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DISTRIBUTED COMPUTER AND COMMUNICATION NETWORKS – 2017

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Российская академия наук (РАН)
Институт проблем управления им. В.А. Трапезникова РАН (ИПУ РАН)
Институт информационных и телекоммуникационных технологий
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РАСПРЕДЕЛЕННЫЕ КОМПЬЮТЕРНЫЕ И ТЕЛЕКОММУНИКАЦИОННЫЕ СЕТИ: УПРАВЛЕНИЕ, ВЫЧИСЛЕНИЕ, СВЯЗЬ

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В научном издании представлены материалы Двадцатой международной научной конференции «Распределенные компьютерные и телекоммуникационные сети: управление, вычисление, связь» по следующим направлениям:

- архитектура и топология компьютерных сетей: управление, проектирование, оптимизация, маршрутизация, резервирование ресурсов;
- аналитическое и имитационное моделирование инфокоммуникационных систем, оценка производительности и качества обслуживания;
- технологии беспроводных сетей сантиметрового и миллиметрового диапазона радиоволн: локальные и сотовые сети 4G/5G;
- RFID-технологии и сенсорные сети;
- приложения распределенных информационных систем: Интернет вещей, анализ больших данных, интеллектуальные транспортные сети;
- распределенные системы и облачные вычисления, программно-определяемые сети, виртуализация;
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- теория очередей, теория надежности и их приложения в компьютерных сетях;
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В материалах конференции DCCN-2017, подготовленных к выпуску Козыревым Д.В., обсуждены перспективы развития и сотрудничества в этой сфере.
Сборник материалов конференции предназначен для научных работников и специалистов в области теории и практики построения компьютерных и телекоммуникационных сетей.

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Some Features of a Finite-Source M/GI/1 Retrial Queuing System with Collisions of Customers

A. A. Nazarov*†, J. Sztrik‡, A. S. Kvach*

* *National Research Tomsk State University*
36 Lenina ave., Tomsk, 634050, Russia
† *Department of Applied Probability and Informatics*
Peoples' Friendship University of Russia
Miklukho-Maklaya str. 6, Moscow, 117198, Russia
‡ *Faculty of Informatics*
University of Debrecen
Egyetem tér 1, Debrecen, 4032, Hungary

Abstract. In this paper a finite-source M/GI/1 retrial queuing system with collisions of customers is considered. The definition of throughput of the system as average number of customers, which are successfully served per unit time is introduced. It is shown that at some combinations of system parameter values and probability distribution of service time of customers the throughput can be arbitrarily small, and at another values of parameters throughput can be greater than the service intensity. It is also demonstrated that there are such values of the system parameters at which probability distribution of number of customers in system is bimodal. That is, for a random process of changing in time the number of customers there are two points of stabilization and the random process alternates from the neighborhood of one stabilization point to the neighborhood of another one and back.

Keywords: closed queuing system, finite-source queuing system, retrial queue, collisions, asymptotic analysis, bistability, bimodal distribution, throughput.

1. Introduction

Retrial queues have been widely used to model many problems arising in telephone switching systems, telecommunication networks, computer networks and systems, call centers, etc. In many practical situations it is important to take into account the fact that the rate of generation of new primary calls decreases as the number of customers in the system increases. This can be done with the help of finite-source, or quasi-random input models, see, for example [1, 2].

Another very important component of queuing models is the collisions of the customers. In the main model it is assumed that if an arriving customer finds the server busy, it involves into collision with customer under service and they both moves into the orbit. See, for example [3, 4].

The aim of the present paper is to investigate such systems which has the above mention properties, that is finite-source, retrial and collisions of customer in the case of non-Markov service. This article paper is a continuation of the paper [5] in which the asymptotic distribution of the number of customers in the system was investigated.

2. Model description and notations

Let us consider a closed retrial queuing system of type M/GI/1//N with collision of the customers. The number of sources is N and each of them can generate a primary request during an exponentially distributed time with rate λ/N. A source cannot generate a new call until end of the successful service of this customer. If a primary customer finds the server idle, he enters into service immediately, in which the required service time has a probability distribution function $B(x)$. Let us denote its hazard rate function by $\mu(y) = B^{'}(y)(1 - B(y))^{-1}$ and Laplace -Stieltjes transform by $B^{*}(y)$, respectively. If the server is busy, an arriving (primary or repeated) customer involves into collision with customer under service and they both moves into the orbit. The retrial time of requests are exponentially distributed with rate σ/N. All random variables involved in the model construction are assumed to be independent of each other.

