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Theory and Methodology

Machine interference problem with a random environment

J. Sztrik *

Department of Mathematics, University of Debrecen, Hungary, and Department of Mathematics, University of Bradford, England, UK

B.D. Bunday

Department of Mathematics, University of Bradford, England, UK

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Abstract: This paper is concerned with a queueing model to analyse the asymptotic behaviour of the machine interference problem with N machines and a single operative. The running and repair times of each machine are supposed to be exponentially distributed random variables with parameter depending on the state of a varying environment. Assuming that the repair rate is much greater than the failure rate ('fast' service), it is shown that the time until the number of stopped machines reaches a certain level converges weakly, under appropriate norming, to an exponentially distributed random variable. Furthermore, some numerical examples illustrate the problem in question in the field of textile winding.

Keywords: Machine interference problem; Random environment; Failure-free operation time; Weak convergence

1. Introduction

The machine repair problem has been analysed in many forms over the past 30 years. In its simplest form, where there are exponential running and repair times, a fixed number of machines in the system and a fixed number of repairmen, the problem is frequently used as a textbook example of a continuous time Markov chain or a finite-source exponential queueing system or birth–death model. Many articles have generalised this basic model by assuming, for example, general repair times, general operating times, etc. For an extensive bibliography on this topic see Stecke and Aronson (1985). In recent years this model has been used, for example, for the mathematical description of terminal computer systems, cf. Takagi (1990), or for modelleing production systems in textile winding, see Bunday (1986). In these papers the steady-state operational characteristics, such as machine availability, operative utilization, mean waiting time, average queue length, have been obtained. In this study an asymptotic approach is

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Correspondence to: J. Sztrik, Department of Mathematics, University of Bradford, Bradford, WY BD7 1DP, England, UK.

presented to analyse the distribution of the time until the number of stopped machines reaches a certain level. This method is quite common in reliability theory, see Anisimov and Sztrik (1989), Gertsbakh (1984, 1989), Osaki et al. (1987), Keilson (1979) or Rukhin and Hsieh (1987).

Refinements in the model are often needed when the the system environment is subject to randomly occuring fluctuations which appear as changes in the parameters of the model. These fluctuations may due to the weather, earthquakes, or other changes in the physical environment, to personnel changes, to alteration of system usage intensity, etc.

This paper is concerned with a queueing model to analyse the asymptotic behavior of the machine interference problem with N machines and a single operative. The running and repair times of each machine are supposed to be exponentially distributed random variables with parameter depending on the state of a varying environment. Assuming that the repair rate is much greater than the failure rate ('fast' service), it is shown that the time until the number of stopped machines first reaches a certain level converges weakly, under appropriate norming, to an exponentially distributed random variable. Furthermore, some numerical examples illustrate the problem in question in the field of textile winding.

2. Preliminary results

In this section a brief survey is given of the most related theoretical results, mainly due to Anisimov, to be applied later on.

Let $(X_\varepsilon(k),\ k \geq 0)$ be a Markov chain with state space

$$\bigcup_{q=0}^{m+1} X_q, \qquad X_i \cap X_j = 0, \quad i \neq j,$$

defined by the transition matrix $\| p_\varepsilon(i^{(q)}, j^{(z)}) \|$ satisfying the following conditions:

(i) $p_\varepsilon(i^{(0)}, j^{(0)}) \to p_0(i^{(0)}, j^{(0)})$, $i^{(0)}, j^{(0)} \in X_0$, and $P_0 = \| p_0(i^{(0)}, j^{(0)}) \|$ *is irreducible*.

(ii) $p_\varepsilon(i^{(q)}, j^{(q+1)}) = \varepsilon\alpha^{(q)}(i^{(q)}, j^{(q+1)}) + o(\varepsilon)$, $i^{(q)} \in X_q$, $j^{(q+1)} \in X_{q+1}$.

