Reliability Analysis of Finite-Source Retrial Queues with Outgoing Calls Using Simulation

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***Abstract*—The aim of this paper is to create a simulation program in order to examine systems of two-way communication by the help of retrial queuing systems with finite source. Primary customers arrive from a finite source to the server according to exponential distribution. If an incoming customer discovers the server in an idle state then its service begins right away. Otherwise, if the server is busy or in failed state arriving primary customers move into the orbit and after some random time they try to reach the server. Whenever the server becomes idle it produces an outgoing call after an exponentially distributed time from the infinite source. If no primary customer arrives either from the finite source or from the orbit and the server functions until its arrival then it enters to the system. Otherwise, the outside call is cancelled. Several scenarios are distinguished when the server is in failed state and in this work we concentrate on comparing various distributions of service time of primary customers and failure probability of the server. The novelty of the investigation is to analyze such system with non-reliable server. Various figures illustrate comparison of using different scenarios showing the mean waiting time of primary customers and the utilization of the server obtained by simulation.**

***Index Terms*—retrial queues, finite-source queuing system, server breakdowns and repairs, simulation, two-way communication.**

I. Introduction

In the real world retrial queues are commonly and powerful tools modeling problems arising in major telecommunications systems such as telephone switching systems, call centers, CSMA-based wireless mesh networks in frame level and computer systems. Their importance can be viewed in the following works like in [1], [2], [3], [4], [5].

It is a peculiar feature of retrial queues that incoming customers move to the so-called orbit instead of the service area when the server is busy or in failed state. In the recent years more and more papers investigate two-way communication structures because with the help of these systems many real life examples can be modeled in various application fields. Particularly, in call centers the service unit not only serves incoming calls, but also manages certain other work in idle state like advertising and promoting products. Increasing utilization of the server is crucial, see for example in [6],[7],[8],[9],[10],[11], [12], [13].

The most significant feature of two-way communication is that an idle server search for customers inside and outside of the system, so it can perform outgoing calls from outside. The novelty of the investigated queuing system resides in the following: idle service unit generate a call from the outside world (infinite source) which arrives after a random time. But it will only be served if no customers from the finite source or from the orbit come because then the external call will be canceled. Results in connection with retrial queueing systems with two-way communication, where the source is infinite, are found in [14],[15],[16],[17],[18],[19],[20],[21].

Many papers deal with the assumption of reliable operation which is unfortunately quite unrealistic in real life applications, thus it is very important to examine retrial queueing systems with server failures. Servers from time to time can break down which has a great influence on the system characteristics and performance measures. Our model contains server breakdowns and repairs. Retrial queuing systems of finite source with server failures and repairs have been investigated in several recent papers, for example in [22],[23],[24],[25],[26],[27],[28],[29],[30],[31],[32],[33].

Our objective is to examine the operation of a system when customers from outside can enter the system containing a non-reliable server. The novelty of this paper is to compare different scenarios in connection with server breakdown using various distribution of failure on performance measures like mean waiting time of primary customers or utilization of the server. We do not have knowledge about that any paper dealt with this model in case of non-reliable server or applying different distributions with the exception of exponential. To achieve this goal a simulation program is developed, which is based on SimPack [34], a collection of C/C++ libraries and executable programs. SimPack toolkit provides a set of utilities that illustrate the basics of building a working simulation from a model description. One of its main advantages is that it really depends on the user what performance measure to calculate and how the model is built up.

II. System model

This paper investigates a retrial queuing model with one server. In this model, N customer resides in the source, which is finite such that the system is stable in every moment. Each customer can produce a call (primary customers) towards