The system state at time t is denoted by $\{k(t), i(t), z(t)\}$, where $i(t)$ is the number of customers in the system at time t, that is, the total number of customers in orbit and in service, $k(t)$ describes the server state as follows

$$k(t) = \begin{cases} 0, & \text{if the server is free,} \\ 1, & \text{if the server is busy,} \end{cases}$$

$z(t)$ is the supplementary random variable, equal to the residual service time, that is time interval from moment t until the end of successful service.

Thus, we will investigate the Markov process $\{k(t), i(t), z(t)\}$, which has a variable number of components, depending on the server state, since the component $z(t)$ is determined only in those moments when $k(t) = 1$.

Let us define the probabilities as follows:

$$P_0(i, t) = P\{k(t) = 0, i(t) = i\},$$
$$P_1(i, z, t) = P\{k(t) = 1, i(t) = i, z(t) < z\}.$$

Assuming that system is operating in steady state, for the stationary probability distribution $P_0(i)$, $P_1(i, z)$ by using standard methods the following system of Kolmogorov equations can be derived

$$\begin{aligned} &\frac{\partial P_1(i+1,0)}{\partial z} - \left[\frac{N-i}{N}\lambda + \frac{i}{N}\sigma\right] P_0(i) + \\ &\qquad + \left(1 - \frac{i-1}{N}\right)\lambda P_1(i-1) + \frac{i-1}{N}\sigma P_1(i) = 0, \\ &\frac{\partial P_1(i,z)}{\partial z} - \frac{\partial P_1(i,0)}{\partial z} - \left[\frac{N-i}{N}\lambda + \frac{i-1}{N}\sigma\right] P_1(i,z) + \\ &\qquad + \left(1 - \frac{i-1}{N}\right)\lambda P_0(i-1)B(z) + \frac{i}{N}\sigma P_0(i)B(z) = 0. \end{aligned} \tag{1}$$

3. Asymptotic analysis

By using asymptotic methods as a consequence of the first order solution to (1) can be obtained as follows

Theorem 1. *Let $i(t)$ be number of customers in a closed retrial queuing system $M/GI/1//N$ with the collisions of customers, then*

$$\lim_{N\to\infty} \mathsf{E}\exp\left\{jw\frac{i(t)}{N}\right\} = \exp\left\{jw\kappa\right\},$$

where value of parameter κ is the positive solution of the equation

$$(1-\kappa)\,\lambda - \delta(\kappa)\frac{B^*(\delta(\kappa))}{2-B^*(\delta(\kappa))} = 0, \tag{2}$$

here $\delta\left(\kappa\right)$ is

$$\delta\left(\kappa\right) = \left(1-\kappa\right)\lambda + \sigma\kappa,$$

and κ has the meaning of the asymptotic average of the normalized number of customers in the system.

It is interesting that equation (2) can have one, two or three roots $0<\kappa<1$. For example, for the gamma distribution function $B(x)$ with a shape parameter α and scale β with parameter values $\alpha=\beta=2$, $\lambda=0.29$, $\sigma=20$, equation (2) has three roots, namely $\kappa_1=0.031$, $\kappa_2=0.188$ and $\kappa_3=0.549$.

In the following let us consider some properties of the system when the main equation (2) has a single root $0<\kappa<1$.

Let us define a measure, called throughput S, as the average number of the number of customers, which are successfully served per unit time. Since all primary customers sooner or later will be successfully served, the throughput S naturally coincides with the intensity of the incoming (generated by the primary sources) flow. For retrial queuing systems with collisions of customers value S can take non-ordinary values.

4. Non-ordinary values of throughput

Let $B(x)$ be the distribution function of the gamma distribution with shape parameter α and scale parameter β, with Laplace-Stieltjes transform $B^*(\delta)$ of the form

$$B^*(\delta) = \left(1+\frac{\delta}{\beta}\right)^{-\alpha},$$

where the average required service time is α/β.

Let us choose the following values of parameters $\alpha=\beta=0.1$ and find values of the root κ of equation (2) at $\sigma=10$ and at the values λ indicated in the Table 1. At $\alpha=\beta$ average value of the required service

Table 1

System throughput S and κ at various values of λ for the case $\alpha = \beta = 0.1$

λ	1	3	5	7	9
κ	0.061	0.219	0.344	0.438	0.510
$S = \lambda(1-\kappa)$	0.939	2.343	3.278	3.931	4.401

time is equal to unit therefore it is possible to tell that intensity of service as an inverse value to average time, is also equal to unit, but in the Table 1 values of throughput significantly greater than service intensity that is a rather non-standard result due to the collisions, since in this case the total time spent under service is not the same as the required service time

Now we will consider the same retrial queuing system and at the same values of parameters in the case of $\alpha = \beta = 2$.