(iii) $p_\varepsilon(i^{(q)}, f^{(q)}) \to 0$ $i^{(q)}, f^{(q)} \in X_q$, $q \geq 1$.

(iv) $p_\varepsilon(i^{(q)}, f^{(z)}) \equiv 0$ $i^{(q)} \in X_q$, $f^{(z)} \in X_z$, $z - q \geq 2$.

In the sequel the set of states X_q is called the q-th level of the chain, $q = 1, \ldots, m+1$. Let us single out the subset of states $\langle \alpha_m \rangle = \bigcup_{q=0}^{m} X_q$. Denote by $\{\pi_\varepsilon(i^{(q)}),\ i^{(q)} \in X_q\}$, $q = 1, \ldots, m$, the stationary distribution of a chain with transition matrix

$$\left\| \frac{p_\varepsilon(i^{(q)}, j^{(z)})}{1 - \sum_{k^{(m+1)} \in X_{m+1}} p_\varepsilon(i^{(q)}, k^{(m+1)})} \right\|, \quad i^{(q)} \in X_q, \quad j^{(z)} \in X_z, \quad q, z \leq m.$$

Furthermore denote by $g_\varepsilon(\langle \alpha_m \rangle)$ the steady state probability of exit from $\langle \alpha_m \rangle$, that is

$$g_\varepsilon(\langle \alpha_m \rangle) = \sum_{i^{(m)} \in X_m} \Pi_\varepsilon(i^{(m)}) \sum_{j^{(m+1)} \in X_{m+1}} p_\varepsilon(i^{(m)}, j^{(m+1)}).$$

Denote by $\{\pi_0(i^{(0)}),\ i^{(0)} \in X_0\}$ the stationary distribution corresponding to P_0 and let

$$\overline{\Pi}_0 = \{\pi_0(i^{(0)}),\ i^{(0)} \in X_0\}, \qquad \overline{\Pi}_\varepsilon^{(q)} = \{\pi_\varepsilon(i^{(q)}),\ i^{(q)} \in X_q\}$$

be row vectors. Finally, let

$$A^{(q)} = \| \alpha^{(q)}(i^{(q)}, j^{(q+1)}) \|, \quad i^{(q)} \in X_q, \quad j^{(q+1)} \in X_{q+1}, \ q = 0, \ldots, m.$$

Conditions (i)–(iv) enable us to compute the main terms of the asymptotic expression for $\overline{\Pi}_\varepsilon^{(q)}$ and $g_\varepsilon(\langle\alpha_m\rangle)$. Namely, we obtain

$$\overline{\Pi}_\varepsilon^{(q)} = \varepsilon^q \overline{\Pi}_0 A^{(0)} A^{(1)} \cdots A^{(q-1)} + o(\varepsilon^q), \quad q = 1, \ldots, m, \tag{1a}$$

$$g_\varepsilon(\langle\alpha_m\rangle) = \varepsilon^{m+1} \overline{\Pi}_0 A^{(0)} A^{(1)} \cdots A^{(m)} \mathbf{1} + o(\varepsilon^{m+1}) \tag{1b}$$

where $\mathbf{1} = (1, \ldots, 1)$ is a column vector, see Anisimov et al. (1987, pp. 141–153). Let $(\eta_\varepsilon(t), t \geq 0)$ be an SMP given by the embedded Markov chain $(X_\varepsilon(k), k \geq 0)$ satisfying conditions (i)–(iv). Let the times $\tau_\varepsilon(j^{(s)}, k^{(z)})$ – transitions times from state $j^{(s)}$ to state $k^{(z)}$ – fulfil the condition

$$E \exp\{i\theta\beta_\varepsilon\tau_\varepsilon(j^{(s)}, k^{(z)})\} = 1 + a_{jk}(s, z, \theta)\varepsilon^{m+1} + o(\varepsilon^{m+1}), \quad i^2 = -1$$

where β_ε is some normalizing factor. Denote by $\Omega_\varepsilon(m)$ the instant at which the SMP reaches the $(m+1)$-st level for the first time, exit time from $\langle\alpha_m\rangle$, provided $\eta_\varepsilon(0) \in \langle\alpha_m\rangle$. Then we have:

