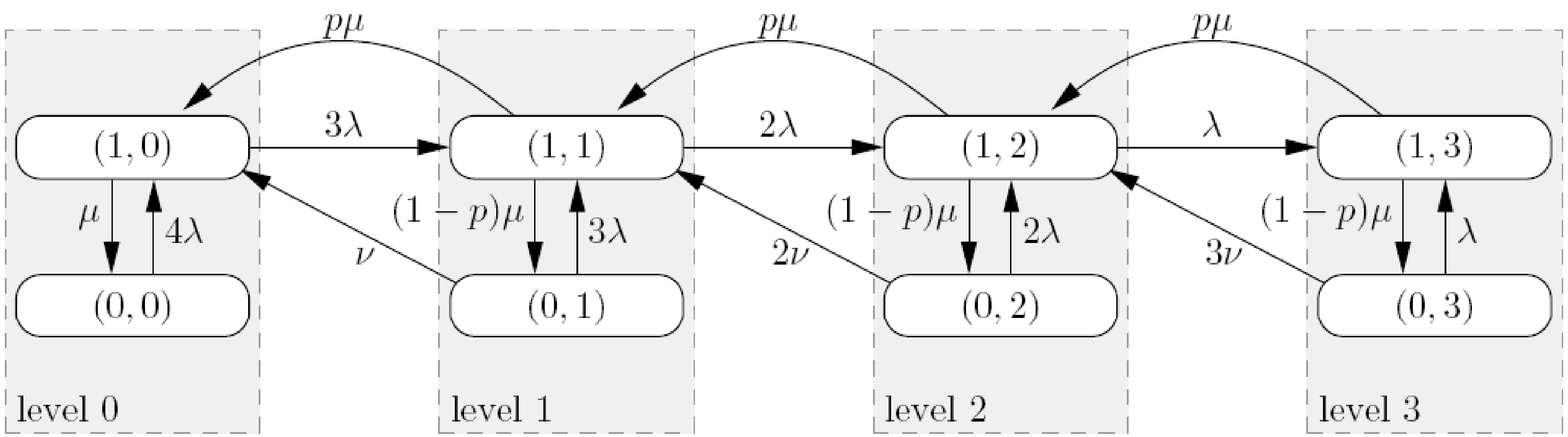
Model Extension - GSPN



Parameters:
- K : Number of Sources
- λ : Arrival Rate
- µ : Service Rate
- ν : Retrial Rate
- p : Search Probability

Underlying Markov Chain

- Two-Dimensional Continuous Markov Chain (CTMC):
  - *X(t) = (C(t), N(t)),* where *C(t)=1* if the server is busy and *0* if idle, and *N(t)* is the number of customers in the orbit at time *t*.
- Graphical Representation of CTMC for K=4:



Parameters: • K=4 Sources • λ : Arrival Rate
• μ : Service Rate • ν : Retrial Rate • p : Search Probability

Performance Measures (1)

- Stationary Probabilities

$$P(r,j) = \lim_{t\to\infty} P(C(t) = r, N(t) = j), \quad r = 0,1, \quad j = 0,...,K-1.$$

- Server Utilization

$$U_S = \sum_{j=0}^{K-1} P(1,j)$$

- Mean Number of Customers in the Orbit

$$N = \sum_{r=0}^{1} \sum_{j=0}^{K-1} jP(r,j)$$

- Mean Number of Customers in the System (Service and Orbit)

$$M = \sum_{r=0}^{1} \sum_{j=0}^{K-1} (r+j)P(r,j)$$

Performance Measures (2)

- Mean Generation Rate of Primary Calls

$$\bar{\lambda} = \lambda(K - M)$$

- Mean Response Time

$$E[T] = M/\bar{\lambda}$$

- Mean Waiting Time

$$E[W] = N/\bar{\lambda}$$

- Blocking Probability of Primary Calls

$$B = \frac{\lambda E[K - C(t) - N(t), C(t) = 1]}{\bar{\lambda}}$$

Numerical Results (1)

