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GEORGIAN TECHNICAL UNIVERSITY

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**შ რ ო მ ე ბ ი**

მართვის ავტომატიზებული სისტემები

**TRANSACTIONS**

**AUTOMATED CONTROL SYSTEMS**

**№ 1(32), Vol. 1.1**



**თბილისი-TBILISI-ТБИЛИСИ**  
**2021**

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ГРУЗИНСКИЙ ТЕХНИЧЕСКИЙ УНИВЕРСИТЕТ

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„მართვის  
ავტომატიზებული  
სისტემების“  
კათედრის დაარსების  
50 წლის იუბილეს  
(1971-2021)



DEDICATED  
TO THE 50th FOUNDATION  
ANNIVERSARY OF THE  
CHAIR "AUTOMATED  
CONTROL SYSTEMS"  
OF GTU  
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*გამოიცემა 2006 წლიდან*

*პერიოდულობა:  
2 ნომერი წელიწადში*

თბილისი-TBILISI-ТБИЛИСИ  
2021

1971 – 2021

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## TEACHING QUEUEING THEORY AND ITS APPLICATIONS

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### Abstract

The aim of the present paper is to a short review of software tools for teaching Queueing Theory and to introduce an own application called Queueing Systems Assistance (QSA). The software is integrated into a lecture note with the goal to calculate and visualize the main performance measures. In addition, it helps to minimize a quite general mean total cost per unit time with linear objective function. Several examples are given to illustrate the advantage of the graphical module included in the package.

**Keywords:** modeling. Queueing. Teaching. Software. Visualization.

### 1. Introduction

The teaching of Queueing Theory (QT) needs innovation and new methods to attract the attention of the students. The field of applications has changed a lot in the past years and I am convinced that more and more students and practitioners need to use the methods and models of QT. The development of computational possibilities has greatly contributed to a better understanding of the theory.

In his lecture note Sztrik [1] discussed a number of basic queueing models that have proved to be useful in analyzing a wide variety of stochastic service systems. The author feels that there is a need for such a treatment in view of the increased use of queueing models in modern technology.

Actually, the application of queueing theory in the performance analysis of computer and communication systems has stimulated much practically oriented research on computational aspects of queueing models.

Furthermore, a software package called **QSA** (Queueing Systems Assistance) developed in 2021 is integrated into to lecture note with the aim to calculate and visualize the main performance measures. In addition, it helps to minimize a quite general mean total cost per unit time with linear objective function.

The greatest advantage of this application that these scripts can run in all modern devices including smart phones, too, thus the application is very convenient for students and improve the efficiency of a teacher.

To solve practical problems the first step is to identify the appropriate queueing system and then to calculate the performance measures. Of course the level of modeling heavily depends on the assumptions. Our advice is to start with a simple system and then if the results do not fit to the problem continue with a more complicated one. Various software packages help the interested readers in different level. The following links worth a visit

<http://web2.uwindsor.ca/math/hlynka/qsoft.html>

We have collected some basic books on QT in which software support is mentioned, for example, **Mathematica** in Allen [2], Harchol-Balter [3], **MatLab** in Bhat[4], Kobayashi and Mark [5], Kulkarni [6], Stidham [7].

A reasonable choice for calculations in teaching is the usage of spreadsheets. We highly recommend an Excel-based software package called QTSPPlus to determine the main performance

measures of basic models. It is associated to the book of Gross, Shortle, Thompson and Harris [8] and can be downloaded here

[ftp://ftp.wiley.com/public/sci\\_tech\\_med/queueing\\_theory/](ftp://ftp.wiley.com/public/sci_tech_med/queueing_theory/)

For application and problem solving oriented teaching courses we have also developed a software package called QSA (**Queueing Systems Assistance**) see, Szilágyi *et. al.* [9] to calculate and visualize the performance measures together with optimal decisions not only for elementary but more advanced queueing systems as well. It is available at

<https://qsa.inf.unideb.hu>

- The **main advantages** of QSA over QTSPlus are the following
- It runs on desktops, laptops, mobile devices
- It calculates not only the mean but the variance of the corresponding random variables
- It gives the distribution function of the waiting/response times (if possible)
- It visualizes all the main performance measures
- It graphically supports the decision making

## 2. QSA in action, problem solving

QSA is a user interface, a web-based application written in TypeScript. Any browser (Firefox, Chrome, Edge, etc.) on every platform (Windows, Linux, Android, iOS) is supported, which means one can use mobile and desktop devices for performing any calculations which are executed on the server. There are no hardware limitations, the source code is available on GitHub, under the MIT license, so anyone interested in can check out the code or help to develop the application.

