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This volume presents selected papers from the XIX International Conference named after A.F. Terpugov. The papers are devoted to new results in the following areas: queuing theory and its applications, data mining and visualization, information technology and software engineering, mathematical and computer modeling of technological processes.

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Simulation of retrial queueing system M/G/1 with impatient customers, collisions and unreliable server*

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In this paper, we consider a retrial queueing system of M/G/1 type with an unreliable server, collisions, and impatient customers. The novelty of our work is to carry out a sensitivity analysis applying different distributions of service time of customers on significant performance measures for example on the probability of abandonment, the mean waiting time of an arbitrary, successfully served, impatient customer, etc. The service, retrial, impatience, operation, and repair times are supposed to be independent of each other.

In the paper [1] a retrial queueing system of M/M/1 type with Poisson flow of arrivals, impatient customers, collisions, and unreliable service device is presented. In that, an asymptotic analysis method is used to define the stationary distribution of the number of customers in the orbit. We investigate the same model as in [1], but the results are gathered by our simulation program package. With this approach, it is possible to calculate performance measures that can not be determined or almost impossible to give exact formulas using numerical or asymptotic analysis. Various software packages exist which are capable to describe and perform an evaluation of complex systems if all the random variables are exponentially distributed but undoubtedly the usage of simulation has a tremendous advantage: besides exponential, any other distribution can be integrated into the code.

In real life in many cases customers encountering the service units in busy state may make a decision to attempt to be served after some random time remaining in the system. Instead of residing in a queue these customers

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are located in a virtual waiting room called orbit and can be modeled with retrial queues. Queuing systems with retrial queues are widely used tools modelling emerging problems in major telecommunication systems, such as telephone switching systems or call centres. Many papers dealt with these types of systems which can be viewed in the following works like in [2–4].

Models with customers impatience in queues like the process of reneging and balking have been studied by various authors in the past. Most recent results about systems having the impatience property can be found for example in [5–7].

System model

We consider a queueing system of M/G/1 with collisions, impatience of the customers, and an unreliable server which is shown in Fig. 1. The system arrival process is characterized by the Poisson process with rate λ. The arriving customer occupies instantly the service unit in idle state and the distribution of its service is according to exponentially, gamma, Pareto, lognormal, hypo-exponentially, and hyper-exponentially distributed random variable with the same mean value and variance but with different parameters. Otherwise, it is forwarded toward the orbit. The retrial time of the requests is assumed to be exponentially distributed with a rate of σ. In the case of a busy server an arriving customer brings about a collision and both requests enter the orbit. It is supposed that the server is unreliable so it breaks down from time to time according to an exponential distribution with parameter γ_0 when the server is idle and with parameter γ_1 when it is busy. In that period generation of new requests continues but each of them is sent

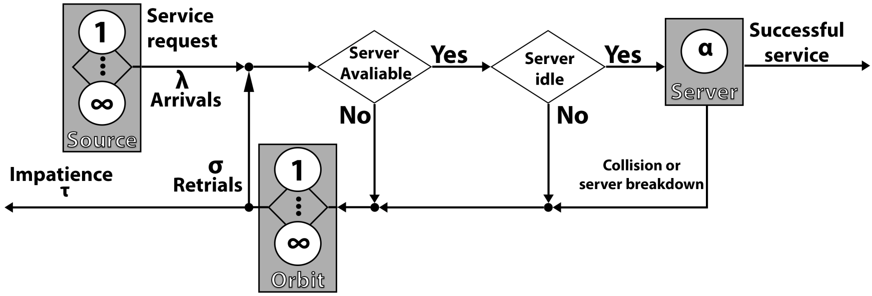

Fig. 1. System model

to orbit. After a breakdown, it is immediately sent for repair and the recovery process is also an exponential random variable with the rate γ_2. Every customer possesses an «impatience» property meaning that a customer may depart from the system earlier after waiting a random time in the orbit. The distribution of the impatient time follows an exponential distribution with parameter τ. In this unreliable model after interruption or breakdown, it is supposed that requests immediately are placed in the orbit. Every service is independent of the other service including the interrupted ones, too.