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the server with rate λ/N, so the inter-request times are exponentially distributed with parameter λ/N. Our model does not contain a queue, therefore the service of primary customers starts instantaneously if the server is idle. The service time of the primary customers follows gamma, hypo-exponential, hyper-exponential, Pareto and lognormal distribution with different parameters but with the same mean value. When the submission (the service of a request) is successful, the request goes back to the source. Customers located in the orbit may retry their requests for service after a random time. The distribution of this period is exponential with parameter σ/N. The server can break down during its operation or in idle state according to an exponentially gamma, hypo-exponential, hyper-exponential, Pareto and lognormal distribution time with the same mean value. Restoration starts instantly upon the breakdown and that time is also an exponentially distributed random variable with rate γ_1. The idle server after some exponentially distributed period can make outgoing calls towards the customers (secondary) from an infinite source. It is performed after an exponentially distributed idle period with parameter γ. The service of these customers can take place if no primary customers arrive from the finite source or from the orbit and the server is not in a failed state upon their arrivals. Otherwise they are cancelled and they return without entering the system. The service time of these type of customers follows gamma distribution with parameters α_2 and β_2. We differentiate four scenarios in case of server failure:

- Scenario 1: Primary customers are forwarded immediately towards the orbit and the secondary customers leave the system without service.
- Scenario 2: Primary customers are forwarded immediately towards the orbit and the secondary customers remain at the service area during the recovery of the service unit.
- Scenario 3: Primary customers remain at the service area during the recovery of the service unit and the secondary customers leave the system without service.
- Scenario 4: Both the primary and the secondary customers remain at the service area during the recovery of the service unit.

All the random variables involved in the model construction are assumed to be totally independent of each other.

III. Applied distributions and its parameters

In this Section the reader gets an insight of the parameters of the applied distributions and the process how to select them in order to execute a valid comparison. To do so our program is integrated with random number generators according to gamma, hyper-exponential, hypo-exponential, lognormal and Pareto distribution. These random number generators need input parameters which are different in every distribution, thus parameter selection is crucial. For valid comparison the goal is to achieve the same mean and variance in case of every distribution hence we take over every distribution and how the fitting process is accomplished.

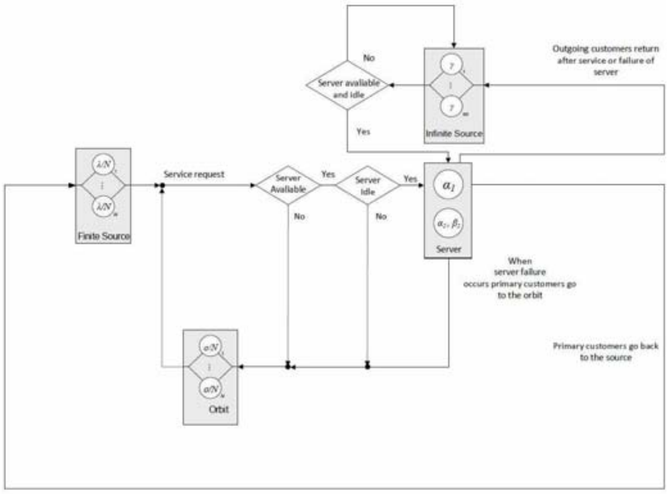


Fig. 1. System model

A. Gamma distribution

Gamma distribution is a general type of statistical distribution and a random variable X has a gamma distribution if its density function is the following:

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{\beta(\beta x)^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)} & \text{if } x \geq 0 \end{cases}$$

where $\beta > 0$ and $\alpha > 0$.

$$\Gamma(\alpha) = \int_0^\infty \mathrm{t}^{\alpha-1}\,\mathrm{e}^{-t}\,\mathrm{d}t$$

This is the so-called complete gamma function, which has two parameters: α is called the shape parameter and β is called the scale parameter. These two parameters are also the input parameters of the random number generator.

The coefficient $C_X^2 = \frac{Var(X)}{(EX)^2}$ is defined as the squared coefficient of variation of random variable X.

The mean value, variation and the squared coefficient of variation can be calculated:

$$\overline{X} = \frac{\alpha}{\beta}, \qquad Var(X) = \frac{\alpha}{\beta^2}, \qquad C_X^2 = \frac{1}{\alpha}$$

For a predetermined mean value and variance to obtain parameters α and β the next calculation has to be done:

$$\alpha = \frac{1}{C_X^2}$$
$$\beta = \frac{\alpha}{\overline{X}}$$

B. Pareto distribution

A random variable X has a Pareto distribution if its density function is the following:

$$f(x) = \begin{cases} 0 & \text{if } x < k \\ \alpha k^\alpha x^{-\alpha-1} & \text{if } x \geq k \end{cases}$$

And the distribution function is:

$$F(x) = \begin{cases} 0 & \text{if } x < k \\ 1 - \left(\frac{k}{x}\right)^{\alpha} & \text{if } x \geq k \end{cases}$$

where $\alpha, k > 0$.

