HETEROGENEOUS FINITE-SOURCE RETRIAL QUEUES WITH SERVER SUBJECT TO BREAKDOWNS AND REPAIRS

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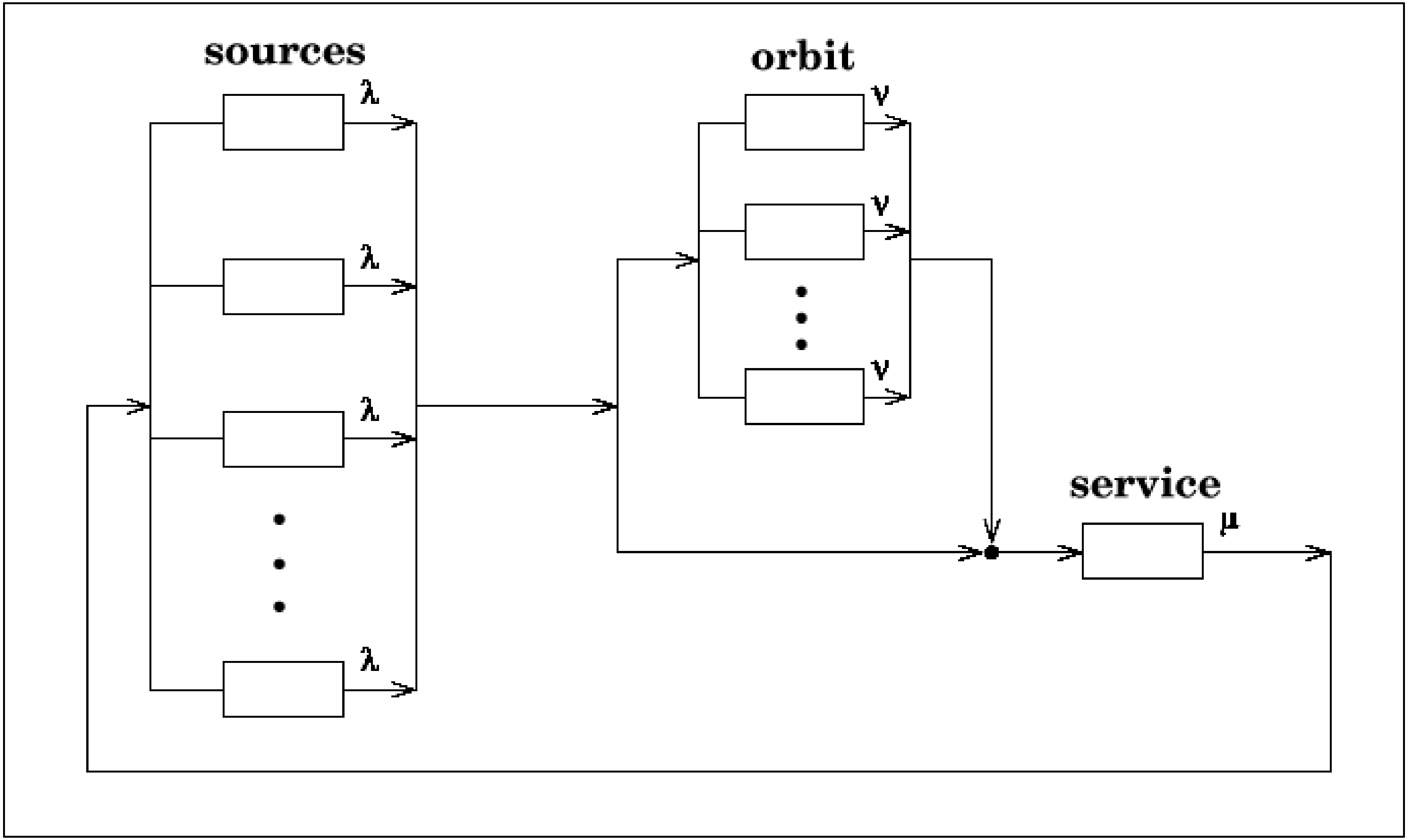
OUTLOOK

- **The queueing model**
- **Applications**
- **Mathematical model**
- **Evaluation Tool MOSEL**
- **Case studies**
- **References**

The queueing model



Applications

- **magnetic disk memory systems**
- **local area networks with CSMA/CD protocols**
- **collision avoidance local area network modeling**

Mathematical model

$$X(t) = ((Y(t); \alpha_{C(t)}; \beta_1, ..., \beta_{N(t)}), t \geq 0)$$

where

$Y(t) = 0$ if the server is up,
$Y(t) = 1$ if the server is down,
$C(t) = 0$ if the server is idle,
$C(t) = 1$ if the server is busy,
$\alpha_{C(t)}$ is the index of the request under service at time t if the server is busy.
$N(t)$ is the number of sources of repeated calls at time t,
β_j, $j = 1, ..., N(t)$ is the indices of request staying in the orbit.

We define the stationary probabilities:

$$P(q;0;0) = \lim_{t\to\infty} P(Y(t) = q; C(t) = 0; N(t) = 0),$$
$$P(q;j;0) = \lim_{t\to\infty} P(Y(t) = q; \alpha_1 = j; N(t) = 0),$$
$$P(q;0;i_1,...,i_k) = \lim_{t\to\infty} P(Y(t) = q; C(t) = 0; \beta_1 = i_1, ..., \beta_k = i_k),$$
$$P(q;j;i_1,...,i_k) = \lim_{t\to\infty} P(Y(t) = q; \alpha_1 = j; \beta_1 = i_1, ..., \beta_k = i_k).$$

Once we have obtained these limiting probabilities the **main system performance measures** can be derived in the following way.

1. The server utilization with respect to source j

$$U_j = \sum_{k=0}^{K-1} \sum_{i_1,\ldots,i_k \neq j} P(0; j; i_1, \ldots, i_k).$$

Hence the *server utilization*

$$U_S = \sum_{j=1}^{K} U_j.$$

2. Utilization of source i

$U^{(i)}$ = P (source i generates a new primary call).