Theorem 1. (cf. Anisimov et al. (1987, p. 153). *If the above conditions are satisfied, then*

$$\lim_{\varepsilon \to 0} E \exp\{i\theta\beta_\varepsilon\Omega_\varepsilon(m)\} = (1 - A(\theta))^{-1}$$

where

$$A(\theta) = \frac{\sum_{j^{(0)}, k^{(0)} \in X_0} \Pi_0(j^{(0)}) p_0(j^{(0)}, k^{(0)}) a_{jk}(0, 0, \theta)}{\overline{\Pi}_0 A^{(0)} A^{(1)} \cdots A^{(m)} \mathbf{1}}.$$

Corollary 1. *In particular, if* $a_{jk}(s, z, \theta) = i\theta m_{jk}(s, z)$, *then the limit is an exponentially distributed random variable with mean*

$$\frac{\sum_{j^{(0)}, k^{(0)} \in X_0} \Pi_0(j^{(0)}) p_0(j^{(0)}, k^{(0)}) m_{jk}(0, 0)}{\overline{\Pi}_0 A^{(0)} A^{(1)} \cdots A^{(m)} \mathbf{1}}.$$

3. The queueing model

Let us consider the machine interference problem with the following assumptions. There are N machines which are looked after by an operative. The system is supposed to operate in a random environment governed by an ergodic Markov chain $(\xi(t), t \geq 0)$ with state space $(1, \ldots, r)$ and with transition density matrix $(a_{ij}, i, j = 1, \ldots, r, a_{ii} = \Sigma_{j \neq i} a_{ij})$.

Whenever the environmental process is in state i, the probability that an operating machine breaks down in the time interval $(t, t+h)$ is $\lambda(i)h + o(h)$. A stopped machine is immediately repaired unless the operative is busy, otherwise it joins the queue of failed machines. Whenever the environmental process is in state i, the probability that the repairman completes the service in the time interval $(t, t+h)$ is $\mu(i, \varepsilon)h + o(h)$. All random variables involved here and the random environment are supposed to be independent of each other.

Let us consider the system under the assumption of 'fast' repair, i.e., $\mu(i, \varepsilon) \to \infty$ as $\varepsilon \to 0$. For simplicity let $\mu(i, \varepsilon) = \mu(i)/\varepsilon$.

Denote by $Y_\varepsilon(t)$ the number of stopped machines at time t, and let

$$\Omega_\varepsilon(m) = \inf\{t\colon t > 0, Y_\varepsilon(t) = m + 1 / Y_\varepsilon(0) \leq m\},$$

that is, the instant at which the number of failed machines reaches the $(m+1)$-st level for the first time, provided that at the beginning their number is not greater than m; $m = 1, \ldots, N-1$.

Denote by $(\pi_k,\ k = 1,\dots,r)$ the steady-state distribution of the governing Markov chain $(\xi(t),\ t \ge 0)$. Now we have

Theorem 2. *For the system in question under the above assumptions, independently of the initial state, the distribution of the normalized random variable $\varepsilon^m \Omega_\varepsilon(m)$ converges weakly to an exponentially distributed random variable with parameter*

$$\Lambda = (m+1)!\binom{N}{m+1}\sum_{i=1}^{r}\pi_i\frac{\lambda(i)^{m+1}}{\mu(i)^m}.$$