It has two parameters: α is called the shape parameter and k is called the location parameter. These two parameters are the input parameters of the random number generator.

The mean value, variation and the squared coefficient of variation can be calculated:

$$\overline{X} = \begin{cases} \frac{k\alpha}{\alpha-1} & \text{if } \alpha > 1 \\ \infty & \text{if } \alpha \leq 1 \end{cases}$$

$$Var(X) = \frac{k^2\alpha}{\alpha-2} - \left(\frac{k\alpha}{\alpha-1}\right)^2$$

$$C_X^2 = \frac{(\alpha-1)^2}{\alpha(\alpha-2)} - 1, \qquad \alpha > 2.$$

For a predetermined mean value and variance to obtain parameters α and k the following interrelation is used:

$$\alpha = 1 + \frac{\sqrt{1+C_X^2}}{\sqrt{C_X^2}}$$

$$k = \frac{\alpha-1}{\alpha} \times \overline{X}$$

C. Lognormal distribution

Let $Y \in N(m, \sigma)$ a random variable with normal distribution, lognormal is a continuous distribution in which the logarithm of a variable having a normal distribution, namely $X = e^Y$ has lognormal distribution with parameters (m, σ). Its distribution and density function are the following:

$$F_x(x) = \Phi\left(\frac{\ln(x) - m}{\sigma}\right), \qquad x > 0.$$

$$f_x(x) = \frac{1}{\sigma x}\Phi\left(\frac{\ln(x) - m}{\sigma}\right), \qquad x > 0.$$

The mean value, variation and the squared coefficient of variation can be calculated:

$$\overline{X} = e^{m+\frac{\sigma^2}{2}}, \qquad Var(X) = e^{2m+\sigma^2}(e^{\sigma^2} - 1)$$

$$C_X^2 = e^{\sigma^2} - 1.$$

To obtain the two parameters of the lognormal distribution the following interrelation is applied:

$$\sigma = \sqrt{\ln(1 + C_X^2)}$$

$$m = \ln(\overline{X}) - \frac{\sigma^2}{2}$$

D. Hypo-exponential distribution

Continuous statistical distribution, let $X_i \in Exp(\mu_i)(i = 1, ..., n)$ be independent exponentially distributed random variables. Then $Y_n = X_1 + ... + X_n$ has n-phase hypo-exponential distribution. Its density function is given by

$$f_{Y_n}(x) = \begin{cases} 0 & \text{if } x < 0 \\ (-1)^{n-1}\left[\prod_{i=1}^{n} \mu_i\right] \sum_{j=1}^{n} \frac{e^{-\mu_j x}}{\prod_{k=1, k\neq j}^{n}(\mu_j - \mu_k)} & \text{if } x \geq 0. \end{cases}$$

The mean value, variation and the squared coefficient of variation can be calculated:

$$\overline{Y_n} = \sum_{i=1}^{n} \frac{1}{\mu_i}, \qquad Var(Y_n) = \sum_{i=1}^{n} \frac{1}{\mu_i^2}$$

$$C_{Y_n}^2 = \frac{\sum_{i=1}^{n}\left(\frac{1}{\mu_i}\right)^2}{\left(\sum_{i=1}^{n}\frac{1}{\mu_i}\right)^2}.$$

In our simulation program we used the 2-phase hypo-exponential distribution where the parameters are the parameters of the two independent exponential distribution (μ_1, μ_2). For a predetermined mean value and variance to obtain parameters μ_1 and μ_2 the next equation system has to be solved:

$$\overline{X} = \frac{1}{\mu_1} + \frac{1}{\mu_2}$$

$$Var(X) = \frac{1}{\mu_1^2} + \frac{1}{\mu_2^2}$$

E. Hyper-exponential distribution

Suppose $X_1, X_2, \cdots, X_n$ are independent exponential random variables, where the rate parameter of X_i is λ_i. The random variable X can be one of the n independent exponential random variables $X_1, X_2, \cdots, X_n$ such that X is X_i with probability p_i with $p_1 + \cdots + p_n = 1$. Such a random variable X is said to follow a hyper-exponential distribution. Its density function is given by

$$f_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ \sum_{i=1}^{n} p_i\lambda_i e^{-\lambda_i x} & \text{if } x \geq 0. \end{cases}$$