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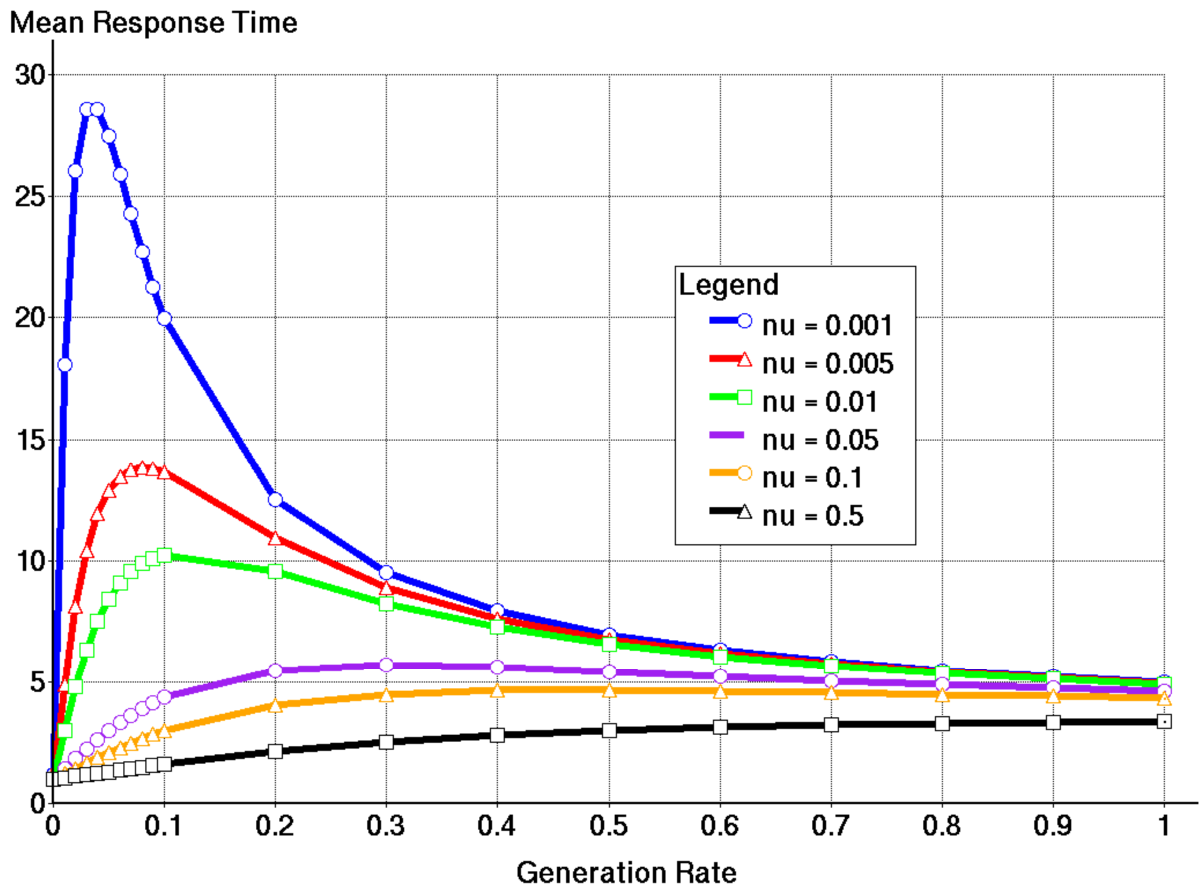
- Obtained using MOSEL-2 with SPNP
- Parameters set:

Parameter	Symbol	Value / Range	
		Slide 11	Slide 12
Number of sources	K	3	
Service rate	μ	1	
Generation rate	λ	0.0001 … 1	
Retrial rate	ν	0.001 … 0.5	0.005
Search probability	p	$1 \cdot 10^{-8}$	$1 \cdot 10^{-8}$ … $(1-1 \cdot 10^{-8})$