Proof. It is easy to see that the process $Z_\varepsilon(t) = (\xi(t),\ Y_\varepsilon(t))$ is a two-dimensional Markov chain with state space $E = ((i, s),\ i = 1,\dots,r,\ s = 0,\dots,N)$. Furthermore, let

$$\langle\alpha_m\rangle = ((i, s),\ i = 1,\dots,r,\ s = 0,\dots,m).$$

Hence our aim is to determine the distribution of the first exit time of $Z_\varepsilon(t)$ from $\langle\alpha_m\rangle$, provided that $Z_\varepsilon(0) \in \langle\alpha_m\rangle$. It can easily be verified that the transition probabilities in any time interval $(t,\ t+h)$ are the following:

$$(i, s) \xrightarrow{h} \begin{cases} (j, s) & a_{ij}h + o(h), \quad i \ne j, \\ (i, s+1) & (N-s)\lambda(i)h + o(h), \quad s = 0,\dots,N-1, \\ (i, s-1) & (\mu(i)/\varepsilon)h + o(h), \quad s = 1,\dots,N. \end{cases}$$

In addition, the sojour time $\tau_\varepsilon(i, s)$ of $Z_\varepsilon(t)$ in state (i, s) is exponentially distributed with parameter $a_{ii} + (N-s)\lambda(i) + \mu(i)/\varepsilon$. Thus, the transition probabilities for the embedded Markov chain are

$$p_\varepsilon[(i,0),(j,0)] = \frac{a_{ij}}{a_{ii} + N\lambda(i)},$$

$$p_\varepsilon[(i,s),(j,s)] = \frac{a_{ij}}{a_{ii} + (N-s)\lambda(i) + \mu(i)/\varepsilon}, \quad s = 1,\dots,N,$$

$$p_\varepsilon[(i,0),(i,1)] = \frac{N\lambda(i)}{a_{ii} + N\lambda(i)},$$

$$p_\varepsilon[(i,s),(i,s+1)] = \frac{(N-s)\lambda(i)}{a_{ii} + (N-s)\lambda(i) + \mu(i)/\varepsilon}, \quad s = 0,\dots,N-1,$$

$$p_\varepsilon[(i,s),(i,s-1)] = \frac{\mu(i)/\varepsilon}{a_{ii} + (N-s)\lambda(i) + \mu(i)/\varepsilon}, \quad s = 1,\dots,N,$$

As $\varepsilon \to 0$ this implies

$$p_\varepsilon[(i,0),(j,0)] = \frac{a_{ij}}{a_{ii} + N\lambda(i)},$$

$$p_\varepsilon[(i,s),(j,s)] = o(1), \quad s = 1,\dots,N,$$

$$p_\varepsilon[(i,0),(i,1)] = \frac{N\lambda(i)}{a_{ii} + N\lambda(i)},$$

$$p_\varepsilon[(i,s),(i,s+1)] = \frac{(N-s)\lambda(i)\varepsilon}{\mu(i)}(1 + o(\varepsilon)), \quad s = 1,\dots,N-1,$$

$$p_\varepsilon[(i,s),(i,s-1)] \to 1, \quad s = 1,\dots,N.$$

This agrees with conditions (i)–(iv), but here the zero level is the set $((i, 0), (i, 1), i = 1,\dots, r)$ while the q-th level is $((i, q+1), i = 1,\dots, r)$. Since level 0 in the limit forms an essential class, the probabilities $\pi_0(i, 0)$, $\pi_0(i, 1)$, $i = 1,\dots, r$, satisfy the following system of equations:

$$\pi_0(j, 0) = \frac{\sum_{i \neq j} \pi_0(i, 0) a_{ij}}{a_{ii} + N\lambda(i)} + \pi_0(j, 1), \tag{2}$$

$$\pi_0(j, 1) = \frac{\pi_0(j, 0) N\lambda(j)}{a_{jj} + N\lambda(j)}. \tag{3}$$

By substituting (3) into (2) we get

$$\frac{\pi_0(j, 0) a_{jj}}{a_{jj} + N\lambda(j)} = \frac{\sum_{i \neq j} \pi_0(i, 0) a_{ij}}{a_{ii} + N\lambda(i)}. \tag{4}$$

Since $\pi_j a_{jj} = \sum_{i \neq j} \pi_i a_{ij}$, from (3) and (4) we have

$$\pi_0(i, 0) = B\pi_i(a_{ii} + N\lambda(i)), \qquad \pi_0(i, 1) = B\pi_i N\lambda(i)$$

where B is the normalizing constant, i.e. $1/B = \sum_{i=1}^{r} \pi_i [a_{ii} + 2N\lambda(i)]$.

