

Multiserver Retrieval Queues with Finite Number of Heterogeneous Sources

J. Roszik, B. Almási, J. Sztrik

Institute of Informatics, University of Debrecen
Debrecen, Hungary

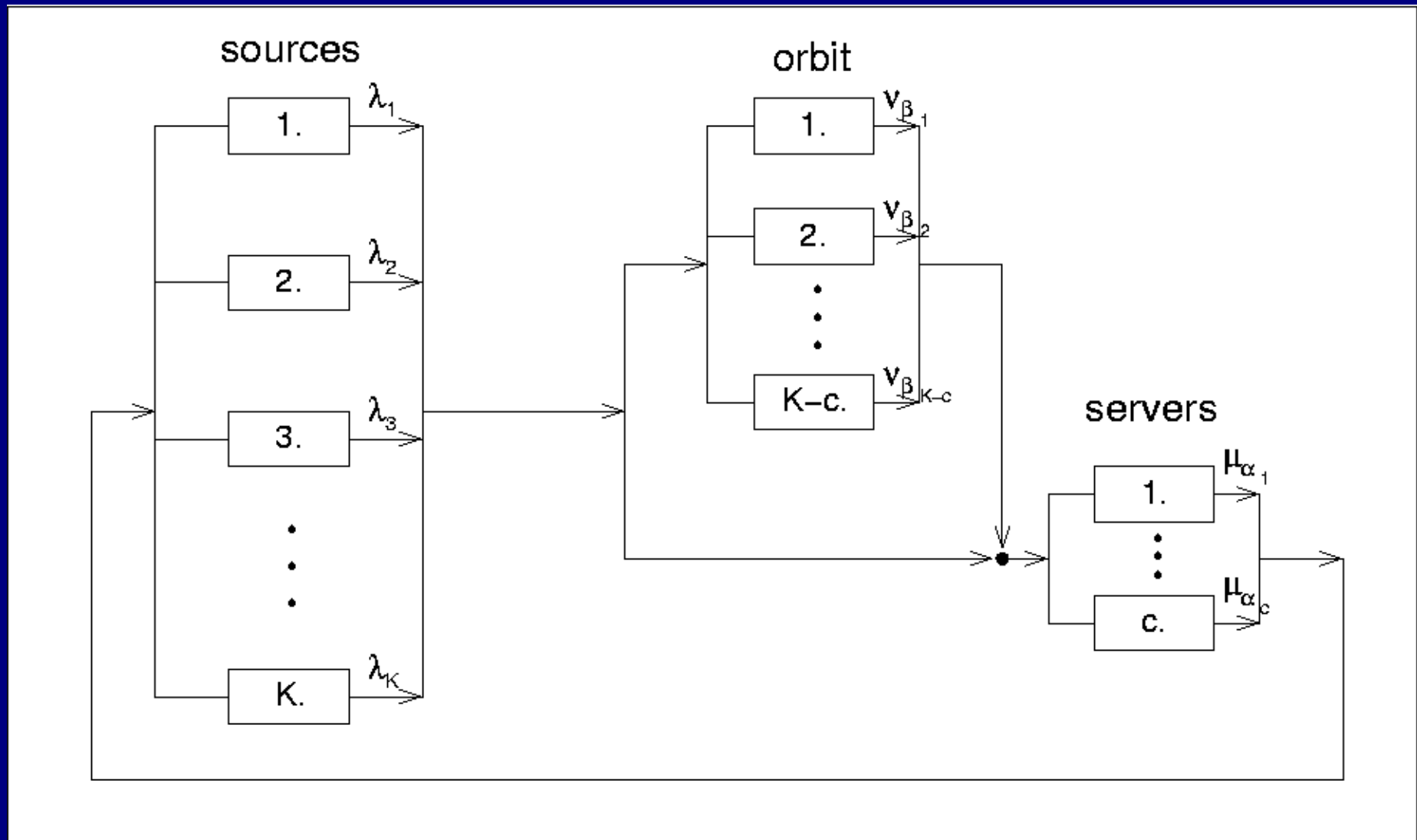
e-mail: jroszik@inf.unideb.hu

www: <http://irh.inf.unideb.hu/user/jroszik/>

OUTLOOK

- **The queueing model**
- **Applications**
- **Mathematical model**
- **The MOSEL tool**
- **Case studies**
- **References**