3. Utilization of the repairman

$$U_R = E[Y(t)] = \sum_{j=0}^{K} \sum_{k=0}^{K-1} \sum_{i_1,...,i_k \neq j} P(1; j; i_1, ..., i_k).$$

4. Availability of the server

$$A_S = 1 - U_R.$$

5. Mean response time of source i

$$P_O^{(i)} = \sum_{q=0}^{1} \sum_{j=0, j \neq i}^{K} \sum_{k=1}^{K-1} \sum_{i \epsilon (i_1,...,i_k)} P(q; j; i_1, ..., i_k).$$

$$P_S^{(i)} = \sum_{q=0}^{1} \sum_{k=0}^{K-1} \sum_{i \neq i_1,...,i_k)} P(q; i; i_1, ..., i_k).$$

Hence, the probability $P^{(i)}$ that request i is at the service facility can be obtained by

$$P^{(i)} = P_S^{(i)} + P_O^{(i)}.$$

The *throughput* of request i is

$$\gamma_i = \frac{1}{E[T_i] + E[S_i]} = \lambda_i U^{(i)} = \mu_i U_i, \qquad i = 1, ..., K.$$

It is easy to see that

$$E[S_i] = E[D_i] + 1/\lambda_i \geq 1/\lambda_i,$$

where $E[D_i]$ denotes the mean delay time due to the failure of the server.

Similarly,

$$U^{(i)} = \frac{1/\lambda_i}{E[T_i] + E[S_i]} = \frac{\mu_i U_i}{\lambda_i} \leq 1 - P^{(i)}, \qquad i = 1, ..., K,$$

$$P^{(i)} = \frac{E[T_i]}{E[T_i] + E[S_i]} = \gamma_i E[T_i] = \lambda_i U^{(i)} E[T_i], \qquad i = 1, ..., K,$$

and hence,

$$E[T_i] = \frac{P^{(i)}}{\lambda_i U^{(i)}} = \frac{P^{(i)}}{\mu_i U_i}, \qquad i = 1, ..., K.$$

6. *Mean waiting time of source i*

$$E[W_i] = E[T_i] - 1/\mu_i = \frac{P^{(i)} - U_i}{\mu_i U_i}, \qquad i = 1, ..., K.$$

7. *Mean number of calls staying at the service facility*

$$M = E[C(t)+N(t)] = \sum_{i=1}^{K} P^{(i)} = \sum_{i=1}^{K} (P_S^{(i)} + P_O^{(i)}) = \sum_{i=1}^{K} P_S^{(i)} + \sum_{i=1}^{K} P_O^{(i)}.$$

8. *Mean rate of generation of primary calls*

$$\overline{\lambda} = \sum_{i=1}^{K} \gamma_i = \sum_{i=1}^{K} \lambda_i U^{(i)} = \sum_{i=1}^{K} \mu_i U_i.$$

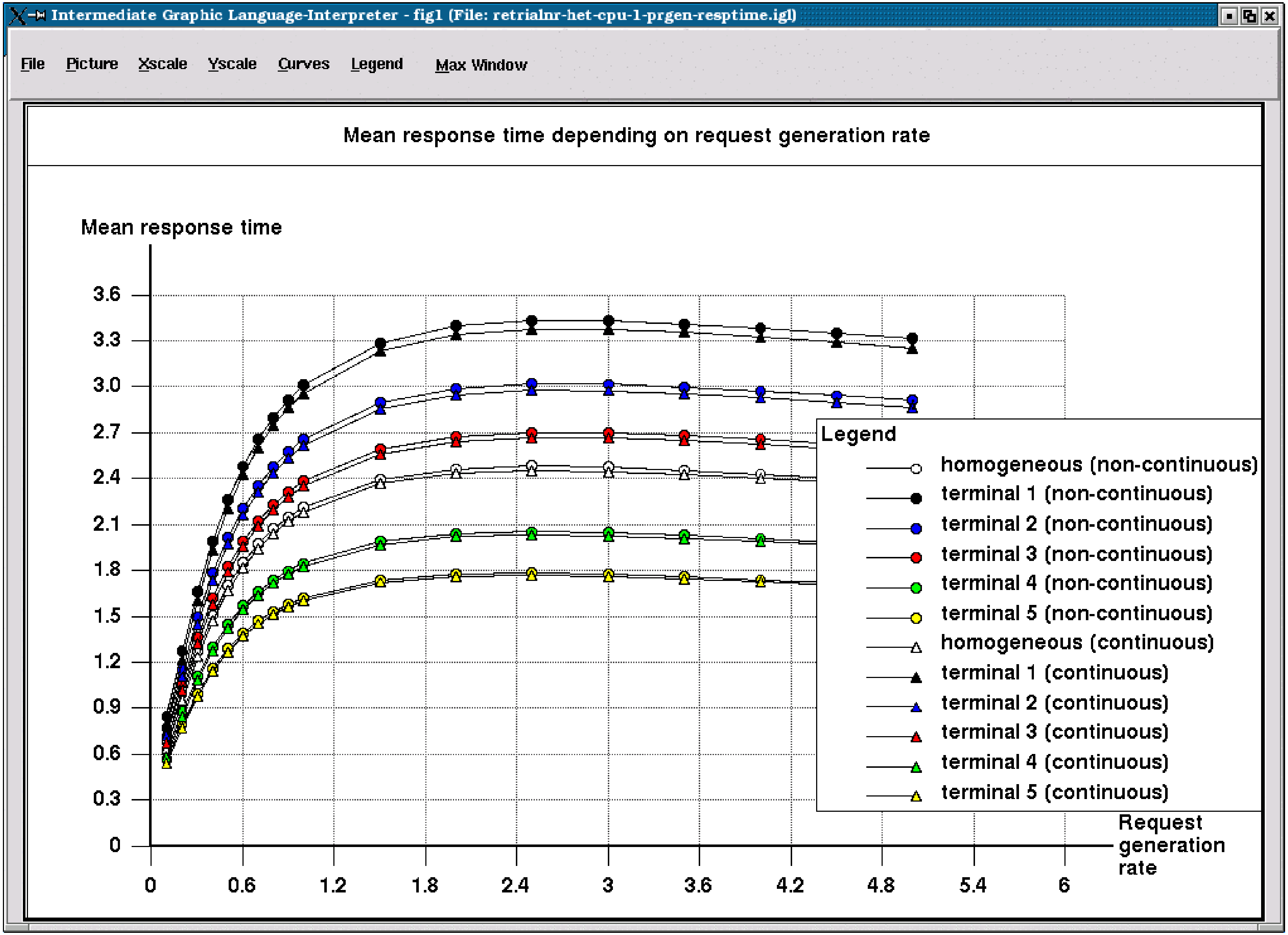
Evaluation Tool MOSEL

MOSEL (Modeling, Specification and Evaluation Language) developed at the University of Erlangen, Germany, is used to formulate and solved the problem.