Its distribution function is

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - \sum_{i=1}^{n} p_i e^{-\lambda_i x} & \text{if } x \geq 0. \end{cases}$$

In the case when for a random variable $X, C_X^2 > 1$ then the the following two-moment fit is suggested

$$f_Y(t) = p\lambda_1 e^{-\lambda_1 t} + (1-p)\lambda_2 e^{-\lambda_2 t}.$$

Y is a 2-phase hyper-exponentially distributed random variable. The most commonly used procedure is the balanced mean method, that is

$$\frac{p}{\lambda_1} = \frac{1-p}{\lambda_2}.$$

To obtain the three parameters of the hyper-exponential distribution the following calculation is used:

$$p = \frac{1}{2}\left(\sqrt{\frac{C_X^2 - 1}{C_X^2 + 1}}\right)$$
$$\lambda_1 = \frac{2p}{\overline{X}}$$
$$\lambda_2 = \frac{2(1-p)}{\overline{X}}$$

IV. Simulation results

A. Different distributions of service time of primary customers

1) Squared coefficient of variation is greater than one: The applied values of the input parameters are presented in Table I, the failure time of the server is exponentially distributed with rate γ_0 in this case. We investigate the effect of different service time distributions where the mean and variance are equal, Table II shows the parameters of service time of primary customers. In this subsection the squared coefficient of variation is greater than one, because using hyper-exponential distribution regardless of the parameters the squared coefficient of variation is always greater than one. Besides hyper-exponential, gamma, lognormal and Pareto distributions are used for comparison.

TABLE I
Numerical values of model parameters

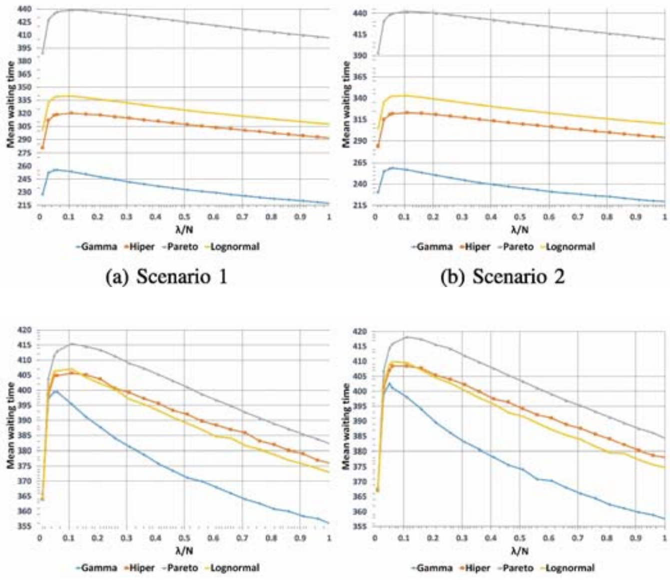
N	λ/N	γ_0	γ_1	σ/N	γ	α_2	β_2
100	0.01	0.05	0.5	0.01	0.8	1	1

TABLE II
Parameters of service time of primary customers

Distribution	Gamma	Hyper-exponential	Pareto	Lognormal
Parameters	$\alpha = 0.037$ $\beta = 0.015$	$p = 0.482$ $\lambda_1 = 0.385$ $\lambda_2 = 0.416$	$\alpha = 2.018$ $k = 1.261$	$m = -0.751$ $\sigma = 1.826$
Mean	2.5			
Variance	169			
Squared coefficient of variation	27.04			

The mean waiting time is shown in function of arrival intensity of primary customers on Figure 2, where all four scenarios appear. Interestingly, pronounced differences can be observed especially on Figure 2a and 2b. Despite the fact that the mean and variance are the same, results clearly illustrate the effect of various distributions. Highest values are experienced in case of Pareto distribution and the lowest in case of gamma distribution. When primary customers remain at the server (Figure 2c and 2d) during failure values of mean mean waiting time are much closer compared to the first two scenarios. With suitable parameter settings, we experience the maximum property characteristic of finite-source retrial queueing systems.

