Finite-Source Retrial Queueing Systems with Non-Reliable Heterogenous Servers and Different Service Policies

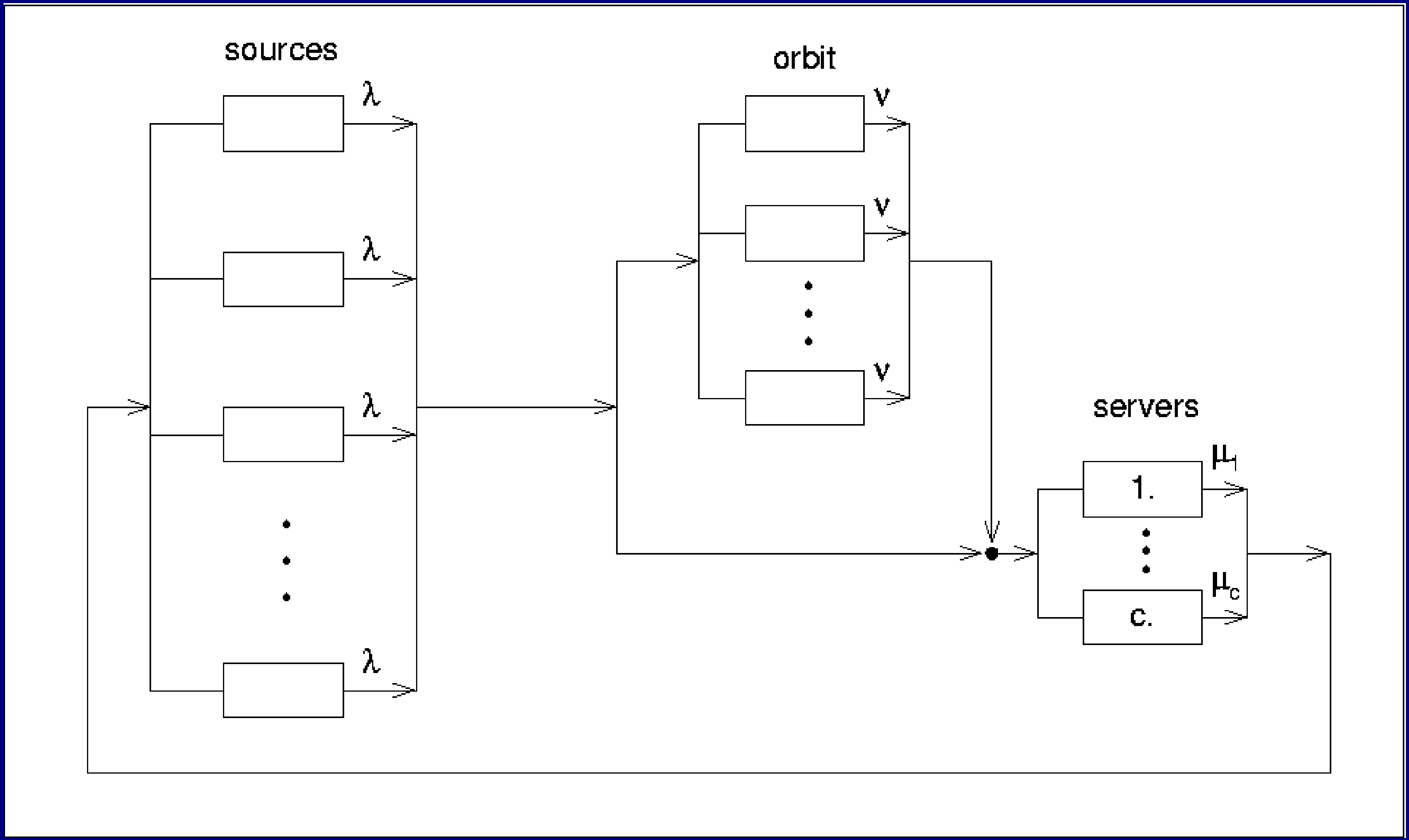
J. Roszik, J. Sztrik
Institute of Informatics, University of Debrecen
Debrecen, Hungary

e-mail: jsztrik@inf.unideb.hu
www: http://irh.inf.unideb.hu/user/jsztrik/index.html

OUTLOOK

- **The queueing model**
- **Applications**
- **Mathematical model**
- **Evaluation Tool MOSEL**
- **Case studies**
- **References**

The queueing model



Applications

- **magnetic disk memory systems**
- **local area networks with CSMA/CD protocols**
- **collision avoidance local area network modeling**