By using formula (1) it is easy to show that the probability of exit from $\langle \alpha_m \rangle$ is

$$\begin{aligned} g_\varepsilon(\langle \alpha_m \rangle) &= \varepsilon^m NB \sum_{i=1}^{r} \pi_i \lambda(i) \prod_{s=1}^{m} \frac{(N-s)\lambda(i)}{\mu(i)} (1 + o(1)) \\ &= \varepsilon^m B(m+1)! \binom{N}{m+1} \sum_{i=1}^{r} \pi_i \frac{\lambda(i)^{m+1}}{\mu(i)^m} (1 + o(1)). \end{aligned} \tag{5}$$

Taking into account the exponentially of $\tau_\varepsilon(j, s)$ for fixed θ, we have

$$E \exp\{i\varepsilon^m \theta \tau_\varepsilon(j, 0)\} = 1 + \varepsilon^m \frac{i\theta}{a_{jj} + N\lambda(j)} (1 + o(1)),$$

$$E \exp\{i\varepsilon^m \theta \tau_\varepsilon(j, s)\} = 1 + o(\varepsilon^m), \quad s > 0.$$

Notice that $\beta_\varepsilon = \varepsilon^m$ and therefore from Corollary 1 we immediately get the statement that $\varepsilon^m \Omega_\varepsilon(m)$ converges weakly to an exponentially distributed random variable with parameter

$$\Lambda = (m+1)! \binom{N}{m=1} \sum_{i=1}^{r} \pi_i \frac{\lambda(i)^{m+1}}{\mu(i)^m},$$

which completes the proof.

Consequently, the asymptotic distribution of $\Omega_\varepsilon(m)$ can be determined as follows:

$$P(\Omega_\varepsilon(m) > t) = P(\varepsilon^m \Omega_\varepsilon(m) > \varepsilon^m t) \approx \exp(-\varepsilon^m \Lambda t),$$

that is, $\Omega_\varepsilon(m)$ is asymptotically an exponentially distributed random variable with parameter

$$\varepsilon^m (m+1)! \binom{N}{m+1} \sum_{i=1}^{r} \pi_i \frac{\lambda(i)^{m+1}}{\mu(i)^m} = (m+1)! \binom{N}{m+1} \sum_{i=1}^{r} \pi_i \frac{\lambda(i)^{m+1}}{(\mu(i)/\varepsilon)^m}.$$

In particular, for $m = N$-1, that is, when all machines are stopped, we have

$$\varepsilon^{N-1}\Lambda = N!\sum_{i=1}^{r}\pi_i\frac{\lambda(i)^N}{(\mu(i)/\varepsilon)^{N-1}}. \tag{6}$$

Hence the steady-state probability Q_W that at least one machine works is

$$Q_W = \frac{\dfrac{1}{\varepsilon^{N-1}\Lambda}}{\dfrac{1}{\varepsilon^{N-1}\Lambda}+\sum_{i=1}^{r}\pi_i\dfrac{1}{\mu(i)/\varepsilon}} = \frac{1}{1+N!\left(\sum_{i=1}^{r}\pi_i\dfrac{\lambda(i)^N}{\mu(i)^{N-1}}\right)\left(\sum_{i=1}^{r}\pi_i\dfrac{1}{\mu(i)/\varepsilon}\right)}. \tag{7}$$