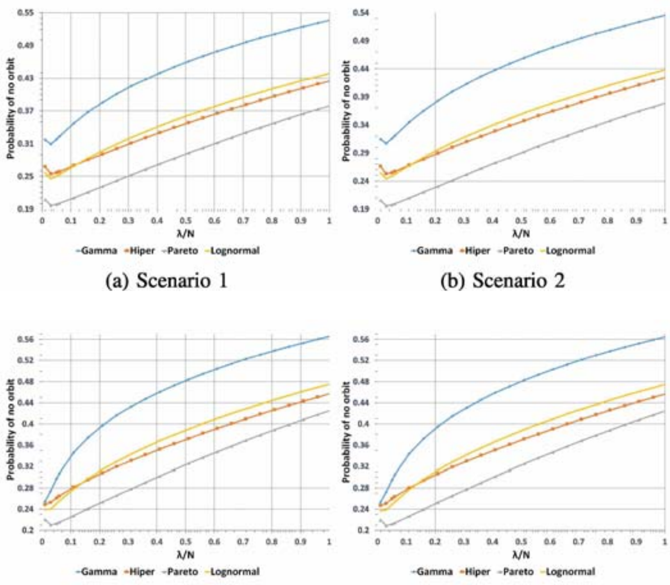
Figure 3 shows the probability of no orbit in function of arrival intensity of primary customers. Under probability of no orbit we mean the following: probability that a primary customer does not get into the orbit throughout its residence



(a) Scenario 1 (b) Scenario 2

(c) Scenario 3 (d) Scenario 4

Fig. 2. Mean waiting time vs. arrival intensity using various distributions



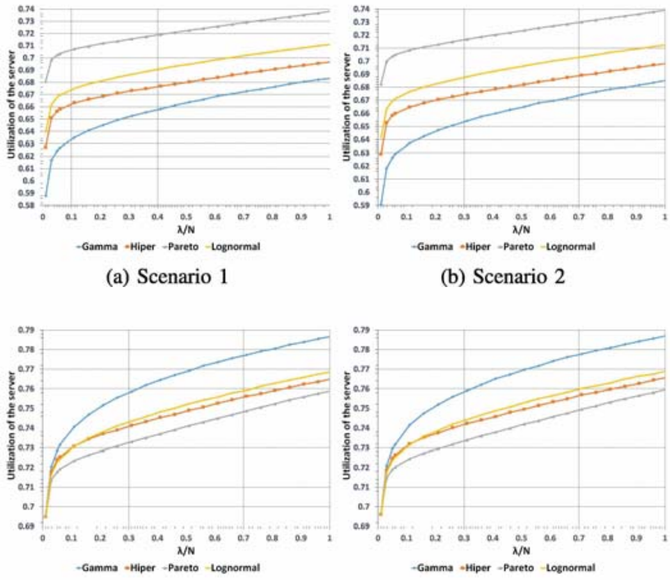
(a) Scenario 1 (b) Scenario 2

(c) Scenario 3 (d) Scenario 4

Fig. 3. Probability of no orbit vs. arrival intensity using various distributions

of the system. In case of gamma distribution this value is the highest and the lowest in case of Pareto distribution. This is logical after seeing the results of Figure 2. Interestingly, in all scenarios it has a minimum value, it starts to decrease when the arrival intensity of primary customers is very low then it increases with the intensity as well.