The queueing model



Applications

- **magnetic disk memory systems**
- **local area networks with CSMA/CD protocols**
- **cellular mobile networks**

Mathematical model

- The system state at time t can be described with the process

$$X(t) = (\alpha_1, \dots, \alpha_c, \beta_1, \dots, \beta_{N(t)})$$

- where

- c is the number of servers
- $N(t)$ is the number of sources of repeated calls at time t
- the indices of the sources at the servers are denoted by α_i ,
 $i=1, \dots, c$
- the indices of the sources in the orbit are denoted by β_j ,
 $j=1, \dots, N(t)$

We define the stationary probabilities:

$$\mathbf{P}(i_1, \dots, i_c, 0) = \lim_{t \rightarrow \infty} \mathbf{P}\{\alpha_1 = i_1, \dots, \alpha_c = i_c, N(t) = 0\}$$

$$\mathbf{P}(i_1, \dots, i_c, j_1, \dots, j_k) = \lim_{t \rightarrow \infty} \mathbf{P}\{\alpha_1 = i_1, \dots, \alpha_c = i_c, \beta_1 = j_1, \dots, \beta_k = j_k\},$$
$$k = 1, \dots, K - c$$

Once we have obtained these limiting probabilities the system performance measures can be derived in the following way.

1. The probability of the i -th source is sending repeated calls

$$N_i = \sum_{i_1, \dots, i_c} \sum_{k=1}^{K-c} \sum_{\substack{j_1, \dots, j_k \notin \{i_1, \dots, i_c\} \\ i \in \{j_1, \dots, j_k\}}} P(i_1, \dots, i_c, j_1, \dots, j_k), \quad i = 1, \dots, K.$$

2. The probability of the i -th source is under service

$$Y_i = \sum_{\substack{i_1, \dots, i_c \\ i \in \{i_1, \dots, i_c\}}} P(i_1, \dots, i_c, 0) +$$

$$\sum_{\substack{i_1, \dots, i_c \\ i \in \{i_1, \dots, i_c\}}} \sum_{k=1}^{K-c} \sum_{j_1, \dots, j_k \notin \{i_1, \dots, i_c\}} P(i_1, \dots, i_c, j_1, \dots, j_k), \quad i = 1, \dots, K.$$