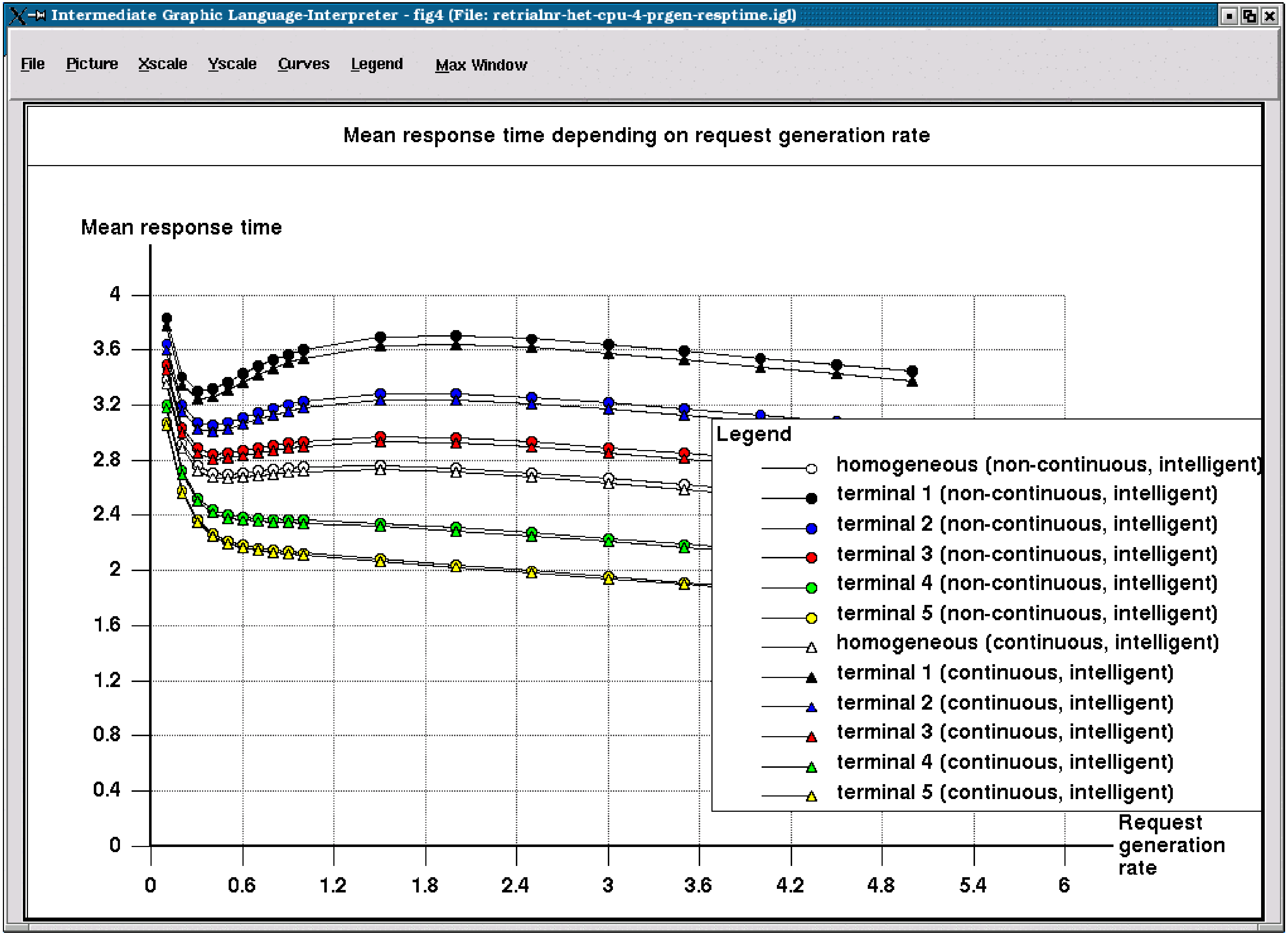
Case studies

Validation of results

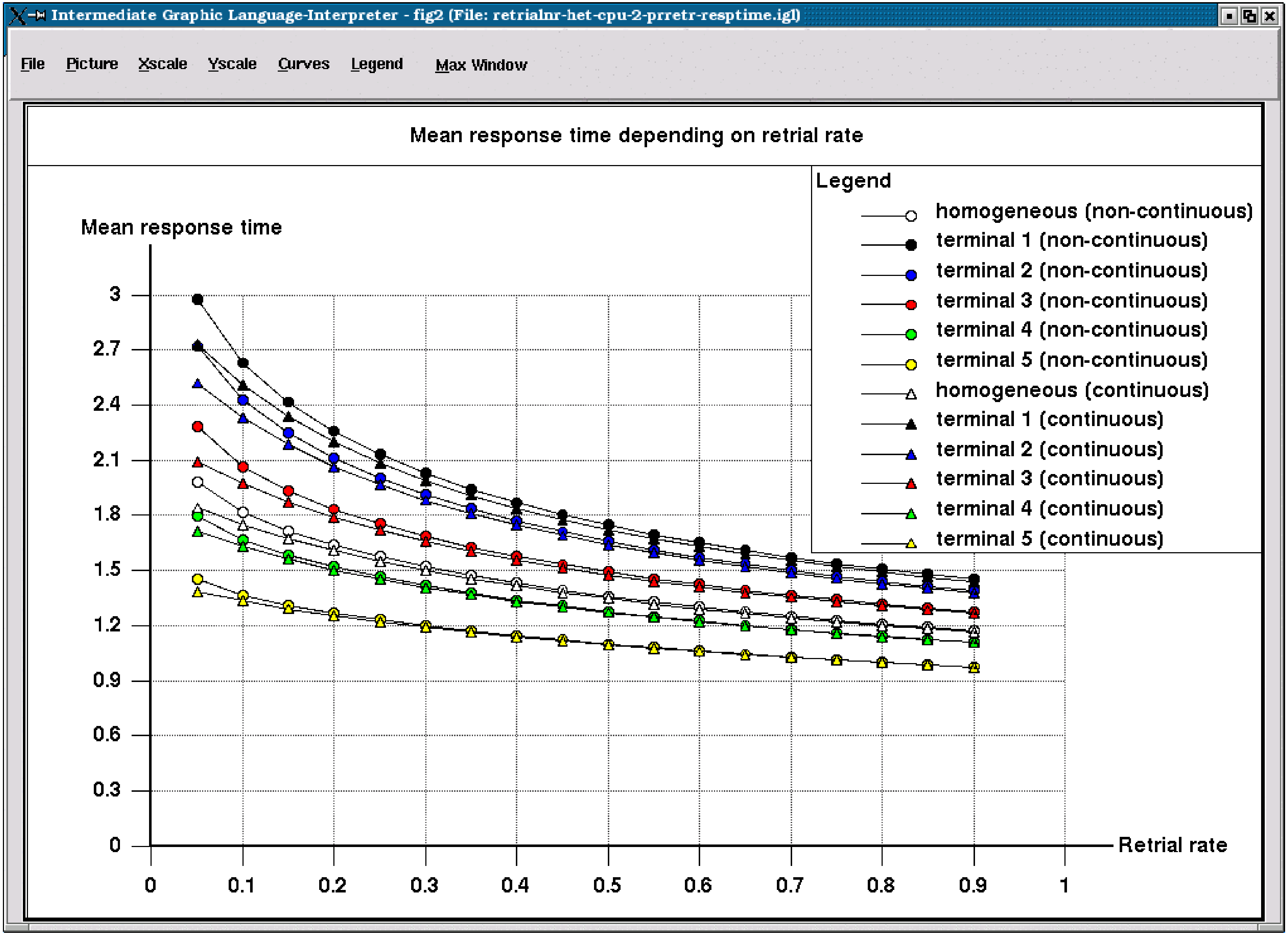
	non–rel. retrial(cont.)	non–rel. retrial(orbit)	non–rel. FIFO
Number of sources:	3	3	3
Request's generation rate:	0.2, 0.3, 0.5	0.2, 0.3, 0.5	0.2, 0.3, 0.5
Service rate:	1, 1.2, 1.1	1, 1.2, 1.1	1, 1.2, 1.1
Retrial rate:	1e+20	1e+20	-
Server's failure rate:	0.002	0.002	0.002
