

TOOL SUPPORTED RELIABILITY ANALYSIS OF FINITE-SOURCE RETRIAL QUEUES

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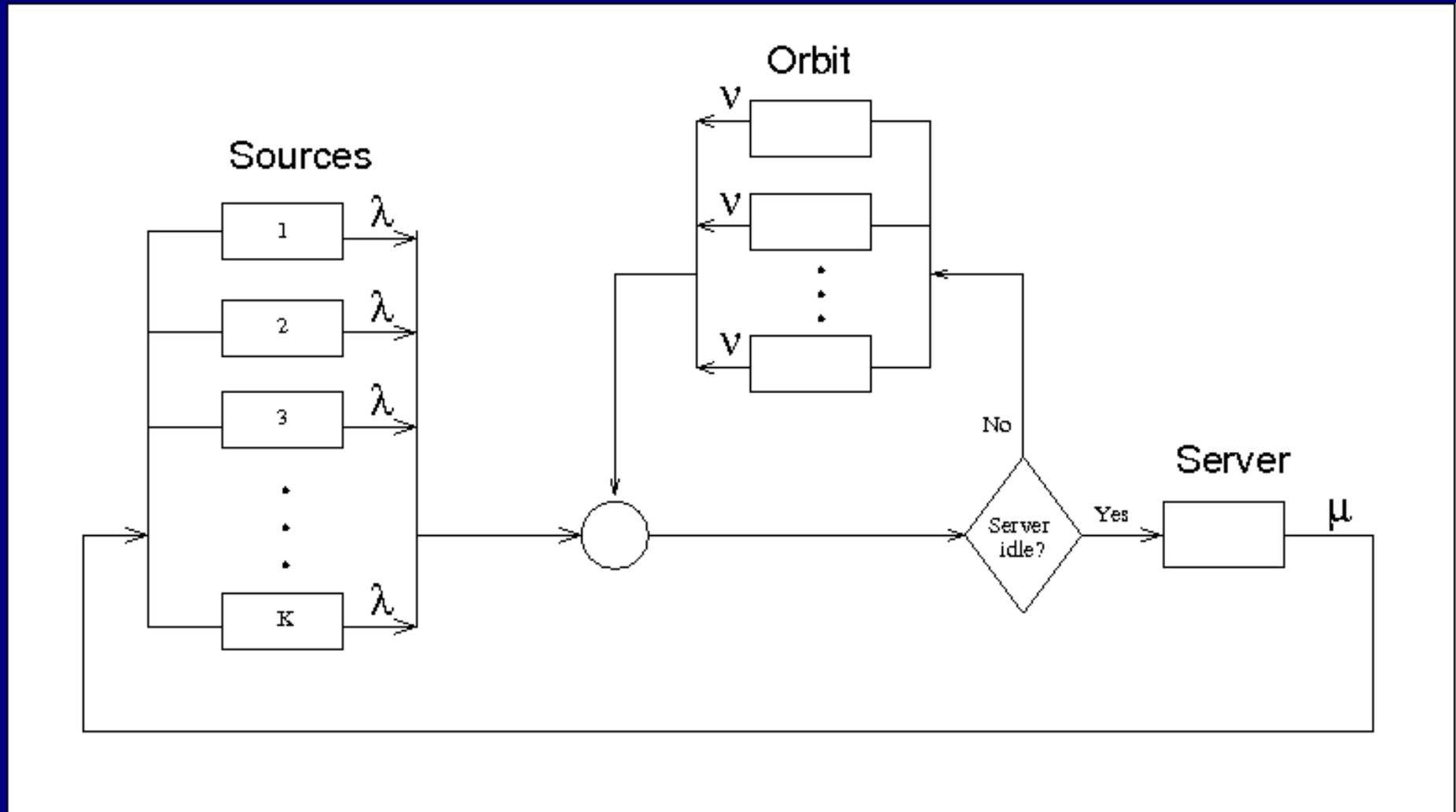
OUTLINE