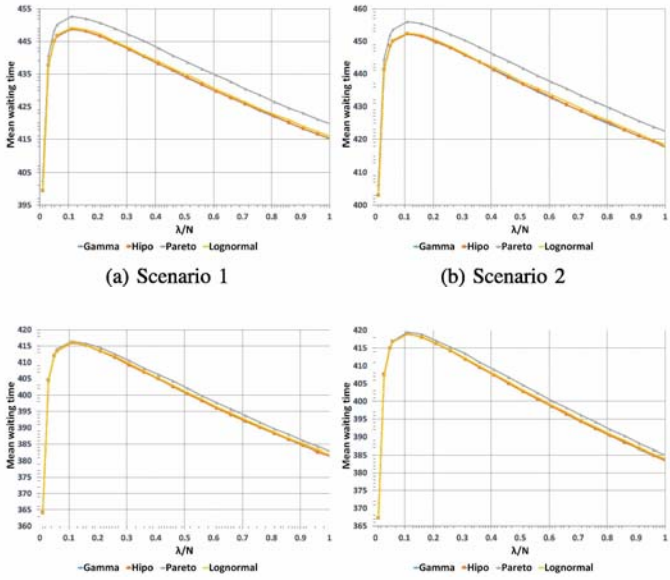
Figure 4 illustrate how the utilization of the server is larger when the arrival intensity of primary customers increases. This performance measure contains the service of outgoing calls, too. In case of Figure 4a and 4b using Pareto distribution we



(a) Scenario 1 (b) Scenario 2

(c) Scenario 3 (d) Scenario 4

Fig. 4. Utilization of server vs. arrival intensity using various distributions



(a) Scenario 1 (b) Scenario 2

(c) Scenario 3 (d) Scenario 4

Fig. 5. Mean waiting time vs. arrival intensity using various distributions

observe the highest values indicating that here happens the most interruptions while in Figure 4c and 4d the highest values can be found at gamma distribution where primary customers spend the least in the system. Naturally in these scenarios the utilization of the server is higher than in Scenario 1 or in Scenario 2.

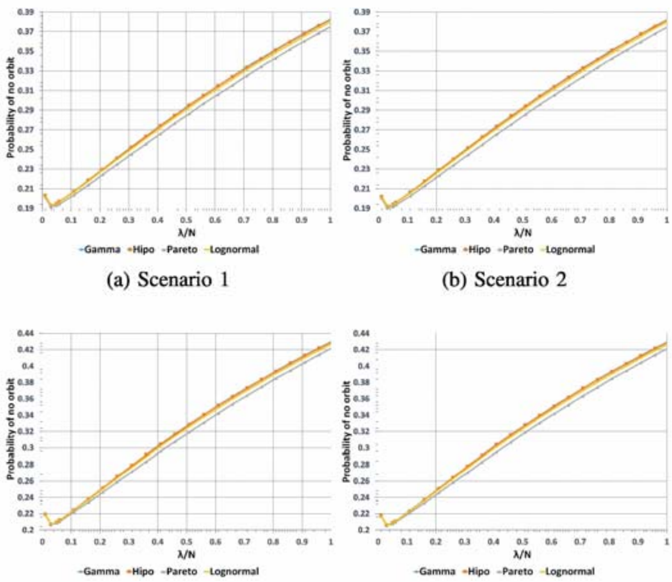
2) Squared coefficient of variation is less than one: In the previous section we see that when the squared coefficient of variation of the service time of primary customers is more than 1, the differences are quite high among the performance measures. The question arises that whether it is true for using other parameter settings for example when the squared coefficient of variation of the service time of primary customers is less than one. We use the same parameters like in the previous section (Table I), and Table III contains the changed parameters of service time of primary customers. Instead of hyper-exponential we use this time hypo-exponential distribution because the squared coefficient of variation is always less or equal to one.

TABLE III
PARAMETERS OF SERVICE TIME OF PRIMARY CUSTOMERS

Distribution	Gamma	Hypo-exponential	Pareto	Lognormal
Parameters	$\alpha = 1.8$ $\beta = 0.72$	$\mu_1 = 0.6$ $\mu_2 = 1.2$	$\alpha = 2.673$ $k = 1.565$	$m = 0.695$ $\sigma = 0.665$
Mean	2.5			
Variance	3.472			
Squared coefficient of variation	0.555			

Comparing Figure 2 with Figure 5 the contrast is quite obvious, namely the values of mean waiting time are almost identical regardless of the distribution. However, when the squared coefficient of variation is more than one it results lower mean waiting time in all Scenarios compared to Figure 2 except the Pareto distribution.