Server's repair rate:	0.04	0.04	0.04
Utilization of the server:	0.578593008176	0.578593460071	0.578595143583
Mean response time			
Source 1:	1.61016598407	1.61027143737	1.6109393482
Source 2:	1.41365083148	1.41357129589	1.41287007613
Source 3:	1.35362123345	1.35362137877	1.35372999206



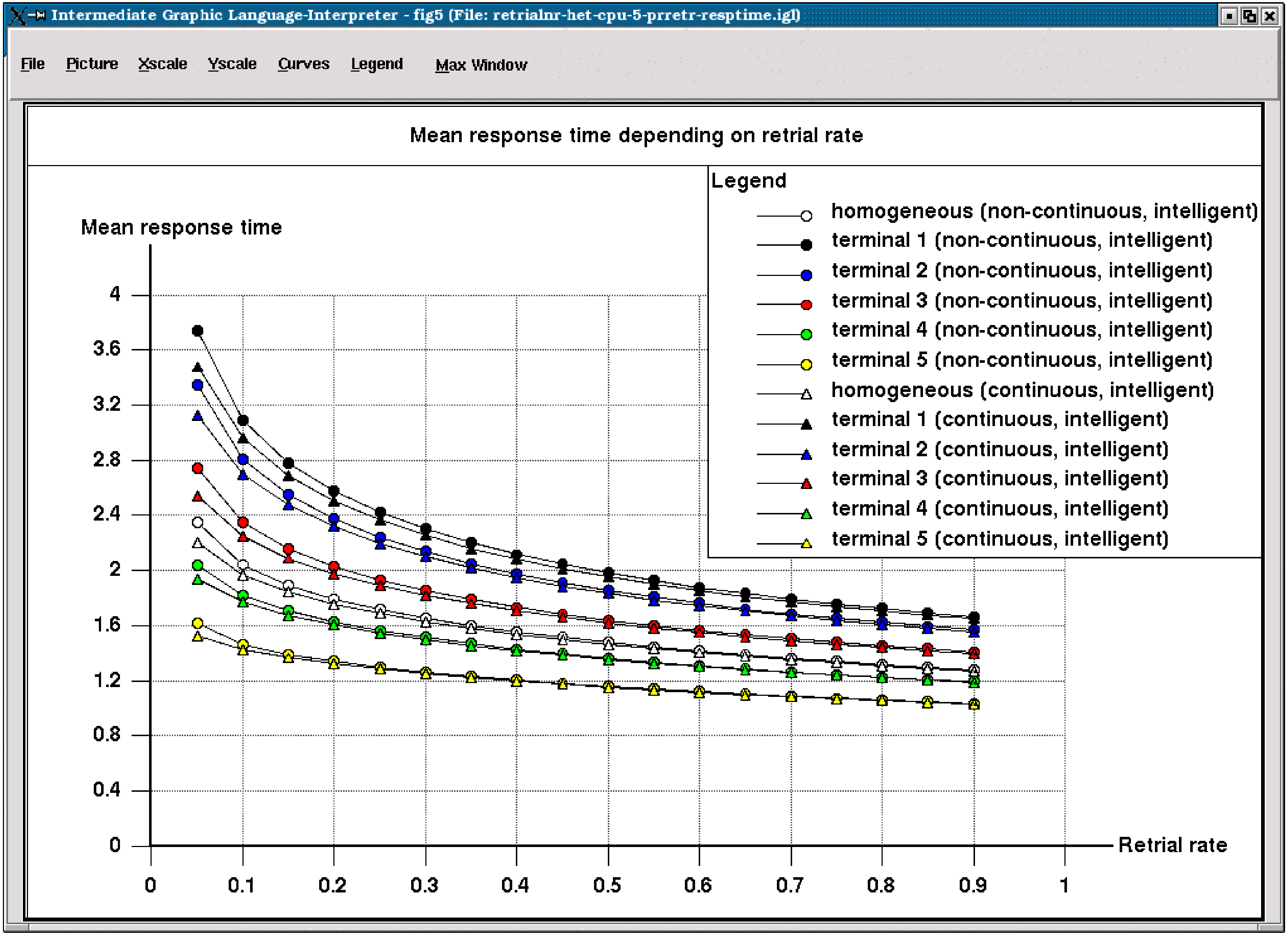
$E[T]$ versus primary request generation rate