- **Modeling tools**
- **Finite-source retrial queueing systems**
- **MOSEL tool**
- **Case studies**
- **References**

Modeling tools

- **University of Dortmund:** *HIT, MACOM, HiQPN, DSPNexpress*
<http://ls4-www.informatik.uni-dortmund.de/tools.html>
- **University of Illinois at Urbana-Champaign:** *MÖBIUS*
<http://www.mobius.uiuc.edu/>
- **University of Erlangen:** *PEPSY, MOSEL*
<http://www4.informatik.uni-erlangen.de/Projects/MOSEL/>
- **University of Oxford:** *PRISM*
<http://www.prismmodelchecker.org/>

Finite-source retrial queueing system



Mathematical model

The system state at time t can be described with the process

$$X(t) = (Y(t); C(t); N(t))$$

where $Y(t) = 0$ if the server is up, $Y(t) = 1$ if the server is failed,

$C(t) = 0$ if the server is idle, $C(t) = 1$ if the server is busy,

$N(t)$ is the number of sources of repeated calls at time t .

We define the stationary probabilities:

$$P(q; r; j) = \lim_{t \rightarrow \infty} P(Y(t) = q, C(t) = r, N(t) = j)$$

$$q = 0, 1, \quad r = 0, 1, \quad j = 0, \dots, K^*,$$

$$\text{where } K^* = \begin{cases} K - 1 & \text{for blocked case,} \\ K - r & \text{for unblocked case.} \end{cases}$$

Once we have obtained these limiting probabilities the **main system performance measures** can be derived in the following way.

1. Utilization of the server

$$U_S = \sum_{j=0}^{K-1} P(0, 1, j)$$

2. Utilization of the repairman

$$U_R = \sum_{q=0}^1 \sum_{j=0}^{K^*} P(1, q, j)$$

3. Availability of the server

$$A_S = \sum_{q=0}^1 \sum_{j=0}^{K^*} P(0, q, j) = 1 - U_R$$

4. The mean number of calls staying in the orbit or in service

$$M = E[N(\infty) + C(\infty)] = \sum_{q=0}^1 \sum_{r=0}^1 \sum_{j=0}^{K^*} j P(q, r, j) + \sum_{q=0}^1 \sum_{j=0}^{K-1} P(q, 1, j).$$

5. Utilization of the sources

$$U_{SO} = \begin{cases} \frac{E[K - C(\infty) - N(\infty); Y(\infty) = 0]}{K} & \text{for blocked case,} \\ \frac{K - M}{K} & \text{for unblocked case.} \end{cases}$$

6. Overall utilization

$$U_O = U_S + KU_{SO} + U_R.$$

7. The mean rate of generation of primary calls

$$\bar{\lambda} = \begin{cases} \lambda E[K - C(\infty) - N(\infty); Y(\infty) = 0] & \text{for blocked case,} \\ \lambda E[K - C(\infty) - N(\infty)] & \text{for unblocked case.} \end{cases}$$

8. The mean response time

$$E[T] = M/\bar{\lambda}$$

9. The blocking probability of a primary call

$$B = \begin{cases} \frac{\lambda E[K - C(\infty) - N(\infty); Y(\infty) = 0; C(\infty) = 1]}{\bar{\lambda}} & \text{for blocked case,} \\ \frac{\lambda E[K - C(\infty) - N(\infty); C(\infty) = 1]}{\bar{\lambda}} & \text{for unblocked case.} \end{cases}$$