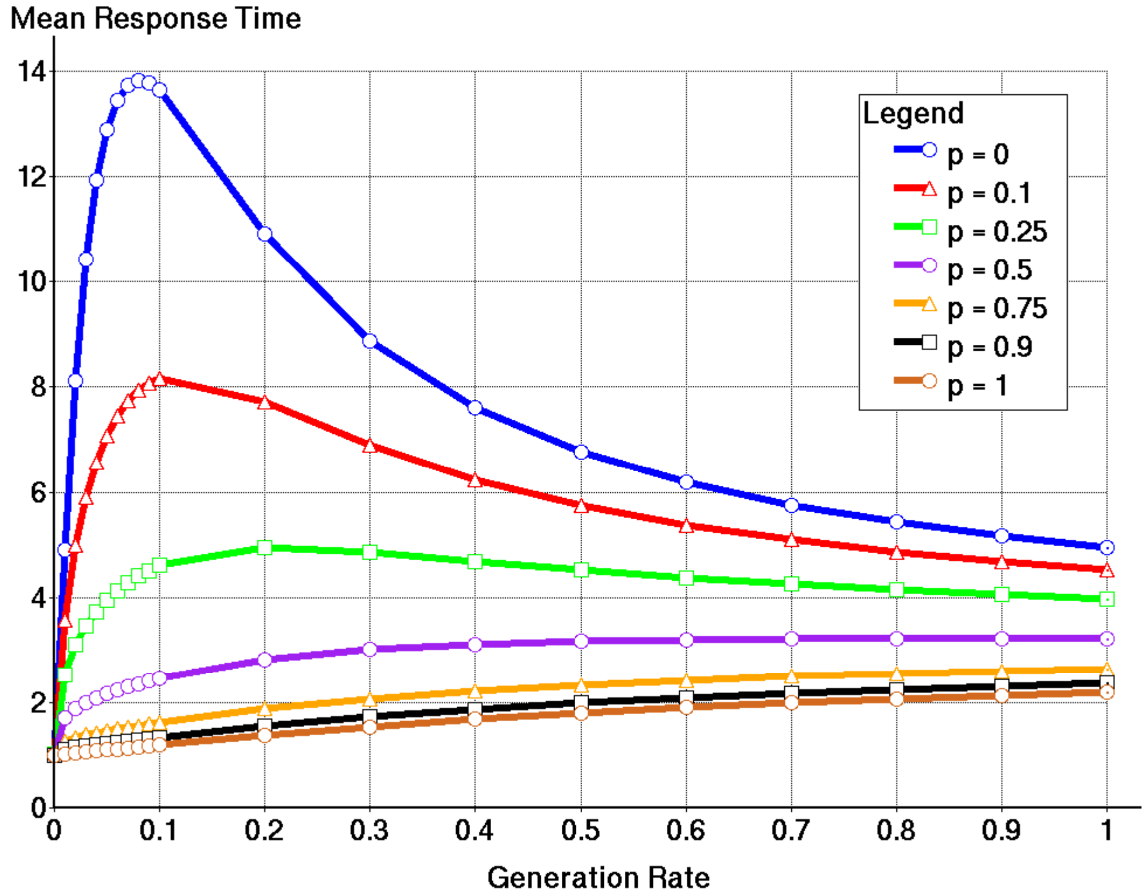
Numerical Results (2)

- Mean response time versus arrival rate (influence of *v*)



Numerical Results (3)

- Mean response time versus arrival rate (influence of *p*)



Conclusion and Future Work

- Conclusion
  - Retrial queues are still an interesting research topic with manifold applications.
  - Results show that retrials should be considered during the modeling process.
  - MOSEL-2 is capable of modeling and evaluating retrial queues in a comfortable way.
- Future Work
  - Derivation of results in explicit closed form (symbolic solutions)
  - Further generalizations of retrial queueing models
    - Multiple heterogeneous and unreliable servers
    - Heterogeneous sources
    - More general arrival, service, and retrial processes (MAP, PH)
  - Upper bounds and/or higher moments of performance measures
- **Questions?**