Mathematical model

The system's state at time t can be described by the process

$$X(t) = (\alpha_1(t), ..., \alpha_c(t); N(t)),$$

where

$N(t) =$ the number of sources of repeated calls,

$$\alpha_i(t) = \begin{cases} 1 & \text{if there is a customer under service at the server,} \\ 0 & \text{if it is operational and idle,} \\ -1 & \text{if the server is failed.} \end{cases}$$

Let us define the stationary probabilities by:

$$P(i_1, ..., i_c, j) = \lim_{t\to\infty} P\{\alpha_1(t) = i_1, ..., \alpha_c(t) = i_c, N(t) = j\},$$

$$i_1, ...i_c = -1, 0, 1, \quad j = 0, ..., K^*,$$

$$\text{where} \quad K^* = K - \sum_{i_k, i_k = 1} i_k.$$

$C(t) =$ the number of busy servers,

$A(t) =$ the number of available servers,

$p_{kj} = \lim_{t\to\infty} P\{C(t) = k, N(t) = j\}.$

Once we have obtained these limiting probabilities the **main system's performance measures** can be derived in the following way.

- *The probability that at least one server is available*

$$A_S = P\{\alpha_k > -1\}, k \in \{1, ..., c\} = 1 - \sum_{j=0}^{K} P(-1, ..., -1, j).$$

- *Mean number of sources of repeated calls*

$$N = E[N(t)] = \sum_{k=0}^{c} \sum_{j=1}^{K} j p_{kj} = \sum_{i_1, ..., i_c} \sum_{j=1}^{K^*} j P(i_1, ..., i_c, j).$$

- *Utilization of the $k-$th server*

$$U_k = \sum_{i_1,\dots,i_c, i_k=1} \sum_{j=0}^{K^*} P(i_1,\dots,i_c,j), \quad k = 1,\dots,c.$$

- *Mean number of busy servers*

$$C = E[C(t)] = \sum_{k=1}^{c} U_k.$$

- *Mean number of calls staying in the orbit or in service*

$$M = E[N(t) + C(t)] = N + C.$$

- *Utilization of the repairman*

$$U_R = \sum_{\substack{i_1,\ldots,i_c \\ -1 \in \{i_1,\ldots,i_c\}}} \sum_{j=0}^{K^*} P(i_1,\ldots,i_c,j).$$

- *Utilization of the sources*

$$U_{SO} = \begin{cases} \frac{E[K-C(t)-N(t);A(t)>0]}{K} & \text{for blocked case,} \\ \frac{E[K-C(t)-N(t)]}{K} & \text{for unblocked case.} \end{cases}$$

- *Overall utilization of the system*

$$U_O = C + KU_{SO} + U_R.$$

- *Mean rate of generation of primary calls*

$$\overline{\lambda} = \begin{cases} \lambda E[K - C(t) - N(t); A(t) > 0] & \text{for blocked case,} \\ \lambda E[K - C(t) - N(t)] & \text{for unblocked case.} \end{cases}$$

- *Mean waiting time*

$$E[W] = N/\overline{\lambda}.$$

- *Mean response time*

$$E[T] = M/\overline{\lambda}.$$

Evaluation Tool MOSEL

MOSEL (Modeling, Specification and Evaluation Language) developed at the University of Erlangen, Germany, is used to formulate and solved the problem.