Figure 6 and Figure 7 present the probability of no orbit and the utilization of server in function of arrival intensity of



(a) Scenario 1 (b) Scenario 2

(c) Scenario 3 (d) Scenario 4

Fig. 6. Probability of no orbit vs. arrival intensity using various distributions

primary customers, respectively. After observing the results in connection of the mean waiting time it is no wonder that the received results are almost identical, this is also true if we take a closer look at the applied Scenarios.

B. Different distributions of failure time of the server

1) Squared coefficient of variation is greater than one: In this section we investigate the effect of failure time of the server on the mean waiting time. We are interested in carrying out sensitivity analysis to identify how much the various distributions alter the characteristics of the system. To do so for

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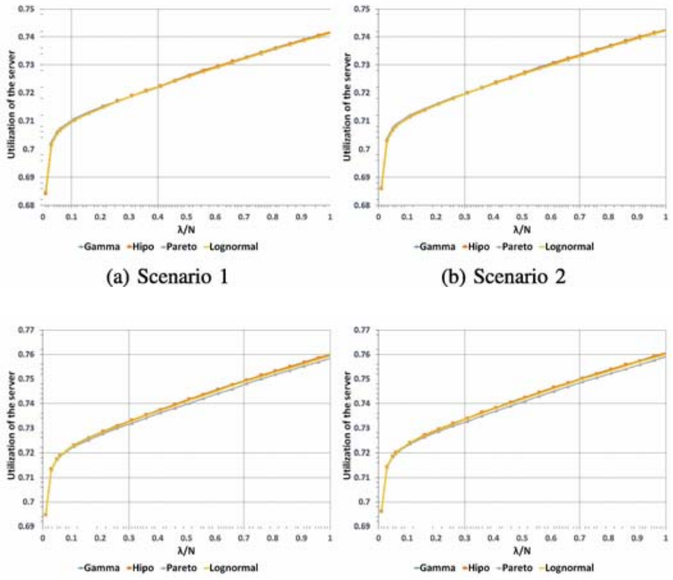
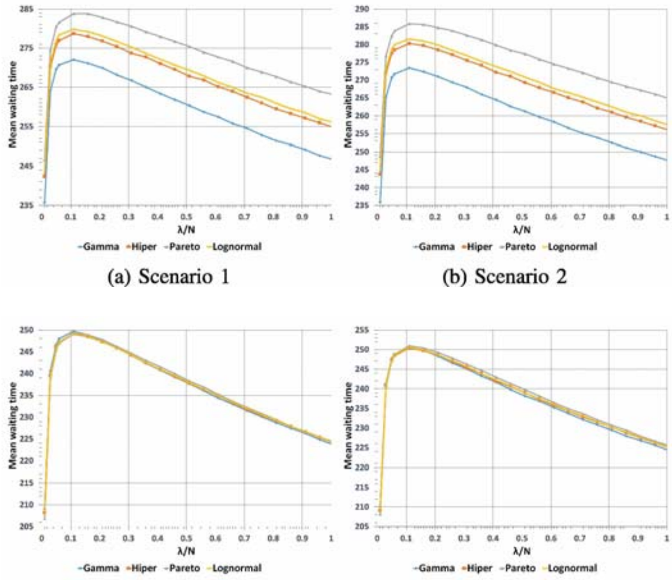


(a) Scenario 1 (b) Scenario 2 (c) Scenario 3 (d) Scenario 4

Fig. 7. Utilization of server vs. arrival intensity using various distributions



(a) Scenario 1 (b) Scenario 2 (c) Scenario 3 (d) Scenario 4

Fig. 8. Mean waiting time vs. arrival intensity using various distributions

failure time we utilize gamma, hyper-exponential, lognormal and Pareto distributions having the same mean and variance so we carefully selected the parameters at the adequate distribution. The service of primary customers in this case is exponential with rate $\alpha_1 = 1$. The same investigation will be carried out as in the previous section so first we examine the case when the squared coefficient of variation is greater than one. For the easier understanding the numerical values of parameters are collected in Table IV and the parameters of failure time in Table V.