$E[T]$ versus primary request generation rate



$E[T]$ versus retrial rate



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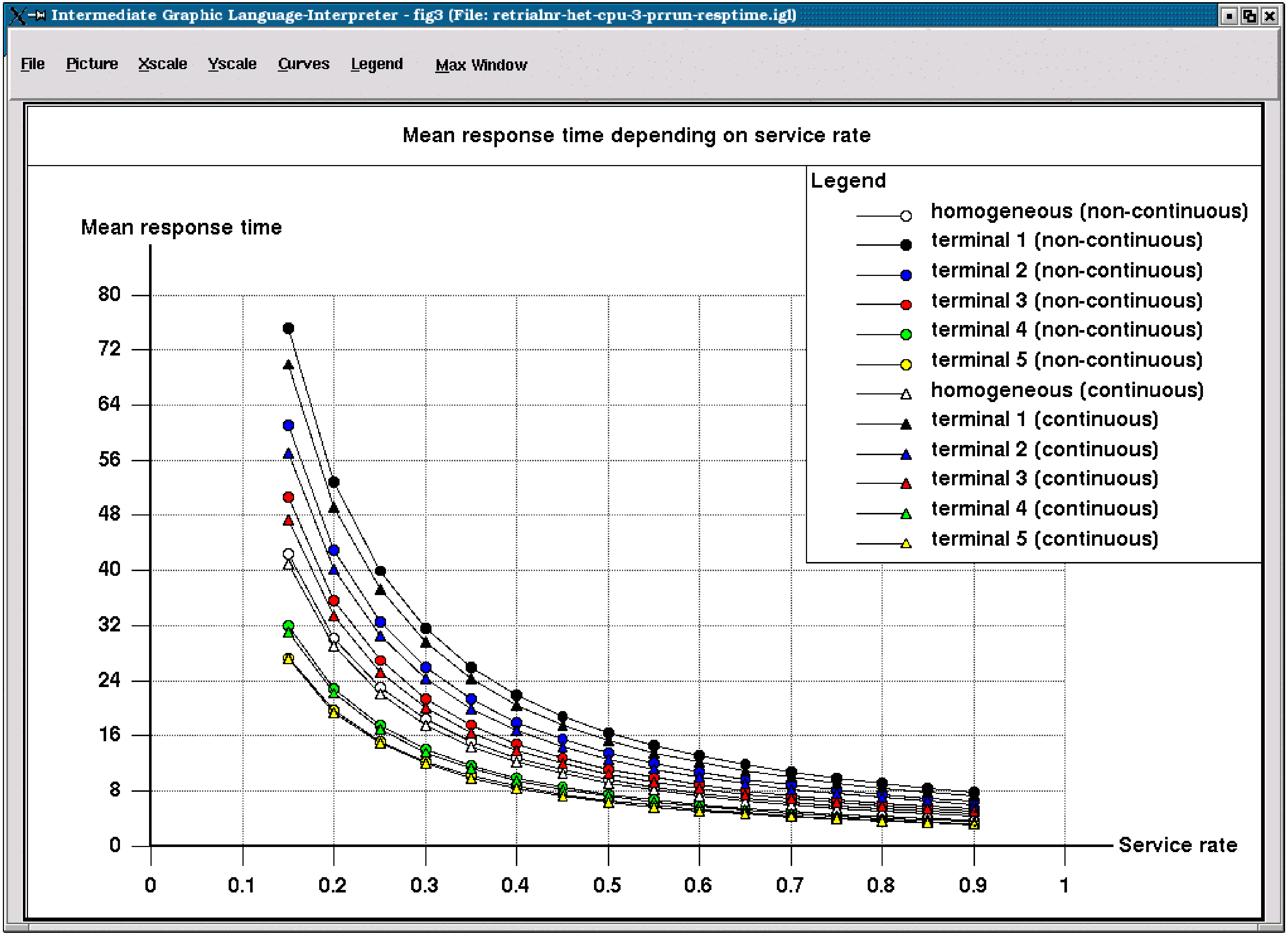
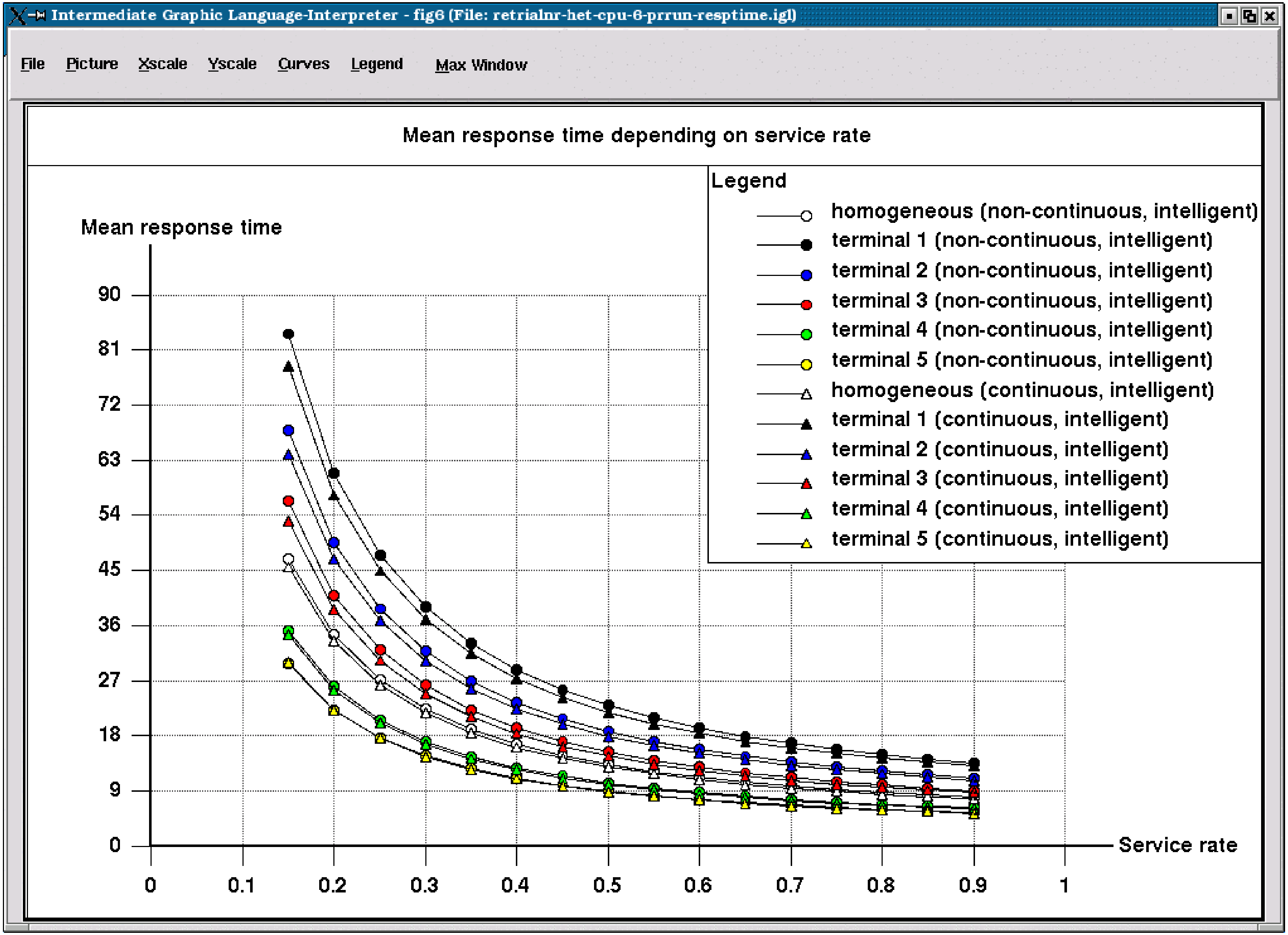