Table 2

System throughput S and κ at various values of λ for a case $\alpha = \beta = 2$

λ	1	3	5	7	9
κ	0.844	0.952	0.972	0.980	0.984
$S = \lambda(1-\kappa)$	0.156	0.144	0.142	0.141	0.141

Dependence of the quantities κ and S on values of parameter $\alpha = \beta$ is shown in Table 3.

Table 3

System throughput S and κ at various values of $\alpha = \beta$ and λ

$\alpha = \beta$		1	2	5	10	100
$\lambda = 0.5$	κ	0.183	0.629	0.954	0.990	0.9992
	S	0.409	0.185	0.023	0.005	0.0003
$\lambda = 2$	κ	0.765	0.927	0.989	0.998	0.9998
	S	0.471	0.147	0.021	0.005	0.0003

For values $\lambda = 0.5$ and $\lambda = 2$ and the increasing values α specified in the Table 3, value of parameter κ significantly increases and practically reaches value $\kappa = 1$, and values of throughput S significantly decrease and are almost equal to zero for sufficiently large values of α. By law of large numbers at $\alpha \to \infty$ the limiting gamma distribution converges to the degenerate, that is servicing time is determined and equal to $\alpha/\beta = 1$. Wherein $B^*(\delta) = e^{-\delta}$.

Further, let us assume $\alpha = \beta = 0.5$, and values of parameters λ and σ are specified in the Table 4 in which the found values of throughput S of system with the collision of customers are given. From the Table 4 it

Table 4
System throughput S at various values of λ and σ for a case $\alpha = \beta = 0.5$

	$\sigma = 2$	$\sigma = 4$	$\sigma = 6$	$\sigma = 8$	$\sigma = 10$
$\lambda = 0.5$	0.196	0.047	0.008	0.0014	0.00026
$\lambda = 1$	0.159	0.041	0.008	0.0014	0.00026
$\lambda = 3$	0.141	0.037	0.007	0.0014	0.00023
$\lambda = 5$	0.138	0.037	0.007	0.0014	0.00023

is visible that throughput S poorly depends on values λ and significantly decreases with increase of intensity σ, reaching almost zero values at $\sigma > 10$.

The presence of extraordinary throughput values S and edge values of κ close to unit and zero, is a consequence of the collision of customers and the admissibility of repeated attempts of service the same customer. Duration of the customer service for repeated attempts has the same probability distribution $B(x)$, but its repeated realization, naturally, accepts various values. If for the distribution $B(x)$ there is a high probability of emergence of small values of service time as in the gamma distribution with the shape parameter $\alpha < 1$, then a small number of retries is sufficient to realize a small value of the service time which will be successful and, as shown in the Table 1 above, the throughput will be greater than intensity of service.

If the small values of the service time are unlikely for the probability distribution $B(x)$, as in the gamma distribution with the shape parameter $\alpha > 1$, then the number of unsuccessful attempts of service becomes big, the server works without results, the number of customers in an orbit increases, the value κ comes nearer to 1, and the throughput S becomes close to zero, as shown in the Tables 3 - 4.

5. Bistability phenomenon

Let us consider such values of the service parameters α, β and the system λ, σ at which the equation (2) has three roots, then the probability distribution $P(i) = P\{i(t) = i\}$ can be of a bimodal type. In particular, for $\alpha = \beta = 2$, $\lambda = 0.29$, $\sigma = 19.7$ by simulation methods, or by a numerical solution of the system of Kolmogorov equations (1) for $P(i)$ at $N = 200$, we obtain the values, the graph of which is shown in the Figure 1.

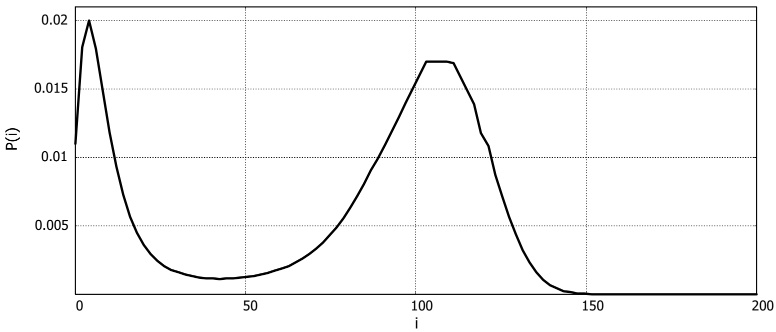

Figure 1. Bimodal probability distribution of the number customer in the system

To clarify the reasons of a two-modality of the probability distribution $P(i)$ at the specified set of values of the system parameters and existence of three roots κ_1, κ_2, κ_3 of the main equation (2), we obtain this equation in another way.