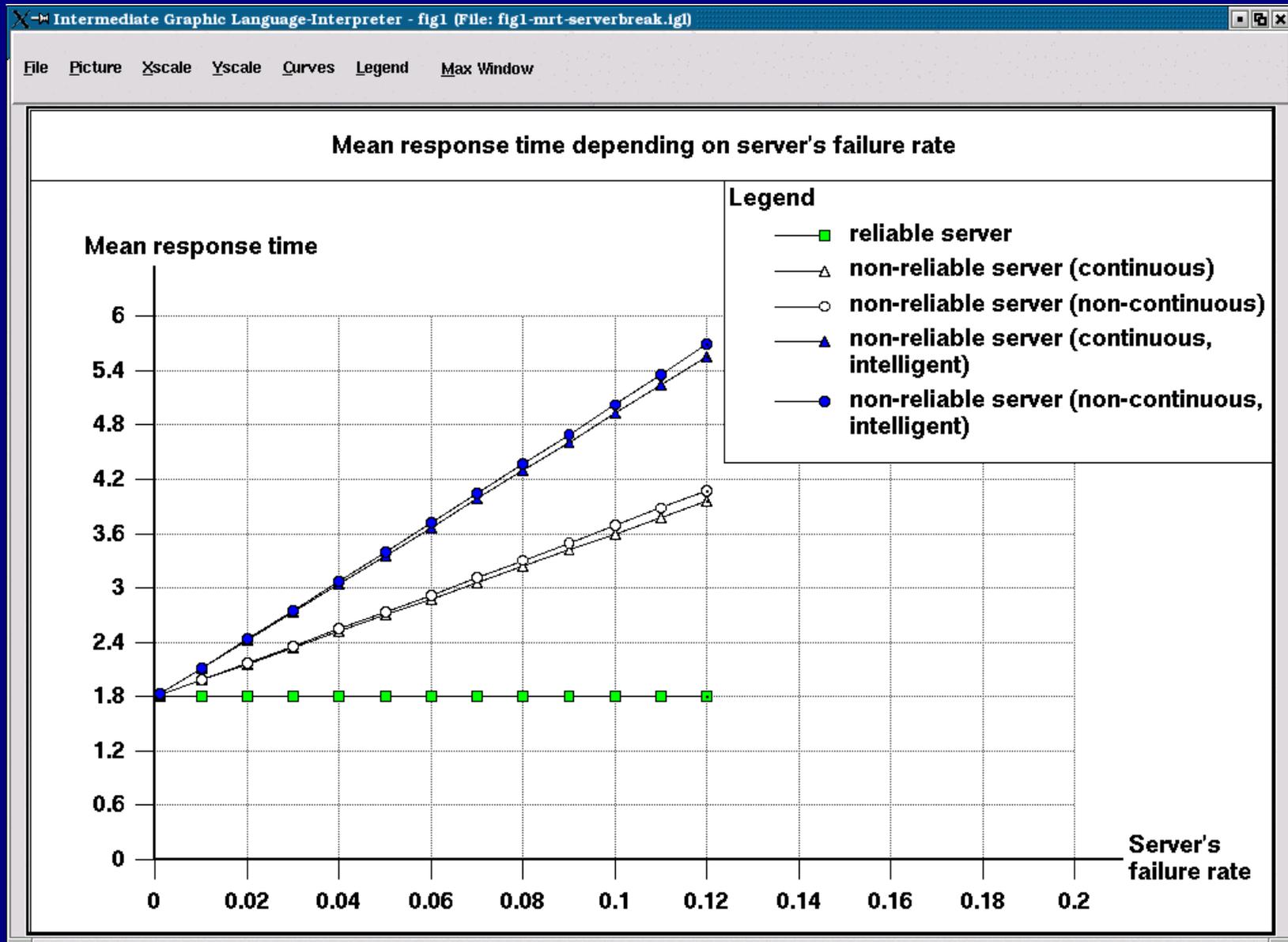
Evaluation Tool MOSEL

MOSEL (Modeling, Specification and Evaluation Language) developed at the University of Erlangen, Germany, is used to formulate and solved the problem.