3. The mean rate of generation of primary calls of the i -th source

$$\bar{\lambda}_i = \lambda_i(1 - Y_i - N_i), \quad i = 1, \dots, K.$$

4. The mean waiting time of the i -th source

$$\bar{W}_i = \frac{N_i}{\bar{\lambda}_i}, \quad i = 1, \dots, K.$$

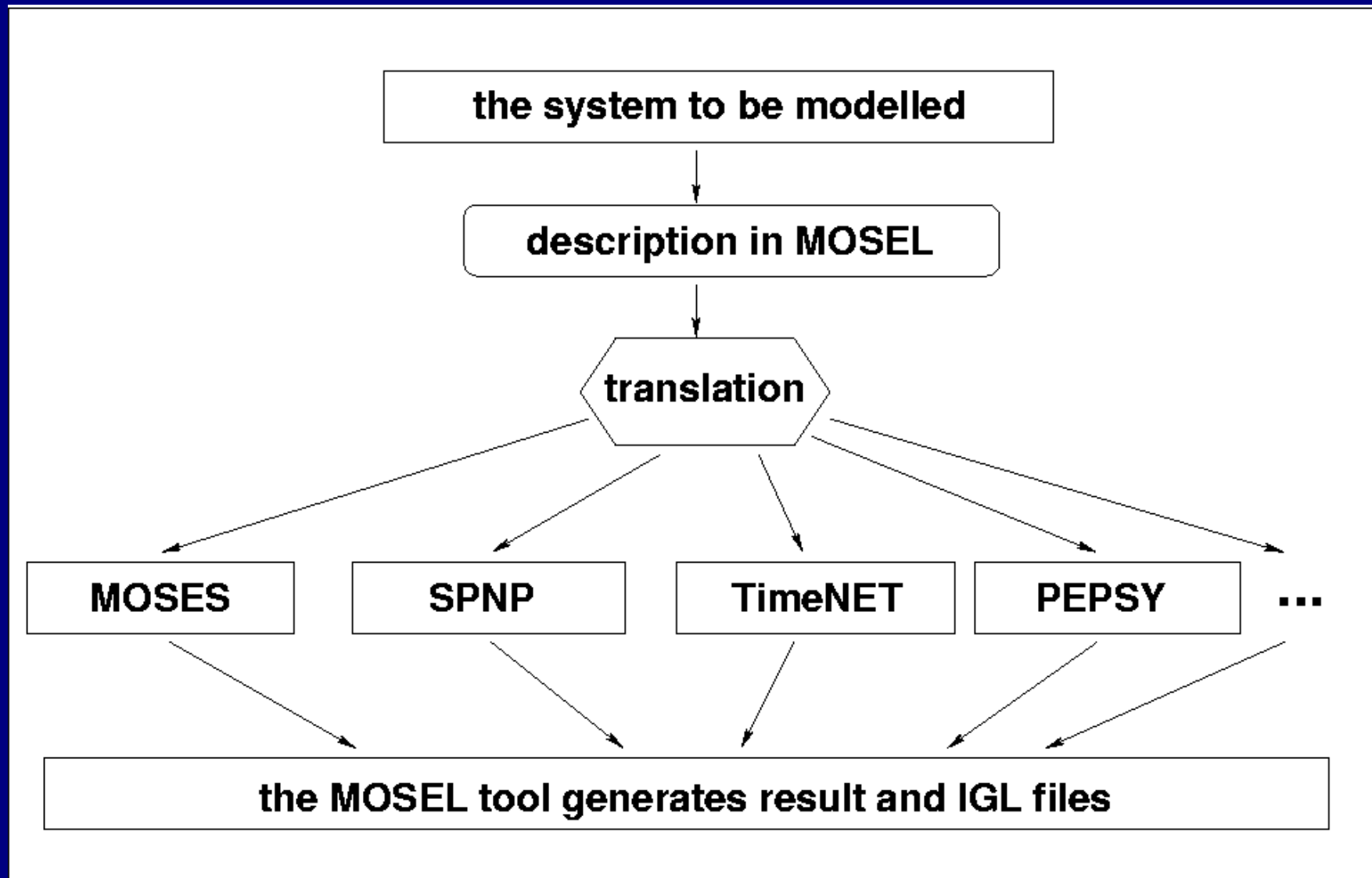
5. The mean response time of the i -th source

$$\bar{T}_i = \frac{N_i}{\bar{\lambda}_i} + \frac{1}{\mu_i}, \quad i = 1, \dots, K.$$