$E[T]$ versus service rate



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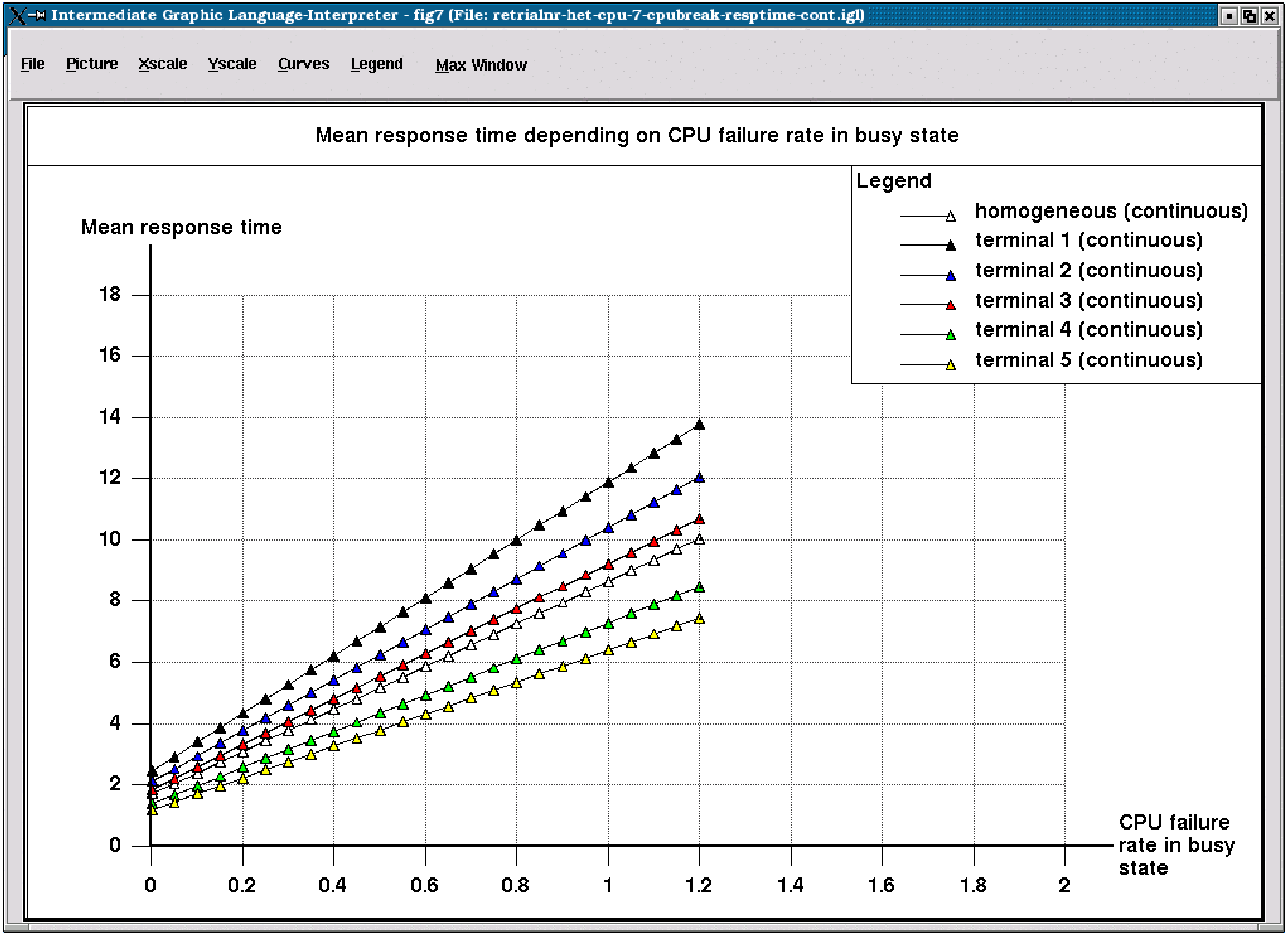
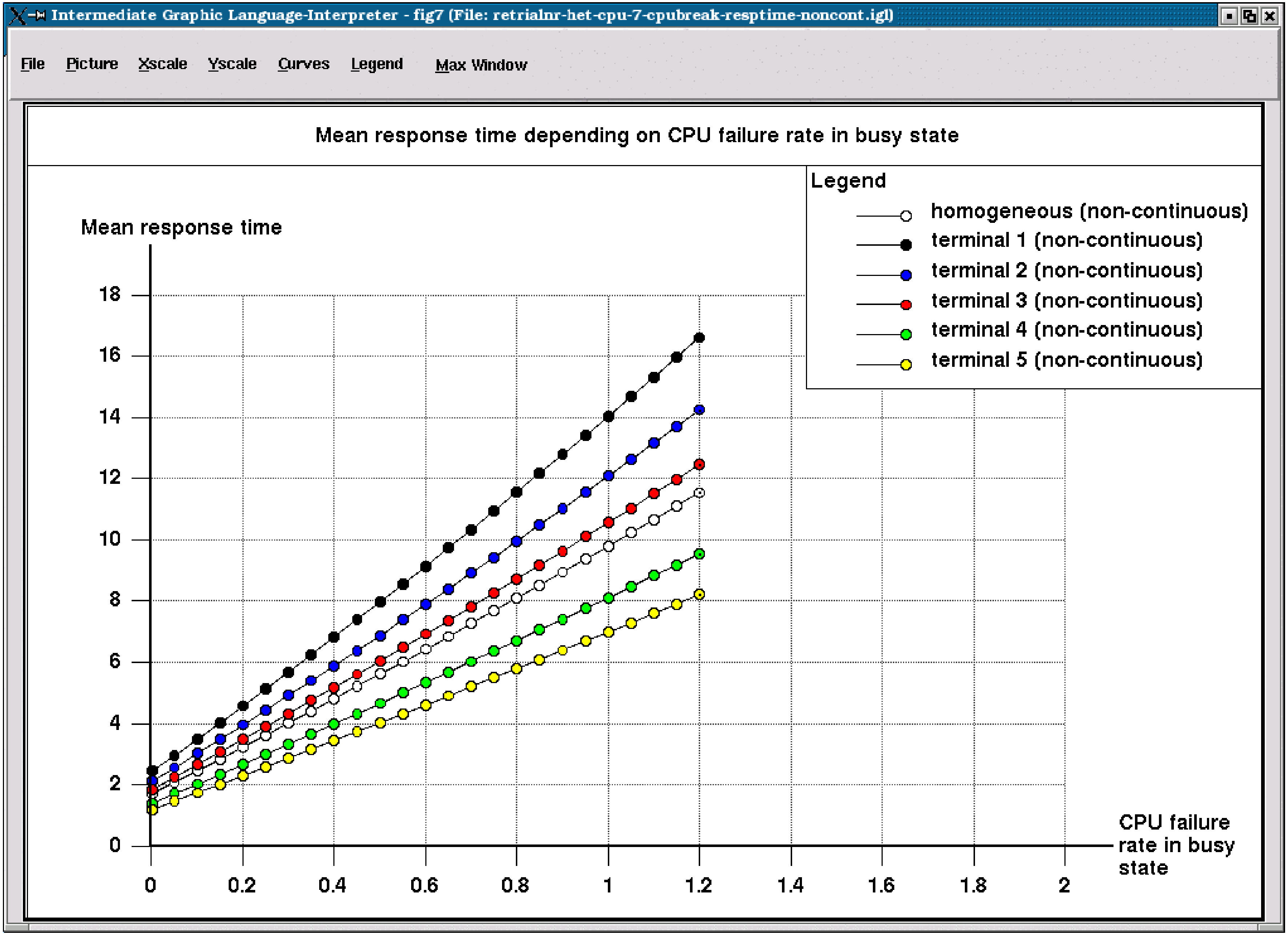