QSA is integrated into the lecture note of Sztrik [1].

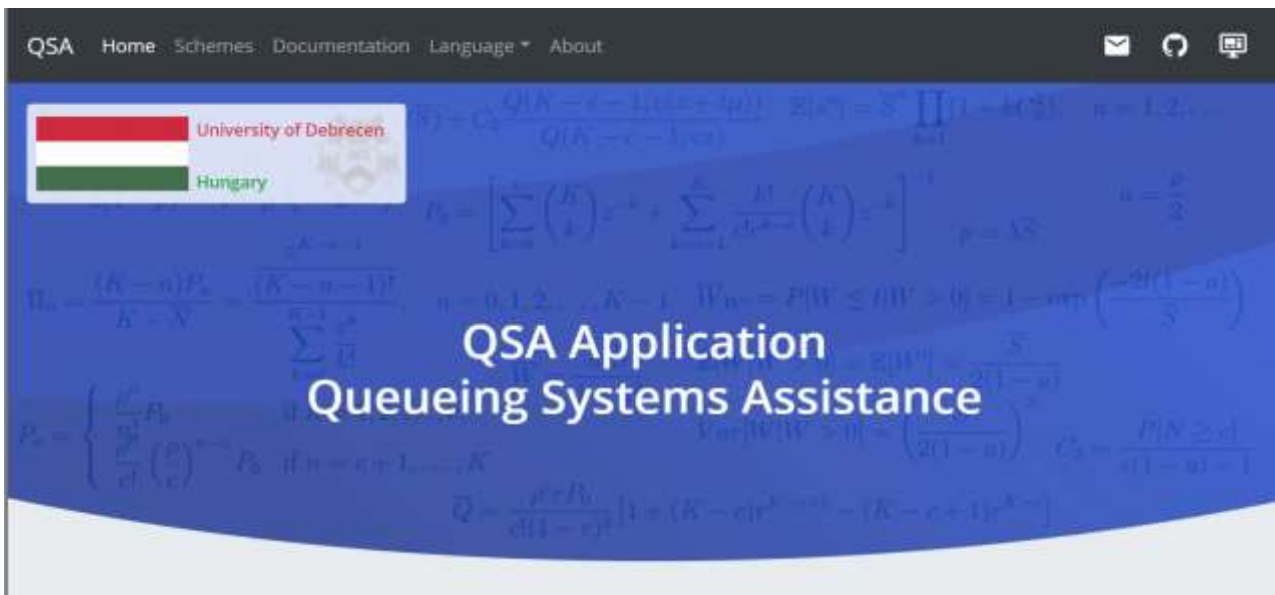


Fig. 1. Opening page of QSA

In this section we show some examples how to use the application. After the opening one can select between the following modules

**Table** - to calculate selected performance measures based on input values. The result is

exportable into different file formats so that one can use it for further work

**Chart** - to generate figures and compare the performance measures with each other. Also, it is useful for demonstration or learning purposes.

**Compare Tables** - to compare two systems' performance measures with each other

In the **Table** module the performance measures can be obtained in a tabular form after giving the required parameters depending on the system (arrival rate, service rate, number of servers, capacity, number of sources), non-required parameters (number of customers in the system, time slot for the distribution functions of waiting/response time, quantile values) and different costs.

The **Chart** module helps to visualize the results and it supports the teaching most. Entering to this part we have to select a systems and to give the variable of the performance measures, the starting, ending values of the interval and the steps. Students can see the effect of different parameters on the performance measures. These measures can be switched on/off by clicking of the mouse to the required one.

Using this function we can solve graphically several decision problems, for example, to find the number of servers, the capacity of the system, service rate, arrival rate, percentage values. The plot of the cost function can be demonstrated. The screen can be downloaded into 4 file forms (PNG, JPEG, PDF, SVG) with and without grids. One can display it in full screen and can print directly. Moving the mouse on the curve the actual values of the variable and the performance measures can be seen to help the decision. Moreover, one can zoom in/out of certain part of the interval.