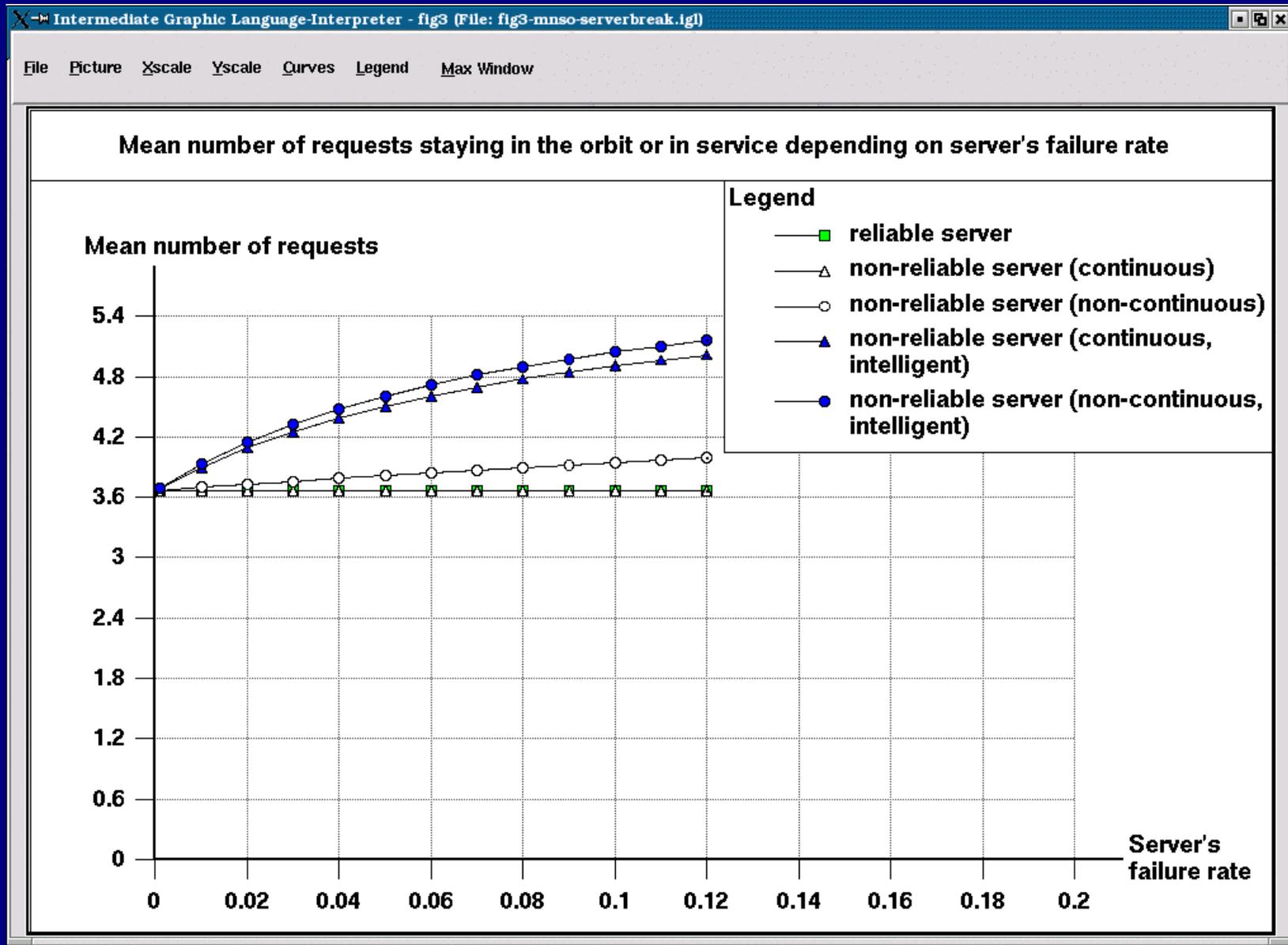
Case studies

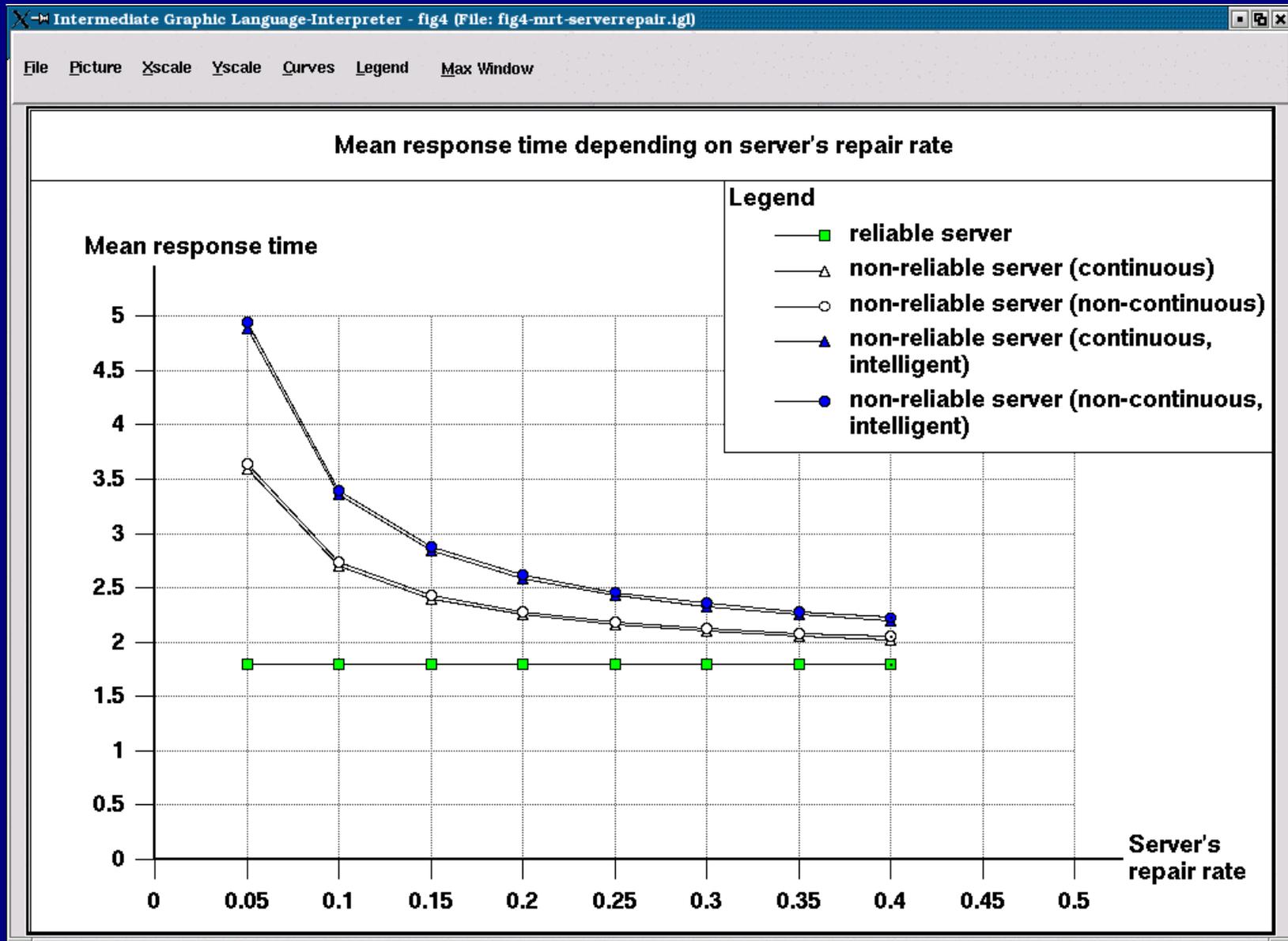
	K	λ	μ	ν	δ, γ	τ
Figure 1	6	0.8	4	0.5	x axis	0.1
Figure 2	6	0.1	0.5	0.05	x axis	0.1
Figure 3	6	0.8	4	0.5	0.05	x axis
Figure 4	6	0.1	0.5	0.05	0.05	x axis