$E[T]$ versus CPU failure rate in busy state



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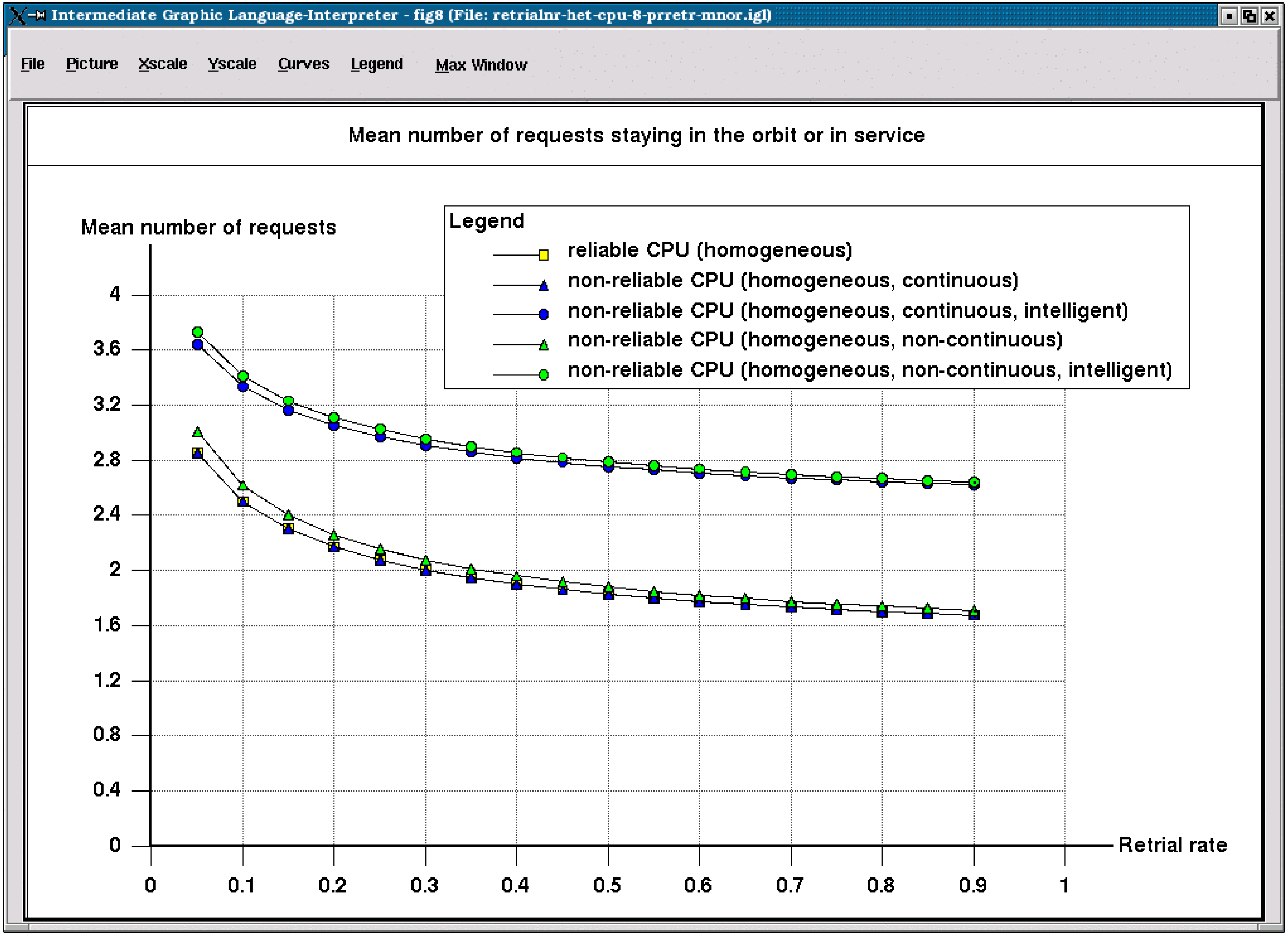