6. The utilization of the i -th source

$$U_i = 1 - N_i - Y_i, \quad i = 1, \dots, K.$$

The MOSEL (Modeling, Specification and Evaluation Language) tool



Validations in the homogeneous multiserver case

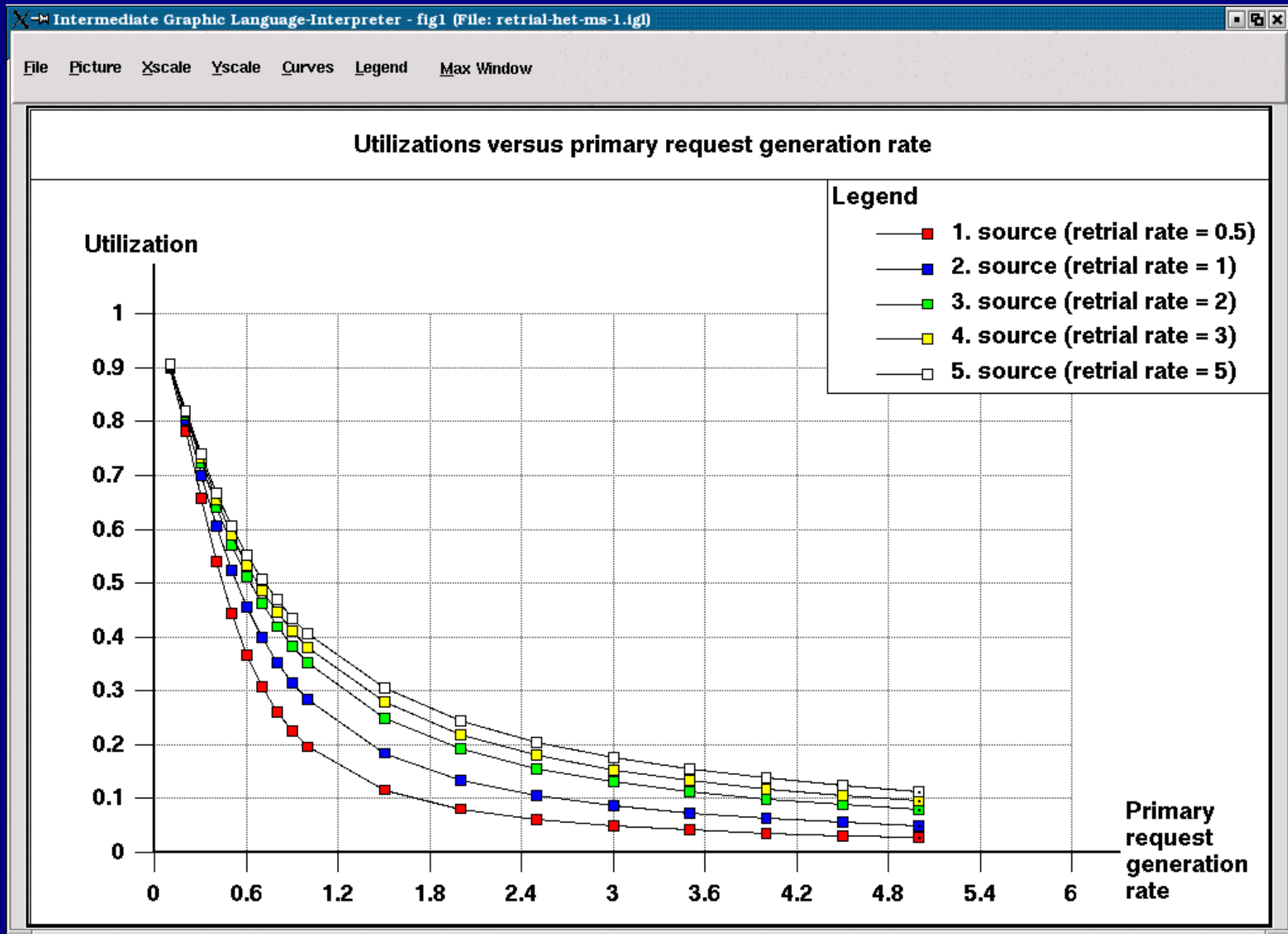
	MOSEL program	Pascal program
Mean no. of busy servers:	1.2263326334	1.2263325906
Mean no. of retrying sources:	0.7078359870	0.7078359330
Mean response time:	0.5771974881	0.5771973594

The results are the same at least up to the 6th decimal digit.