Input system parameters

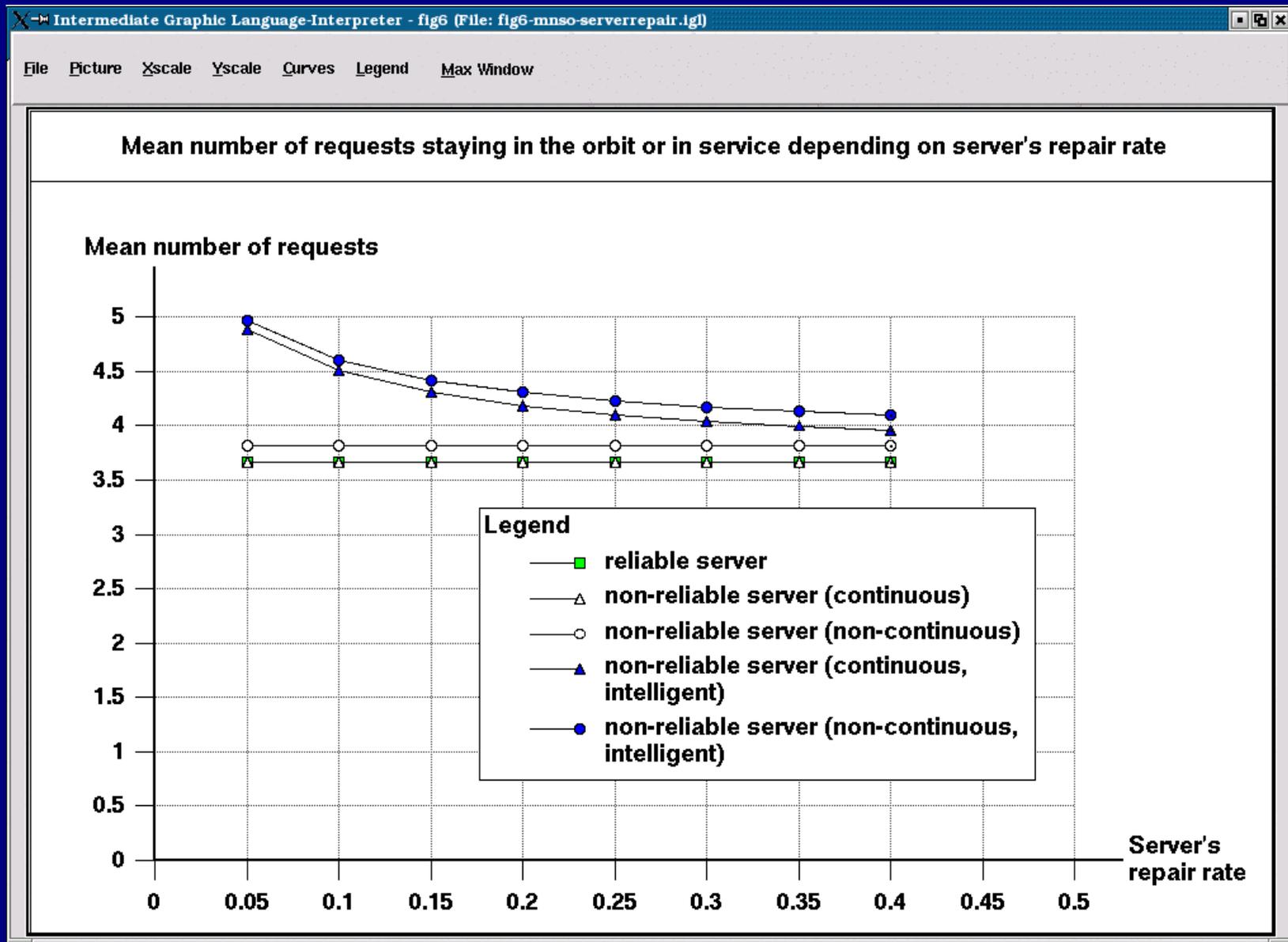


$E[T]$ versus server's failure rate

 M versus server's failure rate



$E[T]$ versus server's repair rate



M versus server's repair rate

References

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*Thank You
for Your
Attention*