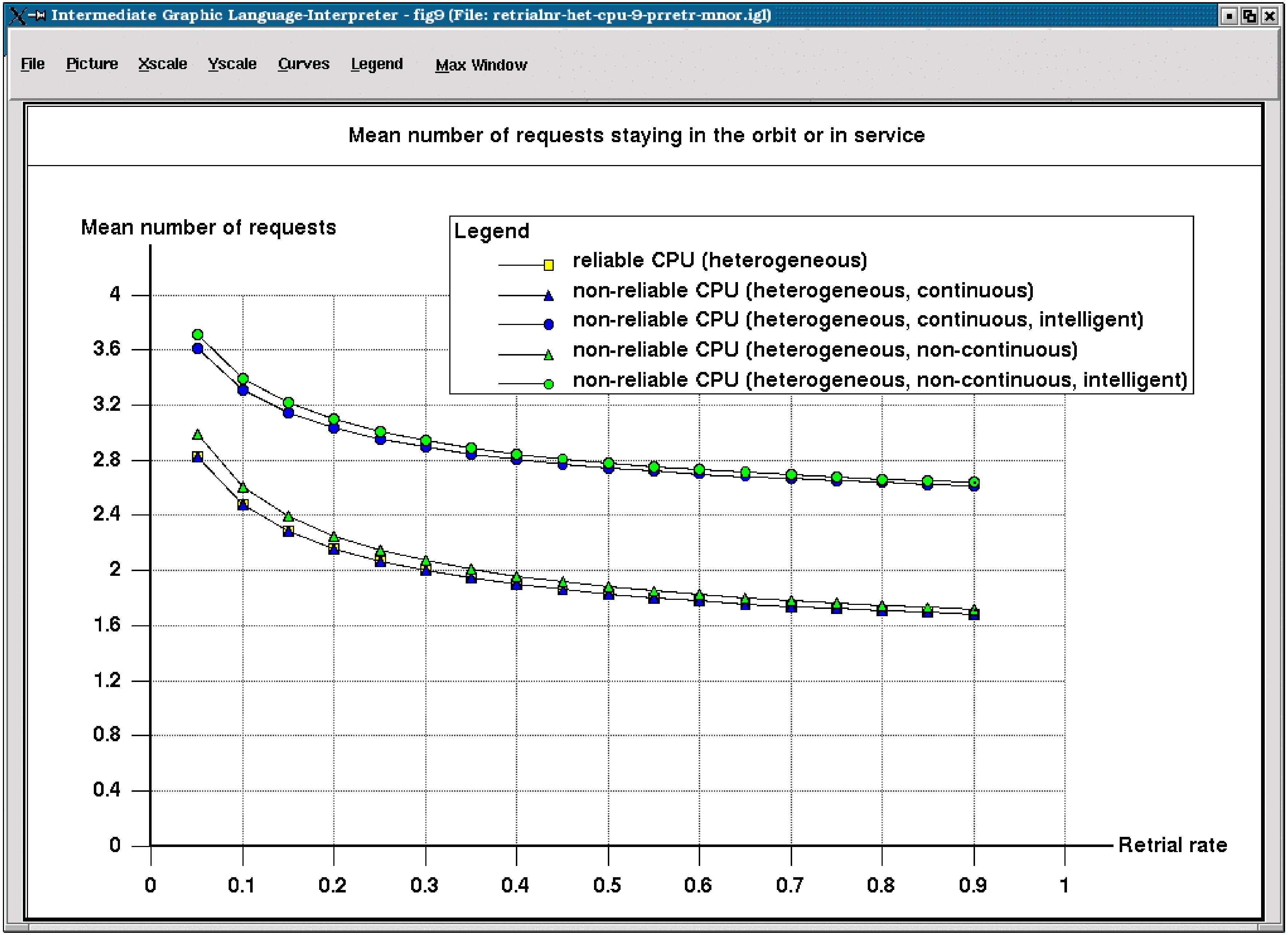


M versus retrial rate



M versus retrial rate

References

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