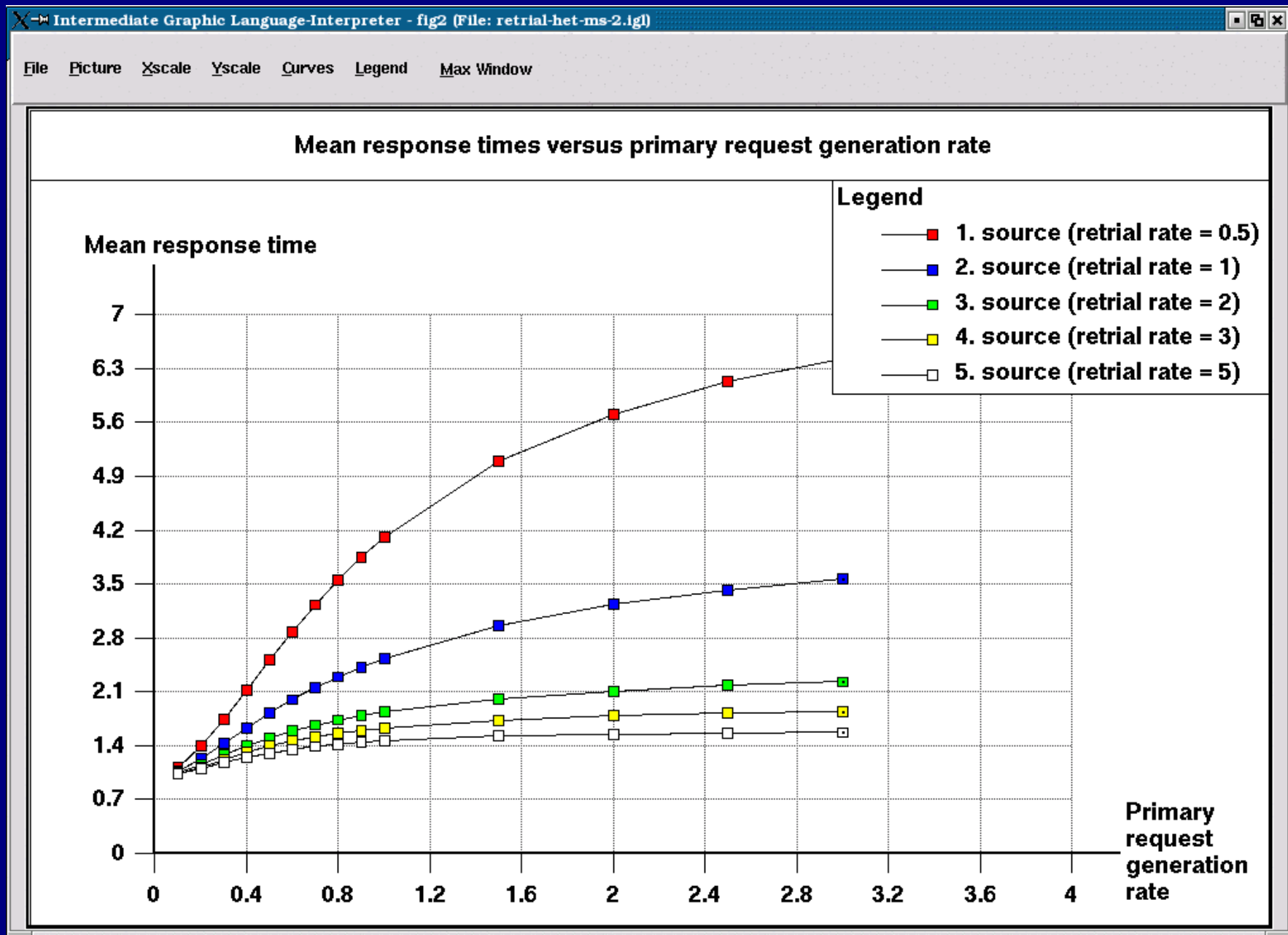
Validations in the heterogeneous case

	repeated attempts	FIFO
Utilization of the server:	0.5454545965	0.5454545840
Utilizations of the sources:		
1. source:	0.8636363668	0.8636362907
2. source:	0.7727272562	0.7727274804
3. source:	0.6090908485	0.6090907576
Mean response times:		
1. source:	1.5789473265	1.5789483457
2. source:	1.4705883740	1.4705864961
3. source:	1.2835824163	1.2835829064