In the case when there is no random environment we get

$$Q_W = \frac{1}{1+N!\left(\dfrac{\lambda}{\mu/\varepsilon}\right)^N}. \tag{8}$$

4. Some numerical results and applications in textile winding

In the context of the production department on the factory floor, most manufacturers will seek to establish a constant and optimal environment in which the various processes can be carried out. They will try to avoid the random environment. However, we do not live in the ideal world and variations in the repair rate and the breakdown rate will occur in spite of their best efforts. Machine operatives will feel 'below par' with physical or mental problems for time to time and this in turn will affect their work rate. Their attitude to work at the start of a shift will be very different from their attitude just prior to the tea-break, just after the tea-break and again before the end of their shift. Of course, one could argue that the latter changes are more deterministic than random, although variations among workers will tend to make the overall effect more random than it might appear to be at first sight.

The use of robots, and there is a marked trend in this direction in many industries, seeks to avoid these effects. The machinery used will suffer from minor faults due to wear and tear. These, although they may not in themselves constitute a breakdown, will have an adverse effect on the stoppage rate of the process. Another reason for variability in the stoppage rate arises from the quality of the raw materials used. This material may have been produced at an earlier stage in the production process, and unless very stringent quality control procedures have been used some variability is inevitable. In the particular case of the textile industry, especially where natural fibres such as wool or cotton are being used, variability between batches of raw yarns is difficult to avoid. Although it is not possible to generalise, because of the great variety of industrial production processes which exist, if the unit of time is taken to be the average repair time, then the average run time between successive stoppages due to yarn breaks of a single machine might be anything from about 20 time units to 100 time units. The idea of 'fast' repair would therefore seem to be reasonable. However, the factors mentioned earlier could easily cause deviations of the order of 10% to 50% of these times. We do not underestimate the practical difficulties of modelling these features of real manufacturing processes. The random environment idea would seem to be a first step in the right direction.

In this section some numerical examples are given to illustrate the problem in question and the asymptotic results are compared to the classical exact formulae as well as the numerical ones obtained by Gaver et al. (1984).

Case 1

In this section we illustrate how 'good' the asymptotic results are by comparing them to the exact ones. Here

$$\rho = \frac{\lambda}{\mu/\varepsilon} \quad \text{and} \quad P_W = 1 - N!\rho^N P_0$$

(from Palm's formula). Using (12) we get the results listed in Table 1.

We can see how Q_W depends on N, ρ and how accurate it is. It should be noted that the greater N, the less ρ is for an acceptable approximation.

Case 2

In this section we show how the system behaves for different ρ (see Table 2).

We can observe that for $\rho = 0.1$, Q_W and $1/\Lambda$ first increase and then decrease. This follows from (8) and it shows that repair is rather 'slow' since Q_W and $1/\Lambda$ should be a non-decreasing function of N. But if $\rho = 0.05$, that is when the service is 'fast' enough, then they increase as N increases.

Case 3

In this section the system operates in a random environment. We compare the asymptotic result to the numerical one obtained by Gaver et al. (1984) and show how it depends on the intensities of the governing Markov chain. See Table 3. $\hat{Q}_W = 1 - g_\varepsilon(\langle \alpha_m \rangle)$ and P_W is the steady-state probability that at least one machine works, obtained by Gaver et al. (1984).

We can observe that the corresponding probabilities are exact up to almost 3 digits while the mean failure-free operation time is very small compared to Gaver et al. (1984) where $1/\Lambda \approx 1500$.

Case 4

In this section we approximate the results of Gaver et al. (1984) by changing the service rate. See Table 4.

This example illustrates that with $\mu/\varepsilon = 1.3$ we get almost the same results as Gaver et al. (1984) but here the formulae are much simpler and we do not need numerical procedures.

Case 5

We show how the system behaves for different N. See Table 5.

We can see that by using (6), (7) for these input parameters, the optimal N-value for $1/\Lambda$ and Q_W is $N = 10$.

Case 6

For different N we investigate the effect of service rate on the performance measures. See Table 6.

We can observe how sensitive $1/\Lambda$ is to changes in μ/ε.