Theorem 2. *Let $P(x)$ be a stationary density distribution of a random process $x(Nt) = i(Nt)/N$, then at $N \to \infty$ we have*

$$-\frac{\partial}{\partial x}\left\{\left[(1-x)\lambda - \delta(x)\frac{B^*(\delta(x))}{2-B^*(\delta(x))}\right]P(x)\right\} = 0. \qquad (3)$$

Due to the page limitation the proof is omitted but will be published in a subsequent paper.

Notice, that the received equality (3) is a degenerate Fokker-Planck equation for the probability distribution $P(x)$ of stationary diffusion process $x(Nt)$ with a drift coefficient $a(x) = (1-x)\lambda - \delta(x)\frac{B^*(\delta(x))}{2-B^*(\delta(x))}$, $0 \le x \le 1$.

The Figure 2 shows the graph of the function $a(x)$.
The zero point κ_k of the function $a(x)$ are the roots of the equation (2). Taking into account the fact that the function $a(x)$ is the drift coefficient and

$$a(x) = \begin{cases} > 0, & \text{if } 0 \le x < \kappa_1, \\ < 0, & \text{if } \kappa_1 < x < \kappa_2, \\ > 0, & \text{if } \kappa_2 < x < \kappa_3, \\ < 0, & \text{if } \kappa_3 < x \le 1, \end{cases}$$

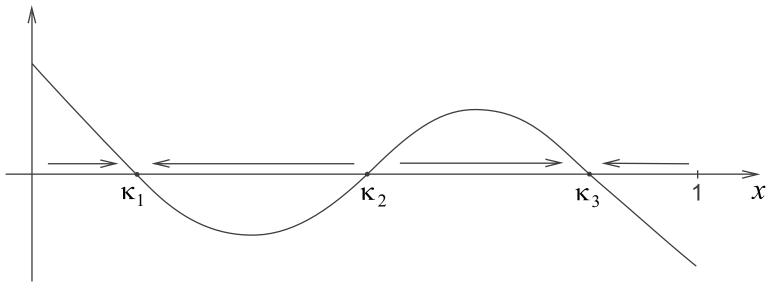

Figure 2. Graph of the function $a(x)$

we can conclude that the random process $i(t)$ of the number of customers in the system can be in the neighborhood of one of the two points $N\kappa_1$ or $N\kappa_3$, which we call the stabilization points of process $i(t)$. The process $i(t)$ periodically passes from the neighborhood of one stabilization point to the neighborhood another and returns. Areas $0 \le i < N\kappa_2$, $N\kappa_2 < i \le N$ will be called areas of stable functioning of the retrial queuing system with collision of the customers.

In the neighborhood of points $N\kappa_1$, $N\kappa_3$ the probability distribution $P(i)$ takes on values of local maxima therefore the probability distribution $P(i)$ is bimodal, and in the neighborhood of a point $N\kappa_2$ distribution $P(i)$ takes a minimum value.

Let us note that this property of the two-modality of the distribution $P(i)$ is realized for sufficiently large values of N, for which the properties of the prelimiting distribution $P(i)$ are close to the properties of the limiting distribution. If in the neighborhood of each of the roots κ_1, κ_2 and κ_3 we consider the probability distribution of the process $\dfrac{i(t) - N\kappa_k}{\sqrt{N}}$ at $N \to \infty$, then the stationary density $P_k(y)$ of the probability distribution of such processes has the form

$$P_k(y) = C \exp\left\{-\frac{y^2}{2\kappa_k^{(2)}}\right\}.$$

For κ_1, κ_3 parameters $\kappa_1^{(2)}$, $\kappa_3^{(2)}$ take positive values, therefore the specified distributions are Gaussian, that is quite natural. It is interesting to note that for κ_2 the distribution $P_2(y)$, determined by the same expression, has parameter $\kappa_2^{(2)} < 0$, this indicates that distribution $P_2(y)$ is not Gaussian, but U-shaped. The class of such distributions sometimes can be encountered in mathematical statistics.

6. Conclusions

In this paper, a finite-source retrial queuing system with collisions of customers was considered. It was shown that at some combinations of system parameter values the throughput takes on exotic values. It has also been demonstrated that there are such values of the system parameters at which a bistability phenomenon arises. Examples, tables and graphical illustrations demonstrated the novelty of the investigations.

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