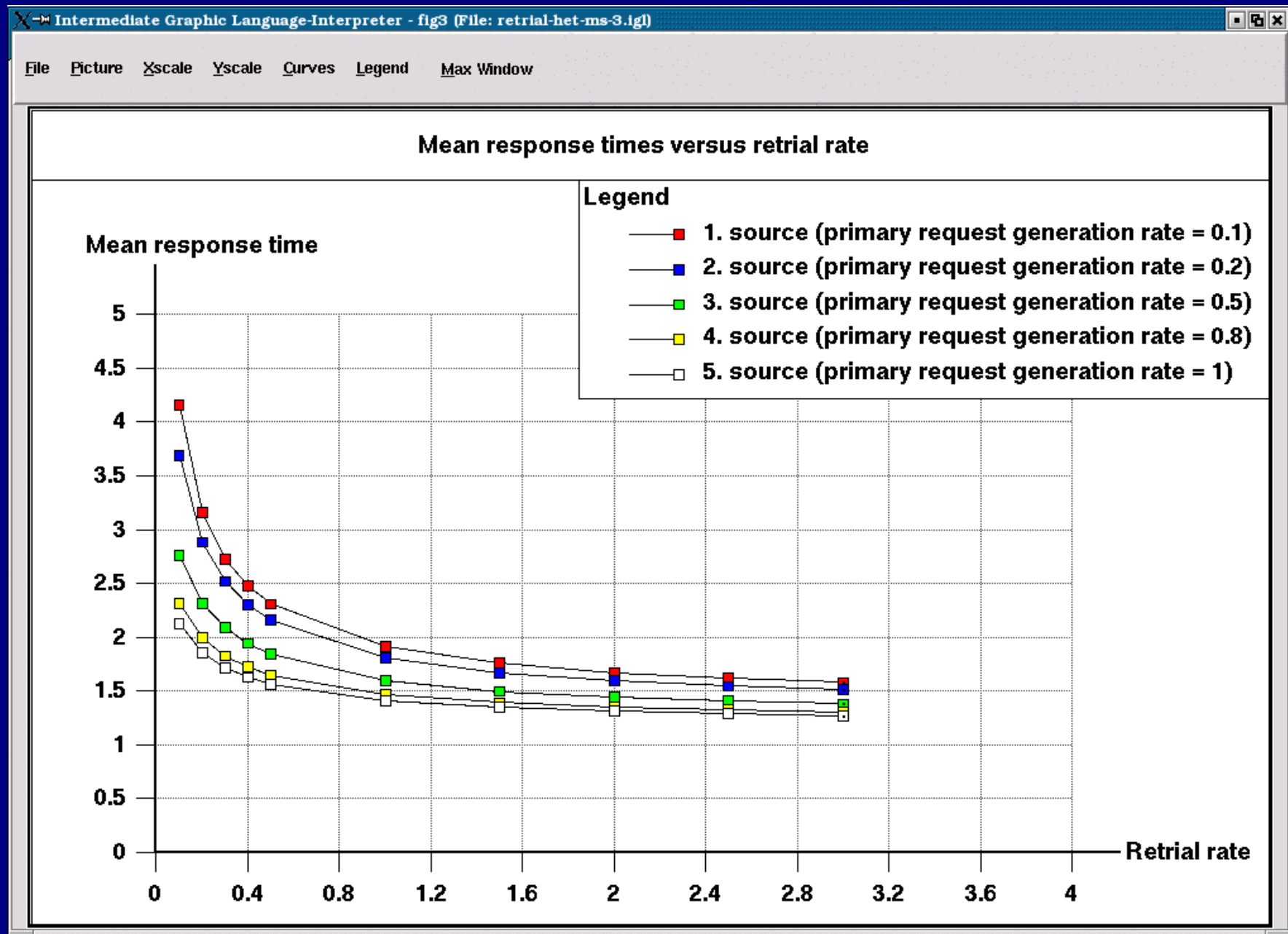
The results are the same at least up to the 5th decimal digit.