The **Compare** module supports comparisons of tables. Before using it you should create several tables and then can select which ones are to be compared. The performance measures are listed and it is easy to analyze the systems.

One of the special features of the software is that the performance measures of  $M/G/1/K/K$  systems with deterministic, Erlang, Hypo-exponential, Hyper-exponential, and gamma distributed service times are calculated. Distribution function of the waiting/response times of the  $M/M/c/K$ ,  $M/M/c/m/K$  systems and the performance measures of  $M/M/c/K$ ,  $M/M/c/m/K$  with balking and reneging are determined as well.

It was our aim determine, where it is possible, the distribution function of the waiting/response time to solve decision problems. In addition, not only the mean but the variances of the measures are derived. What is also unique is the calculation of the mean total cost per unit time in steady-state.

For illustration let us see the following examples.

**Example 1:** Customers arrive to a 3 server system according to a Poisson process with rate 4. The service times are exponentially distributed with parameter 1.5. Find the minimum capacity of the system for which the probability of blocking is less than 0.01 and the probability that the waiting time exceeds 1.5 minutes is less than 0.05.

**Solution:** It is an  $M/M/3/K$  system and the problem is that by increasing the capacity the blocking probability is decreasing but the waiting time is increasing thus the probability that it exceeds a certain level is increasing. First of all we have to switch to the distribution function of the waiting time and that is why its value should be at least 0.95 at 1.5.

It should be mentioned that for this system there is no closed-form analytical expression for the distribution function of the waiting time as in  $M/M/c$  systems. However, it can be computed by the following formula, see Sztrik [1]

$$F_w(t) = 1 - \sum_{n=c}^{K-1} \Pi_n \sum_{i=0}^{n-c} \frac{(c\mu t)^i e^{-c\mu t}}{i!}, \quad \Pi_n = \frac{P_n}{1 - P_K}, \quad (n \leq K-1).$$

Clearly we have to use the **Chart** module and to visualize the curves as the function of the capacity  $K$ .

Of course the step is 1, after giving the required parameters  $\lambda$ ,  $\mu$ ,  $c$  and time slot  $t = 1.5$  we generate the chart showing only the measures in question. We can switch on/off the grid, too. Then we get the following Figure 1 showing that there is no solution on these conditions. However, if we change the blocking probability to 0.08 the solution is  $K = 8$ . Similar questions could be put for the service intensity, and the number of servers, too.