TABLE IV
NUMERICAL VALUES OF MODEL PARAMETERS

N	λ/N	α_2	β_2	γ_1	σ/N	γ	α_1
100	0.01	1	2.5	0.5	0.01	0.8	1

TABLE V
PARAMETERS OF FAILURE TIME OF THE SERVER

Distribution	Gamma	Hyper-exponential	Pareto	Lognormal
Parameters	$\alpha = 0.312$ $\beta = 0.056$	$p = 0.362$ $\lambda_1 = 0.129$ $\lambda_2 = 0.228$	$\alpha = 2.146$ $k = 2.984$	$m = 1.003$ $\sigma = 1.198$
Mean	5.588			
Variance	100			
Squared coefficient of variation	3.2			

Figure 8 shows the mean waiting time of primary customers vs. the arrival of primary customers in respects of all Scenarios. On Figure 8a and 8b the difference is evident among the applied distributions apparently in case of Pareto where the values of mean waiting time is the highest. But in Scenario 3 and Scenario 4 this divergence disappears and the received graphs reflect near identity.

2) Squared coefficient of variation is less than one: In this section we change the parameters of failure of the server to examine the mean waiting time when the squared coefficient of variation is less than one. Table VI contains the overview of these parameters, mean value remains the same as in the previous section. The value of other parameters are unchanged and Table IV contains them.

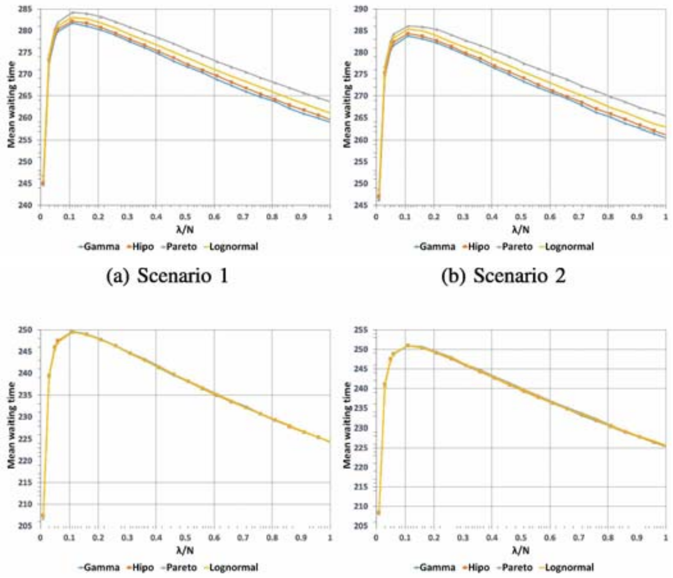
TABLE VI
PARAMETERS OF FAILURE TIME OF THE SERVER

Distribution	Gamma	Hypo-exponential	Pareto	Lognormal
Parameters	$\alpha = 1.232$ $\beta = 0.2204$	$\mu_1 = 0.2$ $\mu_2 = 1.7$	$\alpha = 2.494$ $k = 3.347$	$m = 1.426$ $\sigma = 0.771$
Mean	5.588			
Variance	25.346			
Squared coefficient of variation	0.8116			

On Figure 9 the mean waiting time of the primary jobs are displayed as a function of the primary generation rate. On Figure 9a and 9b the difference is very moderate compared to Figure 8a and 8b. The lines are identical in case of Scenario 3 and 4. These graphs show that the distribution of the failure time has minor impact on the performance measures when the squared coefficient of variation is less than one.

V. CONCLUSION

In this paper a finite-source retrial queueing system is presented with a non-reliable server which can produce an outgoing call from an infinite source. Several figures showed the effect of using various distributions on the mean waiting time of primary customers, on the probability of no orbit of primary customers and on the utilization of the server. Results clearly presented when the squared coefficient of variation is greater than one then the disparity is huge among the performance measures having the same mean and variance and very little when it is less than one. Using stochastic simulation method we studied the effect of distribution of the failure and



(a) Scenario 1 (b) Scenario 2

(c) Scenario 3 (d) Scenario 4

Fig. 9. Mean waiting time vs. arrival intensity using various distributions

service times on the main performance measures. The authors intend to resume their research work, examining finite-source retrial queuing system with two-way communication in case of outgoing calls toward the customers from the finite source or/and orbit when the server is not reliable.

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