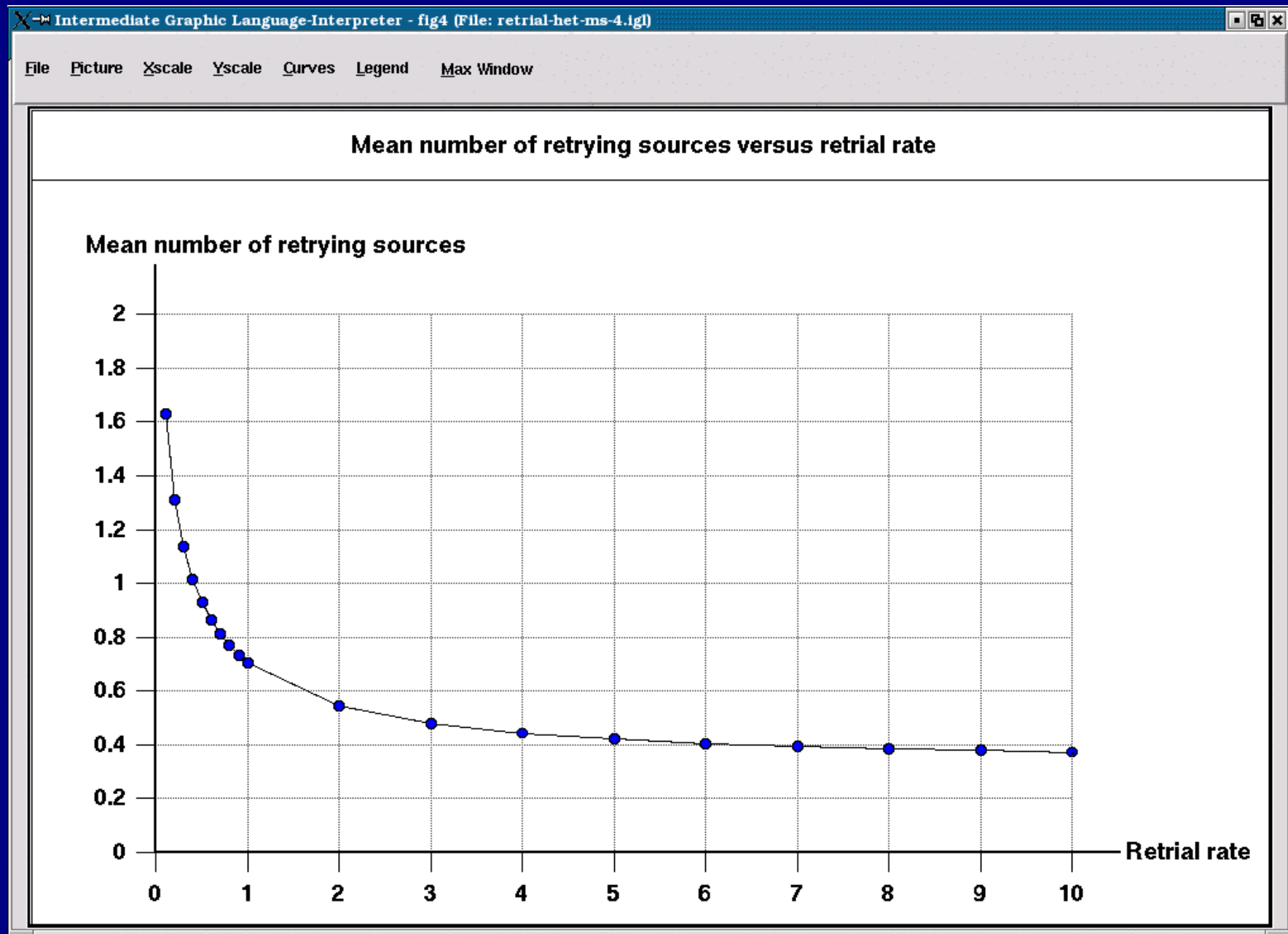
Utilizations versus primary request generation rate



Mean response times versus primary request generation rate



Mean response times versus retrial rate



Mean number of retrying sources versus retrieval rate

References

- [1] **Begain K., Bolch G., Herold H.** *Practical Performance Modeling, Application of the MOSEL Language*, Kluwer Academic Publisher, Boston, 2001.
- [2] **Falin G.I. and Templeton J.G.C.** *Retrieval queues*, Chapman and Hall, London, 1997.
- [3] **Artalejo J.R.** Retrieval queues with a finite number of sources, *J. Korean Math. Soc.*, 35(1998), 503-525.
- [4] **Falin G.I. and Artalejo J.R.** A finite source retrieval queue, *European Journal of Operational Research* 108(1998) 409-424.