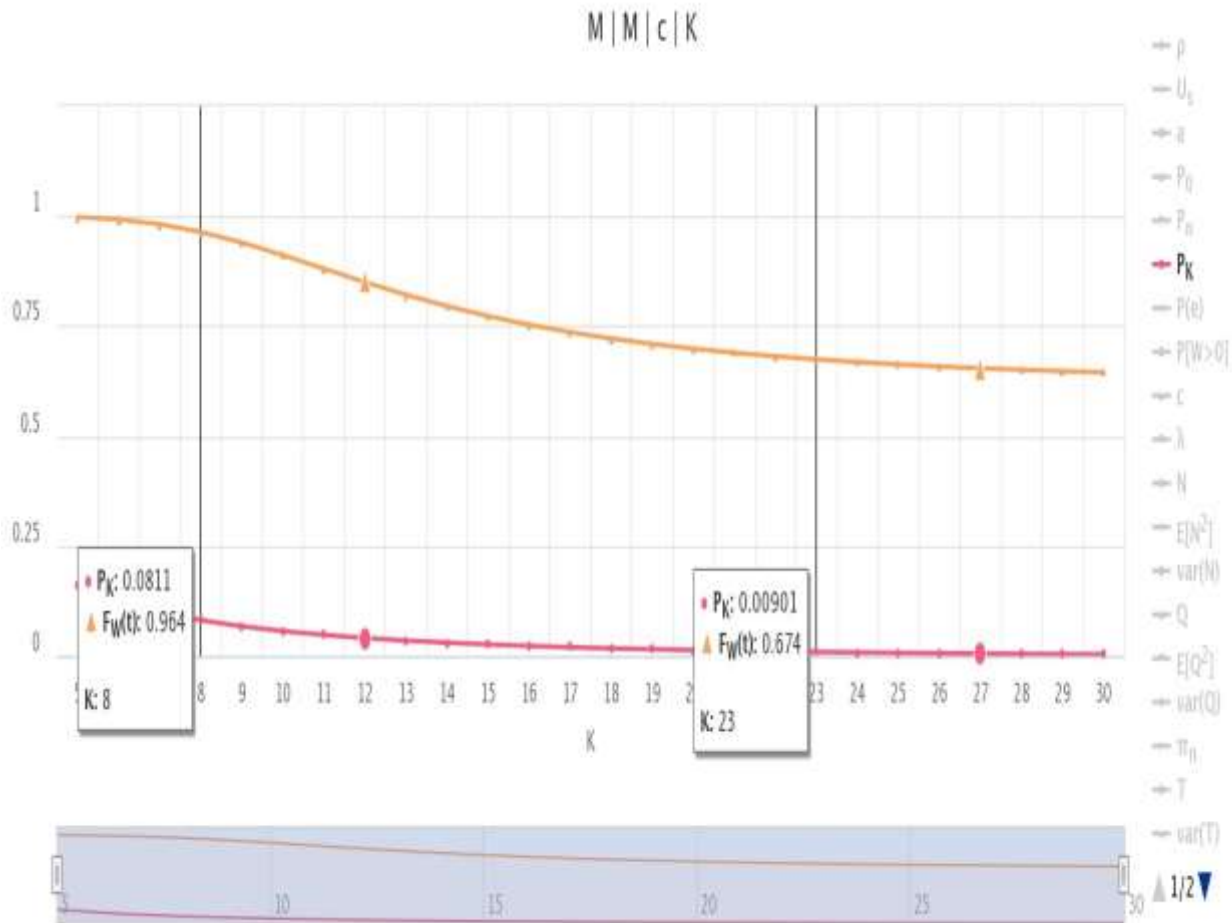


Fig. 2. Solution to the  $M/M/3/K$  system

**Example 2:** We have a finite-source system with 30 sources, the request generation times are exponentially distributed with rate 1. The service times are exponentially distributed for all the 5 servers with intensity 2. Find the minimum capacity of the system for which the probability of blocking is less than 0.01 and the probability that the waiting time exceeds 2.4 minutes is less than 0.05.

**Solution:** It is an  $M/M/5/m/30$  system and the problem is that by increasing the capacity the blocking probability is decreasing but the waiting time is increasing thus the probability that it exceeds a certain level is increasing. First of all we have to switch to the distribution function of the waiting time and that is why its value should be at least 0.95 at 2.4.

It should be mentioned that for this system there is no closed-form analytical expression for the distribution function of the waiting time as in  $M/M/c/K/K$  systems. However, it can be computed by the following formula, see Sztrik [1]

$$P(W > x) = \sum_{k=r}^{K-1} \sum_{j=0}^{k-r} \frac{(r\mu x)^j}{j!} e^{-r\mu x} \Pi_k(n, r, K), \quad P(W \leq x) = 1 - P(W > x),$$

$$P(W = 0) = \sum_{k=0}^{r-1} \Pi_k(n, r, K).$$

where the probability that a customer arriving into the systems finds  $k$  customers there is

$$\Pi_k(n, r, K) = \frac{(n-k)P_k(n, r, K)}{\sum_{i=0}^{K-1} (n-i)P_i(n, r, K)}, \quad k=0, \dots, K-1.$$

It should be noted that the notation of the lecture note and QSA is different concerning the number of servers, capacity and sources but the meaning is the same according to the Kendall's notation. Here  $P_k(n, r, K)$  denotes the steady-state probability of a M/M/r/K/n system which is denoted by M/M/c/m/K in QSA.

Clearly we have to use the **Chart** module and to visualize the curves as the function of the capacity  $m$ . Of course the step is 1, after giving the required parameters  $\lambda, \mu, c, K$  and time slot  $t=2.4$  we generate the chart showing only the measures in question. Then we get the following Figure 3 showing that there is no solution under these conditions. However, if we change the blocking probability to 0.04 the solution is  $m=24$ . Similar questions could be put for the service intensity, the number of servers, and number of sources, too.