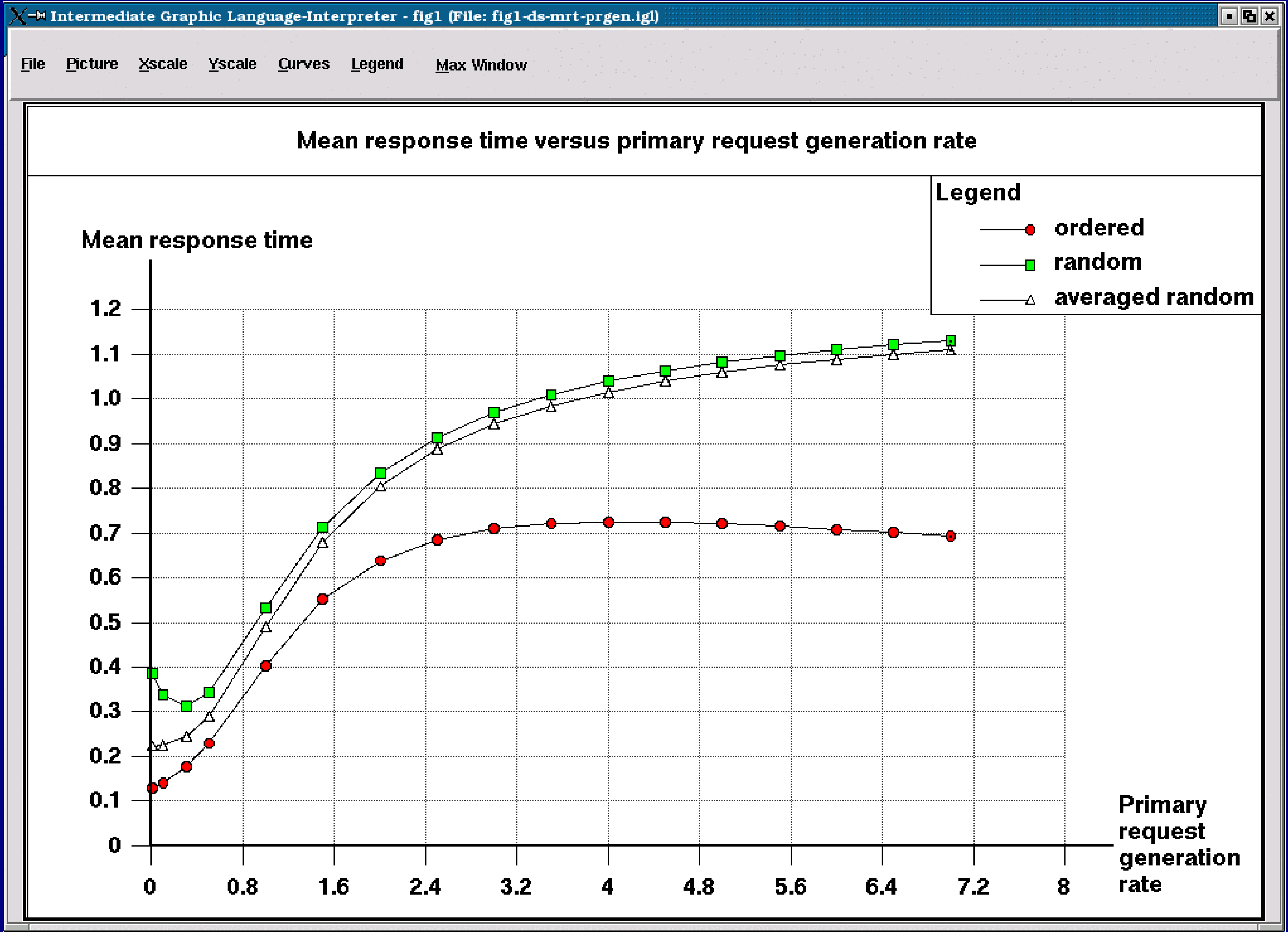
Case studies

Validation of results

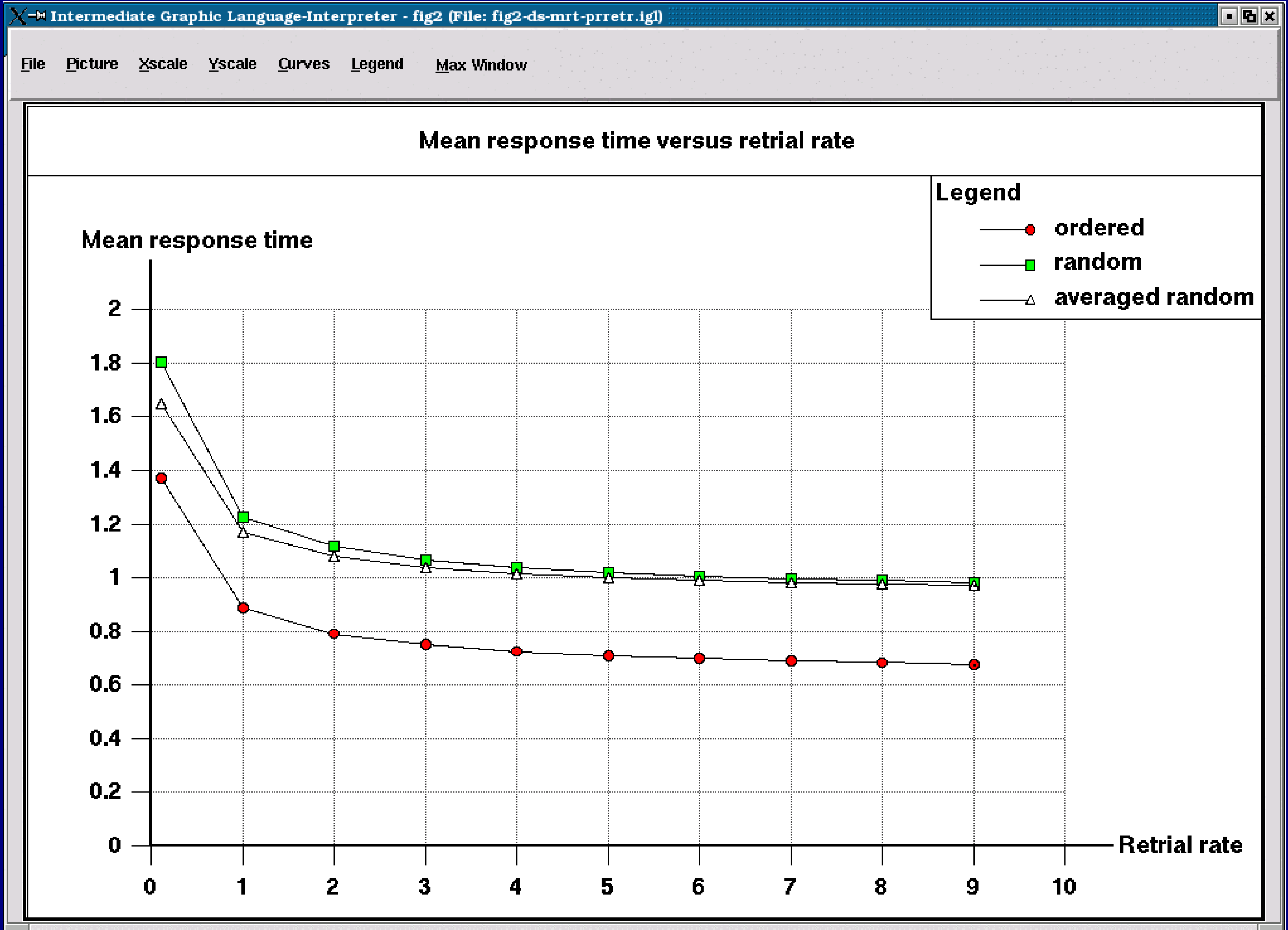
	Pascal	random	FFS
Number of servers:	4	4	4
Number of sources:	20	20	20
Request's generation rate:	0.1	0.1	0.1
Service rate:	1	1	1
Retrial rate:	1.2	1.2	1.2
Server's failure rate:	–	1e-25	1e-25
Server's repair rate:	–	1e+25	1e+25
Mean waiting time:	0.1064954794	0.1064959317	0.1064959929
Mean number of busy servers:	1.8007480431	1.8007485102	1.8007485548
Mean number of sources of repeated calls:	0.1917715262	0.1917717923	0.1917718470