Case 7

In this section we show the behaviour of the mean failure-free operation time of the system. See Table 7.

We can see that $1/\Lambda$ sharply increases for increasing failure level.

Table 1

ρ	$N=5$		$N=10$		$N=15$		$N=20$		$N=25$		$N=30$	
	P_W	Q_W	P_W	Q_W	P_W	Q_W	P_W	Q_W	P_W	Q_W	P_W	Q_W
1	0.631901845	8.26446281 E-3	0.632120555	2.75573116 E-7	0.632120559	7.64716373 E-13	0.632120559	4.11031762 E-19	0.632120559	6.44695028 E-26	0.632120559	3.76998763 E-33
2^{-1}	0.862385321	0.210526316	0.864663592	2.82107342 E-4	0.864664717	2.50582255 E-8	0.864664717	4.30998041 E-13	0.864664717	2.16323755 E-18	0.864664717	4.04799339 E-24
2^{-2}	0.976671851	0.895104895	0.981632201	0.224180395	0.981684271	8.20434288 E-4	0.981684361	4.51933998 E-7	0.981684361	7.25862072 E-11	0.981684361	4.34649981 E-15
2^{-3}	0.998245819	0.996351253	0.999588836	0.996631800	0.999661753	0.9641165497	0.999664506	0.321522100	0.999664537	2.42967127 E-3	0.999664537	4.66699685 E-6
2^{-4}	0.999918676	0.999885572	0.999998546	0.999996700	0.999999759	0.9999988660	0.999999870	0.999997987	0.999999886	0.9999987764	0.999999887	0.999800486
2^{-5}	0.999996963	0.999996424	0.999999998	0.999999997	1	1	1	1	1	1	1	1
2^{-6}	1	1	1	1	–	–	–	–	–	–	–	–

Table 2

N	$\rho = 0.1$		$\rho = 0.05$	
	Q_W	$1/\Lambda$	Q_W	$1/\Lambda$
5	0.998801438	83.33	0.999962500	13333.33
10	0.999637251	275.57	0.999999640	141093.45
15	0.998694000	76.47	0.999999960	1252911.33
20	0.976240000	4.11	0.999999970	2154990.25

Table 3

$N = 5$, $m = 4$, $r = 2$;
$\lambda(1) = 0.12$, $\mu(1)/\varepsilon = 1.00$, $\pi_1 = \frac{2}{3}$;
$\lambda(2) = 0.06$, $\mu(2)/\varepsilon = 1.00$, $\pi_2 = \frac{1}{3}$.

a_{11}	a_{22}	Q_W	$1/\Lambda$	P_W	$\hat{Q}_W$
50	100			0.99932	0.99997
0.5	1	0.99798	494.61	0.99925	0.99879
0.05	0.1			0.99908	0.99810

Table 4

$N = 5$, $m = 4$, $r = 2$;
$\lambda(1) = 0.12$, $\mu(1)/\varepsilon = 1.30$, $\pi_1 = \frac{2}{3}$;
$\lambda(2) = 0.06$, $\mu(2)/\varepsilon = 1.30$, $\pi_2 = \frac{1}{3}$.

a_{11}	a_{22}	Q_W	$1/\Lambda$	P_W	$\hat{Q}_W$
50	100	0.99946	1412.68	0.99932	0.99999
0.5	1	0.99946	1412.68	0.99925	0.99958
0.05	0.1	0.99946	1412.68	0.99908	0.99934

Table 5

$\lambda(1) = 0.12$, $\mu(1)/\varepsilon = 1.30$, $\pi_1 = \frac{2}{3}$;
$\lambda(2) = 0.06$, $\mu(2)/\varepsilon = 1.30$, $\pi_2 = \frac{1}{3}$.