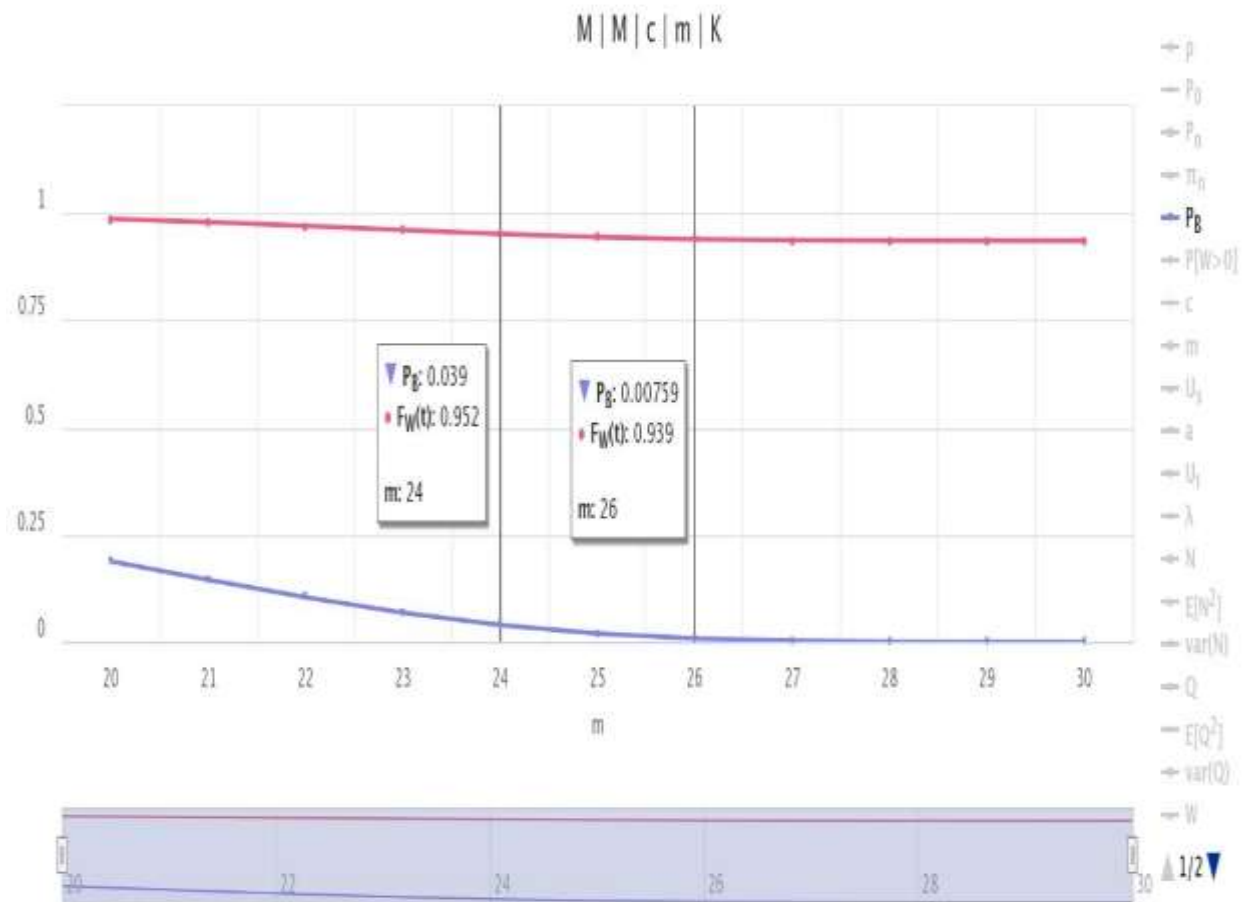


Fig. 3. Solution to the M/M/5/m/30 system

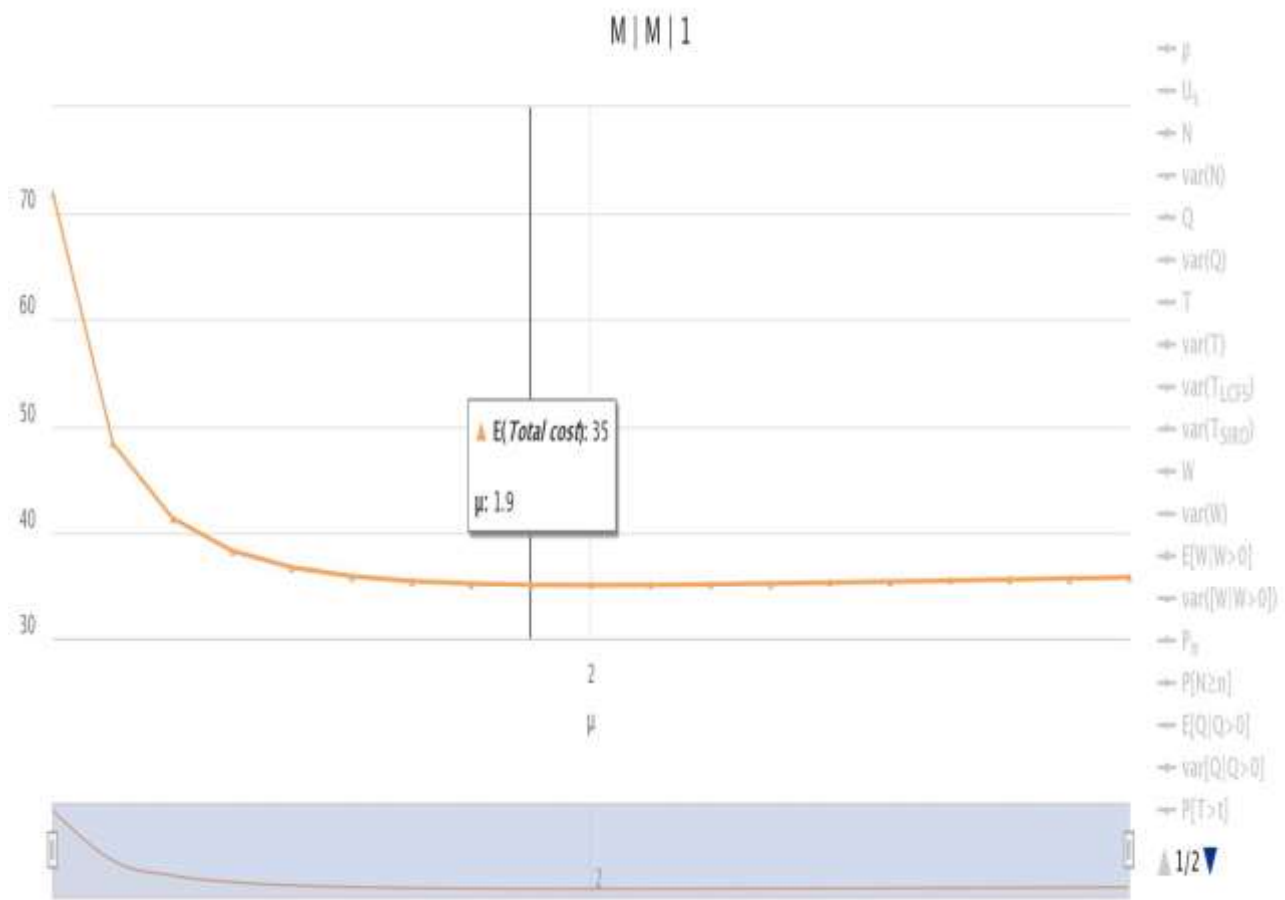


**Example 3:** Let us see an M/M/1 system with  $\lambda = 1$  arrival intensity and the following costs, cost of service per server per unit time  $CS = 5$ , cost of waiting in the system per customer per unit time  $CWS = 5$ , cost of idleness per server per unit time  $CI = 20$ , cost of service rate per server per unit time  $CSR = 25$ , reward per customer per unit time  $R = 10$ .

Find the optimal value for  $\mu$  which minimize the expected total cost per unit time with linear objective function.

**Solution:** It is an M/M/1 system and clearly we have to use the **Chart** module and to visualize the curves as the function of service intensity  $\mu$ . Of course the step is quite fine, let us say 0.1, after giving the required parameter  $\lambda$  and the costs mentioned before we generate the chart showing only the expected total cost.

Then we get the following Figure 4 showing the optimal value at  $\mu = 1.9$  which equals 35.