Input parameters

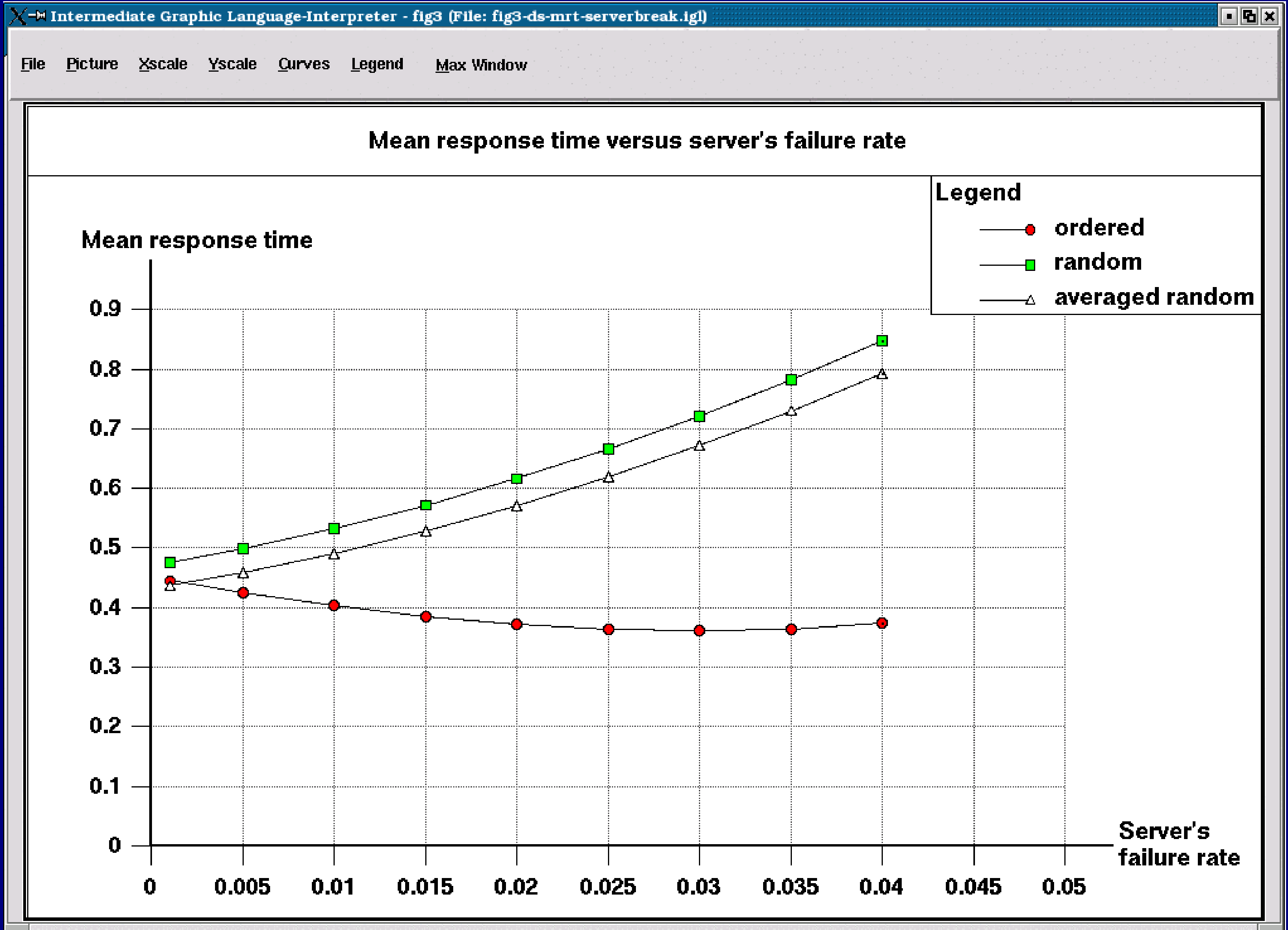
	c	K	λ	$\mu_1, \ldots, \mu_c - \mu_{avg}$	ν	δ,γ	τ
Figure 1,5	4	20	x axis	8,5,4,1 – 4.5	4	0.01	0.2
Figure 2,6	4	20	4	8,5,4,1 – 4.5	x axis	0.01	0.2
Figure 3,4,7	4	20	1	8,5,4,1 – 4.5	4	x axis	0.2