N	Q_W	$2/\Lambda$
5	0.99946	1412.68
10	0.99989	7076.09
15	0.99974	2931.39
20	0.99674	253.10

Table 6

$N = 5$, $m = 4$, $r = 2$;
$\lambda(1) = 0.12$, $\pi_1 = \frac{2}{3}$;
$\lambda(2) = 0.06$, $\pi_2 = \frac{1}{3}$.

μ/ε	W_W	$1/\Lambda$
1.2	0.99919	1025.64
1.3	0.99946	1412.68
1.4	0.99962	1900.12
1.45	0.99968	2186.46

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/ = 5, $r = 2$; (1) = 0.12, $\mu(1)/\varepsilon = 1.30$, $\pi_1 = \frac{2}{3}$; (2) = 0.06, $\mu(2)/\varepsilon = 1.30$, $\pi_2 = \frac{1}{3}$.	
ɨ	$1/\Lambda$
[illegible]	23.01
[illegible]	128.42
[illegible]	1412.68

ſable 8

V = 6, $m = 5$, $r = 2$							
ι(1)	$\lambda(2)$	$\mu(1)/\varepsilon$	$\mu(2)/\varepsilon$	π_1	π_2	Q_W	$1/\Lambda$
).12	0.06	1.2	1.45	2/3	1/3	0.9995464	1730.86
).12	0.06	1.283	1.263	2/3	1/3	0.9996767	2409.84
).1	0.1	1.2	1.45	2/3	1/3	0.9998191	4341.32
).12	0.06	1.2	1.2	2/3	1/3	0.9995164	1722.65
).12	0.06	1.45	1.45	2/3	1/3	0.9998446	4437.43
).12	0.12	1.2	1.45	2/3	1/3	0.9994600	1453.90
).06	0.06	1.2	1.45	2/3	1/3	0.9999915	93049.71

Case 8

For given N, m, r, we illustrate the system's behaviour for different failure and repair rates. See Table 8.

It is a general case and we can observe that Q_W is the same up to 3 digits but $1/\Lambda$ differs very much in different cases.

5. Conclusion

In this paper the machine interference problem has been treated supposing that the system is embedded in a random environment. Assuming that the repair rate is much greater than the failure rate ('fast' service), it is shown that the time until the number of stopped machines reaches a certain level converges weakly, under appropriate norming, to an exponentially distributed random variable. Furthermore, some numerical examples illustrate the problem in question in the field of textile winding.

References

Anisimov, V.V., Zakusilo, O.K., and Donchenko, V.S. (1987), *Elements of Queueing Theory and Asymptotic Analysis of Systems*, Visha Skola, Kiev (in Russian).

Anisimov, V.V., and Sztrik, J. (1989), "Asymptotic analysis of some controlled finite-source queueing systems", *Acta Cybernetica* 9, 27–38.

Bunday, B.D. (1986), *Basic Queueing Theory*, Edward Arnold, London.

Gaver, D.P., Jacobs, P.A., and Latouche, G. (1984), "Finite birth-and-death models in randomly changing environments", *Advances in Applied Probability* 16, 715–731.

Gertsbakh, I.B. (1984), "Asymptotic methods in reliability theory: A review", *Advances in Applied Probability 16, 147–175.*

Gertsbakh, I.B. (1989), Statistical Reliability Theory, Marcel Dekker, New York.

Keilson, J. (1979), *Markov Chain Models – Rarity and Exponentiality*, Springer-Velag, Berlin.

Osaki, S., and Cao, J. (eds.) (1987), *Reliability Theory and Applications*, World Scientific, Singapore.

Rukhin, A.L., and Hsieh, H.K. (1987), "Survey of Soviet works on reliability theory", *Statistical Science* 2, 484–503.
Stecke, K.E., and Aronson, J.E. (1985), "Review of operator/machine interference models", *International Journal of Production Research* 23, 129–151.
Takagi, H. (1990), "Bibliography on Performance Evaluation", in: H. Takagi (ed.), *Stochastic Analysis of Computer and Communication Systems*, IFIP 1990, North-Holland, Amsterdam.