**Fig. 4. Solution to the M/M/1 system**

Besides these options we can ask for statistics concerning the usage of the software by clicking the monitor icon. The application was released on 19.02.2021 and at present we have the following data:

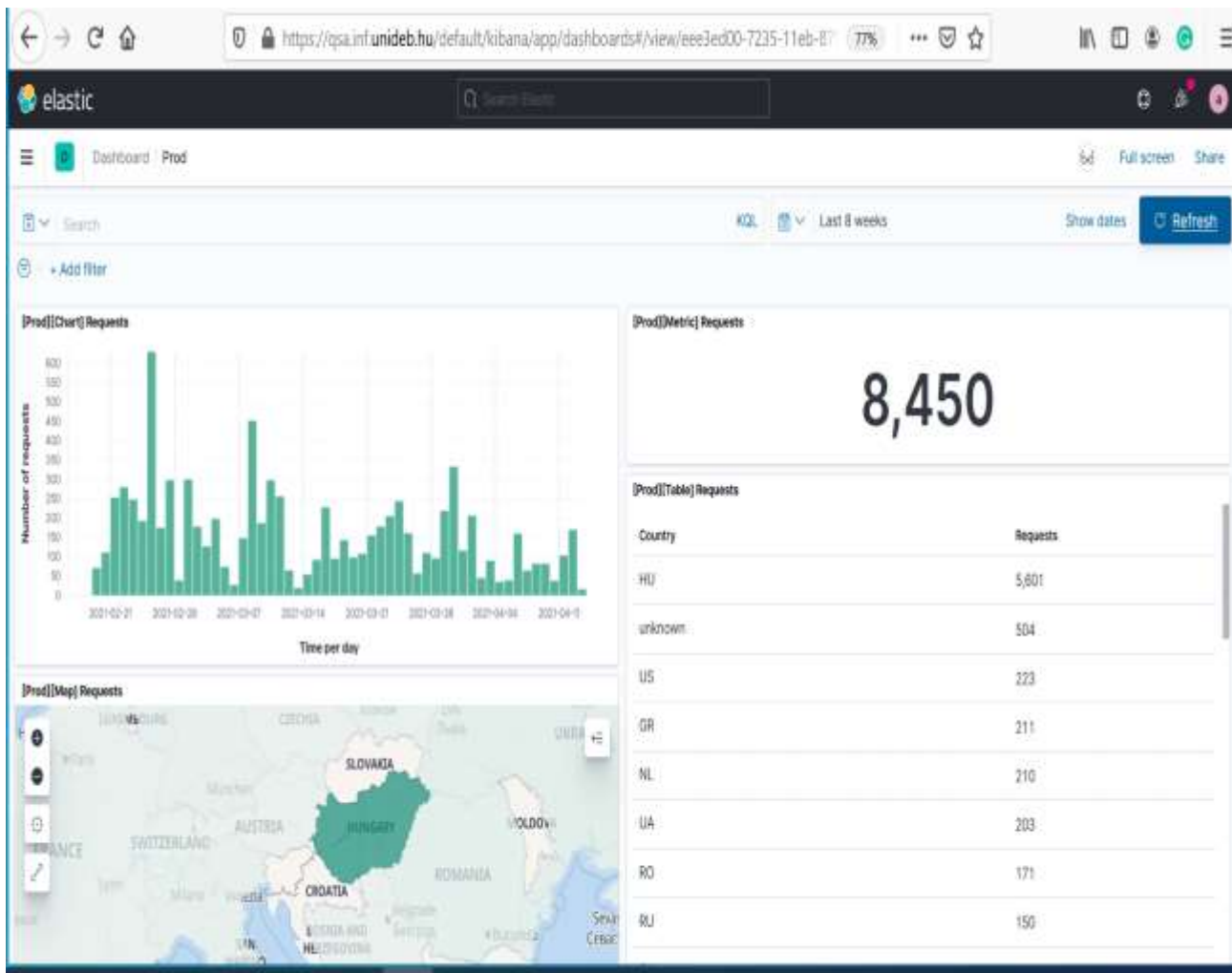


Fig. 5. Elastic statistics

### 3. Conclusion

In this paper we introduced a new application to help teaching Queuing Theory. One of the main advantages of the software is that it runs on most platforms including smart phones and became very popular among the students. It is easy-to use and in addition to the calculation of the main steady-state performance measures it visualizes the results and thus support decision making and optimization of cost functions. The software is integrated into a lecture note where the theoretical part, formulas, and proofs can be found.

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## რიგების თეორიის და მისი გამოყენებითი

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