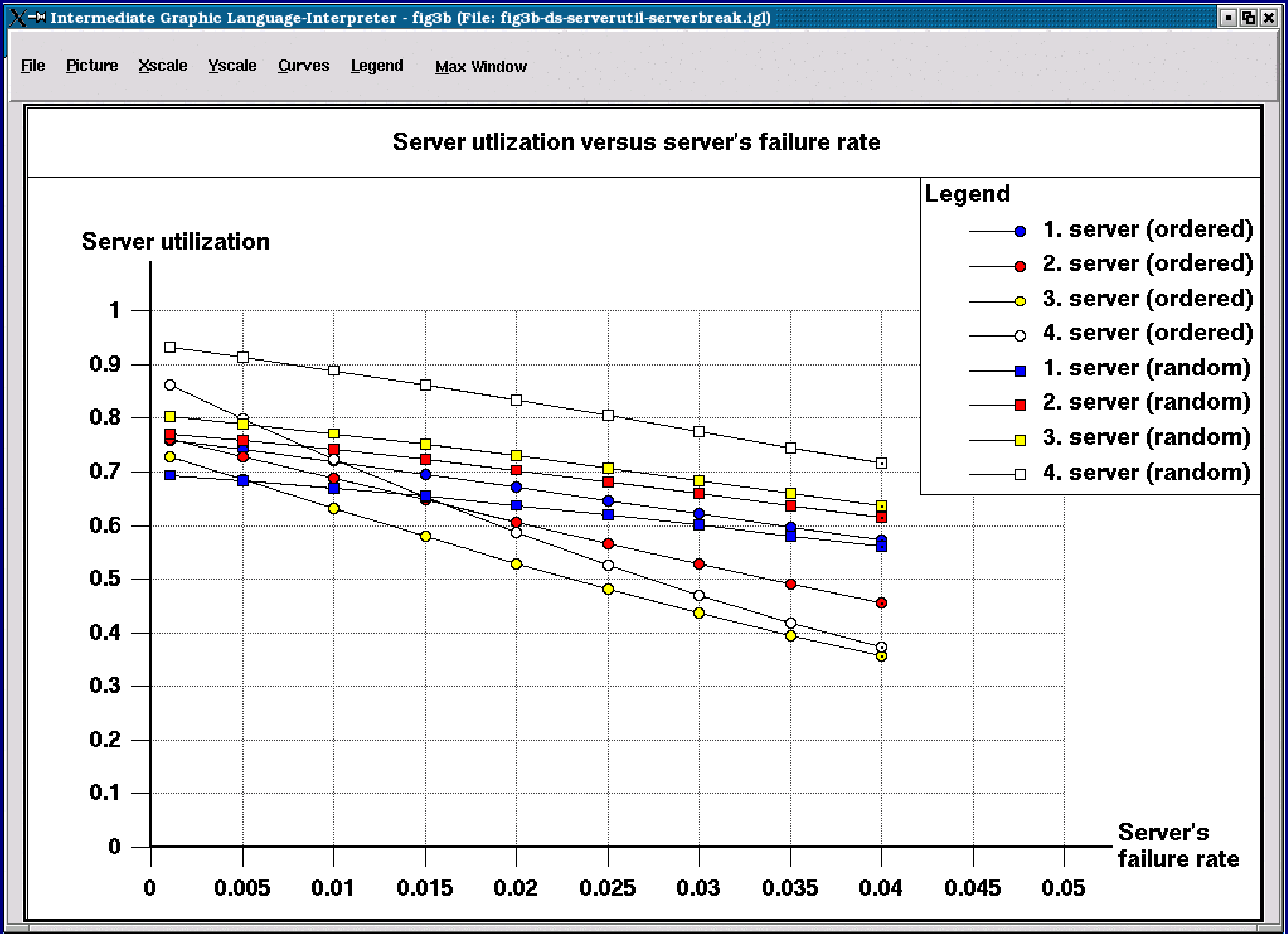
$E[T]$ versus primary request generation



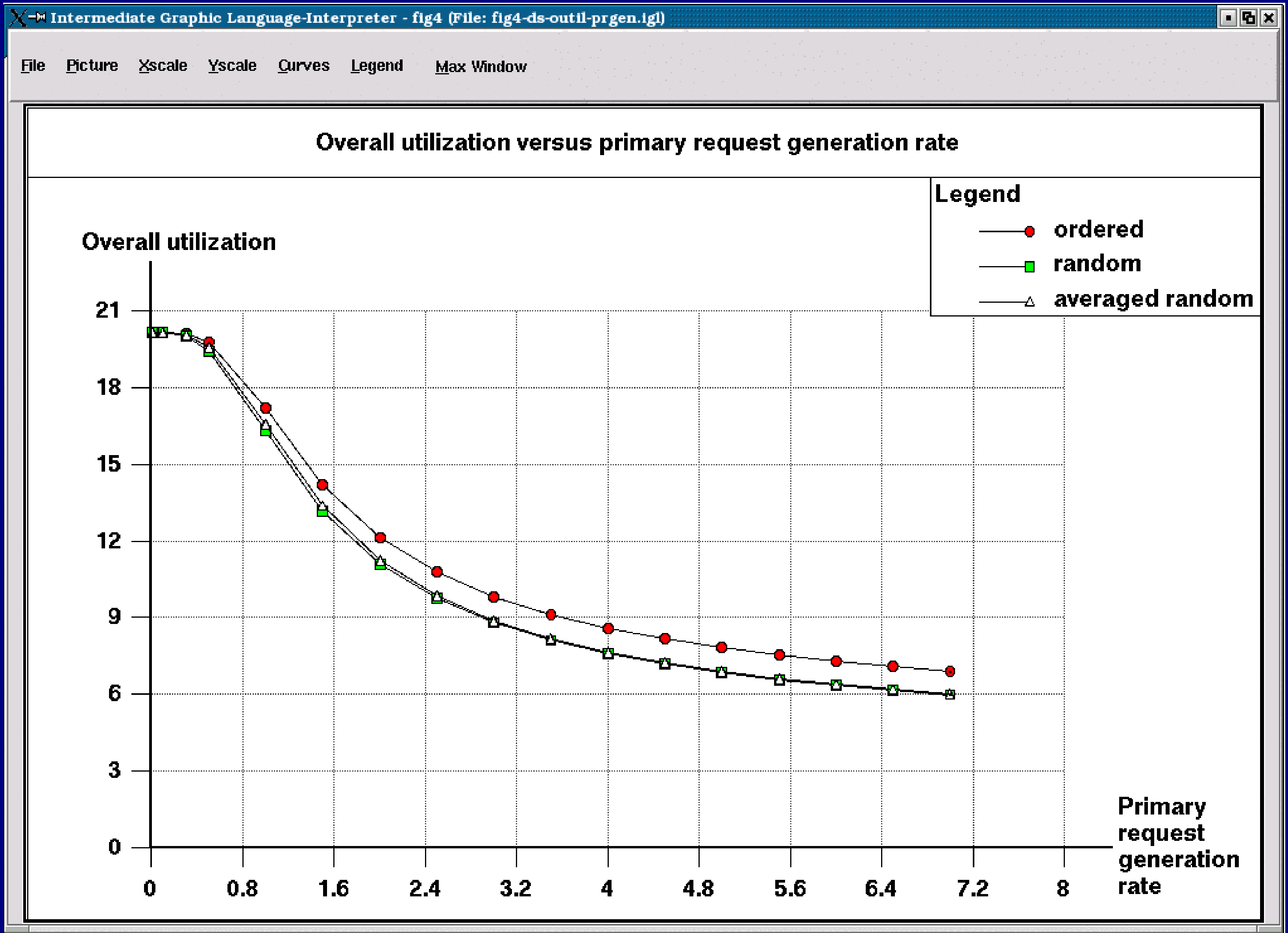
$E[T]$ versus retrial rate



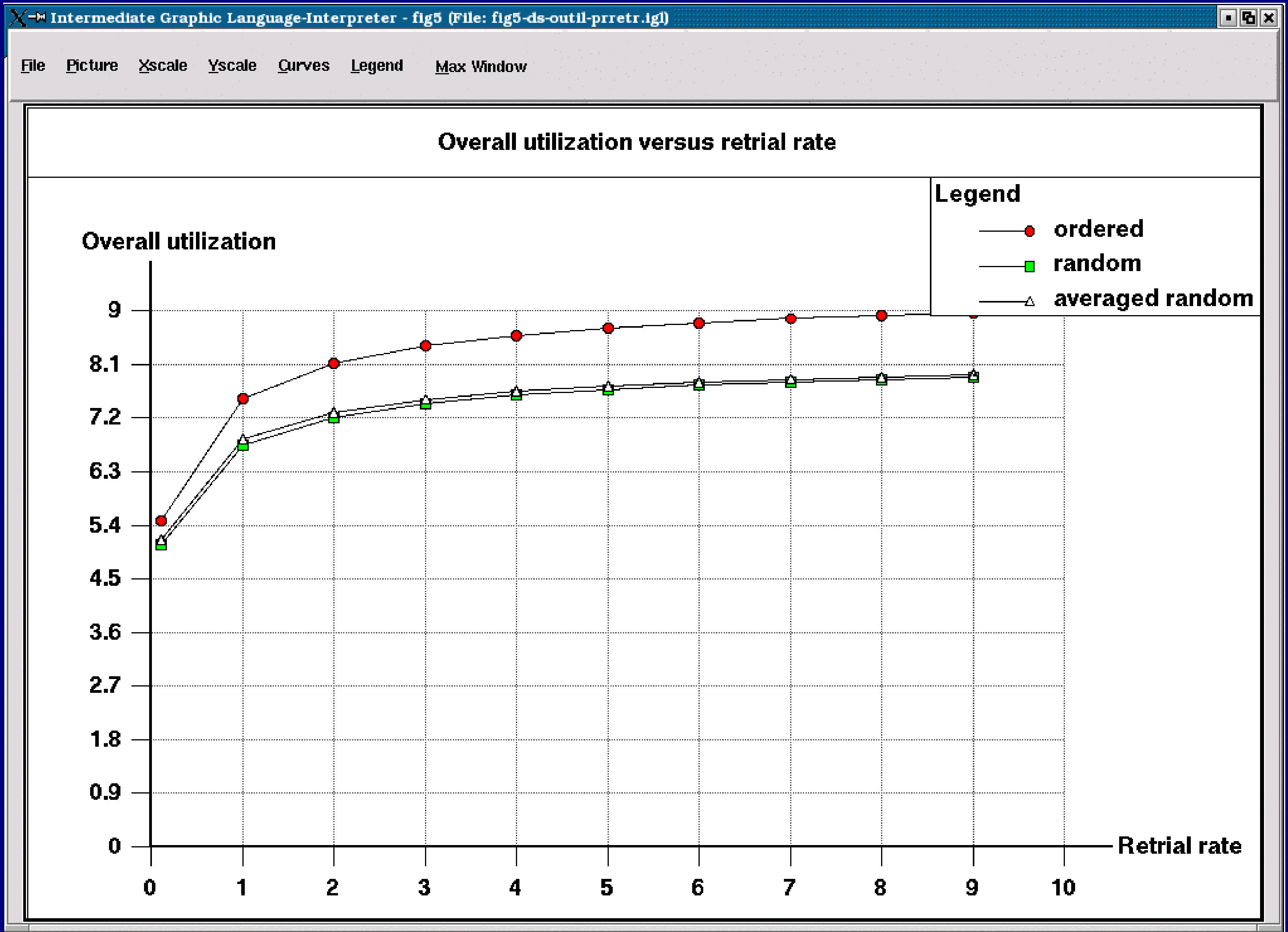
$E[T]$ versus server's failure rate



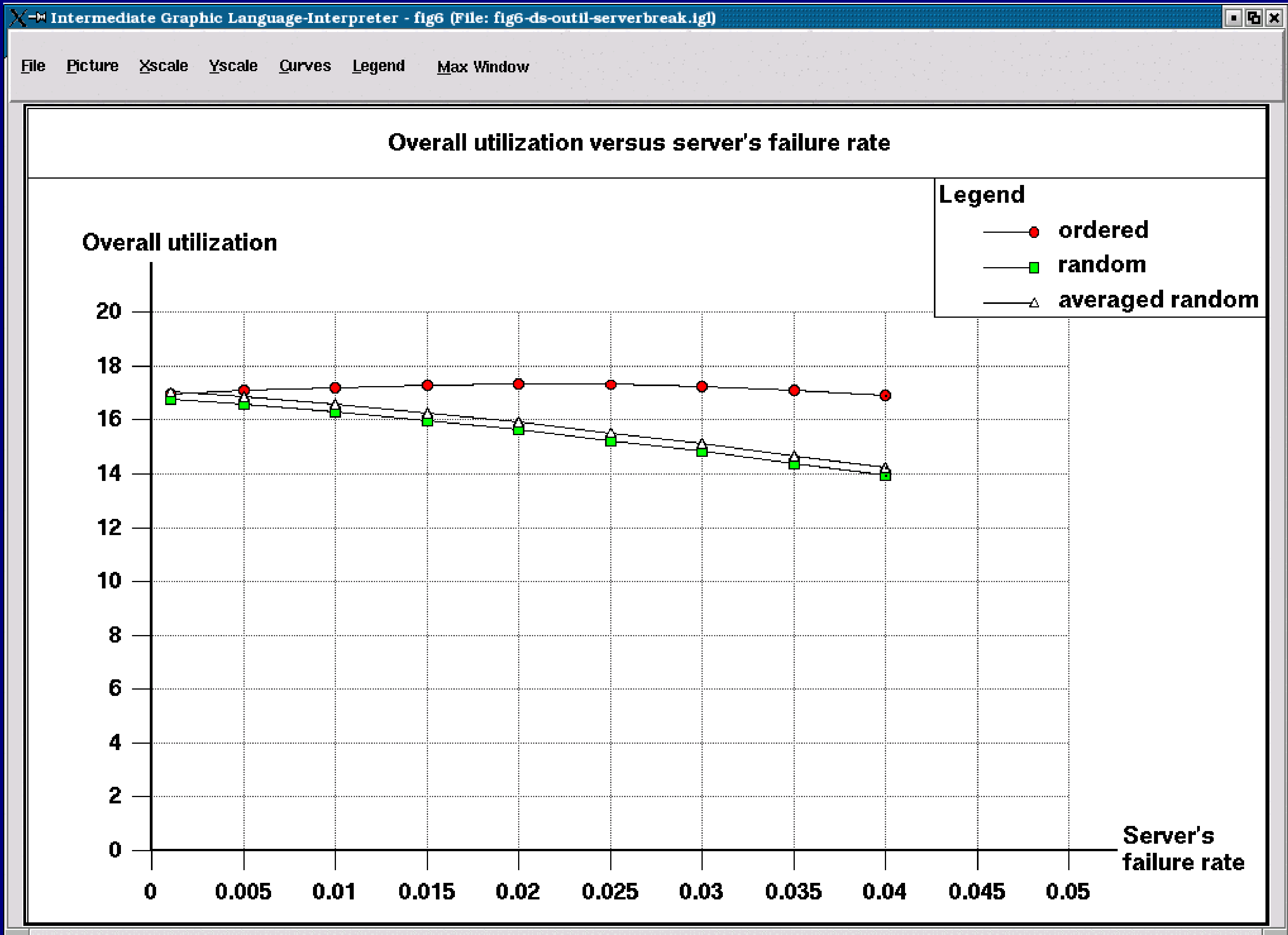
Server utilization versus server's failure rate



Overall utilization versus primary request generation rate



Overall utilization versus retrial rate



Overall utilization versus server’s failure rate

References

[1] **Almási B., Roszik J., and Sztrik J.** Homogeneous finite-source retrial queues with server subject to breakdowns and repairs, *Computers and Mathematics wit Applications* (accepted).

[2] **Begain K., Bolch G., Herold H.** *Practical Performance Modeling, Application of the MOSEL Language*, Kluwer Academic Publisher, Boston, 2001.

[3] **Falin G.I. and Templeton J.G.C.** *Retrial queues*, Chapman and Hall, London, 1997.

[4] **Pourbabai B.** Markovian queueing systems with retrials and heterogeneous servers, *Computers and Mathematics with Applications* 13(1987), 917-923.

[5] **Wang Jinting, Cao Jinhua and Li Quanlin** Reliability analysis of the retrial queue with server breakdowns and repairs, *Queueing Systems Theory and Applications* 38(2001), 363–380.