Simulation results

To obtain the results of our simulation program a statistic package is used that was developed by Andrea Francini in 1994 [8]. With the help of this tool, it is possible to make a quantitative estimation of the mean and variance values of the desired variables using the method of batch means. There are *n* observations in every batch and the useful run is divided into a predetermined number of batches. In order for the estimation to work correctly, the batches are necessary to be long enough and approximately independent. It is one of the most popular confidence interval techniques for a steady-state mean of a process. The following works contain more detailed information about this method in [9]. The simulations are performed with a confidence level of 99.9%. The relative half-width of the confidence interval required to stop the simulation run is 0.00001.

The realization of the sensitivity analysis includes four different distributions of service time to compare the performance measures with each other. In every case, the parameters are selected in a way that the mean and variance would be equal. To accomplish that we applied a fitting process that is required to be done and [10] contains detailed information about the whole process describing every used distribution. Two scenarios are developed to investigate the effect of the various distributions. Table 2 shows the chosen parameters of the distribution of service time while Table 1 the values of other parameters. In the first one, the squared coefficient of variation is greater than one and the following distributions are used: hyper-exponential, gamma, Pareto, and lognormal. Results in connection with the second scenario (when the squared coefficient of variation is less than one) were also examined but because of the page limitation, these will be intended to be published in the extended version of the paper.

Table 1

Numerical values of model parameters

σ	γ_0	γ_1	γ_2	τ
0.01; 0.001	0.1	0.2	1	0.02; 0.002

Table 2

Parameters of service time of incoming customers

Distribution	Gamma	Hyper-exponential	Pareto	Lognormal
Parameters	$\alpha = 0.0816326$	$p = 0.4607213$	$\alpha = 2.0400156$	$m = -1.291998$
	$\beta = 0.0816326$	$\lambda_1 = 0.9214426$	$k = 0.5098076$	$\sigma = 1.6074817$
		$\lambda_2 = 1.0785573$		
Mean	1			
Variance	12.25			
Squared coefficient of variation	12.25			

In Figures 2 and 3 the comparison of steady-state distribution can be seen when the distribution of service time of the incoming customers is different. It demonstrates the probability ($P(i)$) of how many customers (i) residing in the orbit. Taking a closer look at the curves in more detail they coincide with normal distribution regardless of the used parameter setting. The figures also show the case of exponential distribution with the same mean as the other applied distributions. The mean number of customers in the orbit significantly differs from each other, at gamma distribution customers spend the fewest at Pareto distribution the highest time for waiting which is quite interesting.

The mean waiting time of an arbitrary customer is presented in the function of the arrival intensity of incoming customers in Figures 4 and 5. Even though the mean and the variance are identical huge gaps develop among the applied distributions. With the increment of the arrival intensity, the mean waiting time of an arbitrary customer increases as well. The same tendency is observable when we use other values of retrial and impatience time. The usage of gamma distribution results in lower mean waiting time compared to the others, especially versus gamma and Pareto distributions.

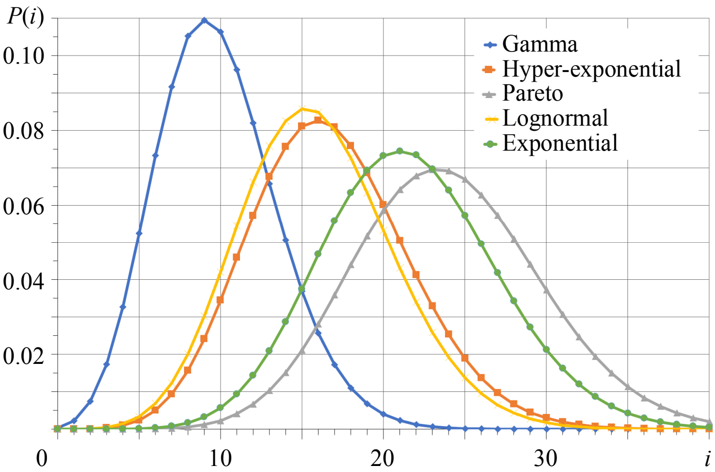

Fig. 2. Distribution of the number of customers in the orbit using various distributions, $\sigma = 0.01$, $\tau = 0.02$, $\lambda = 0.7$

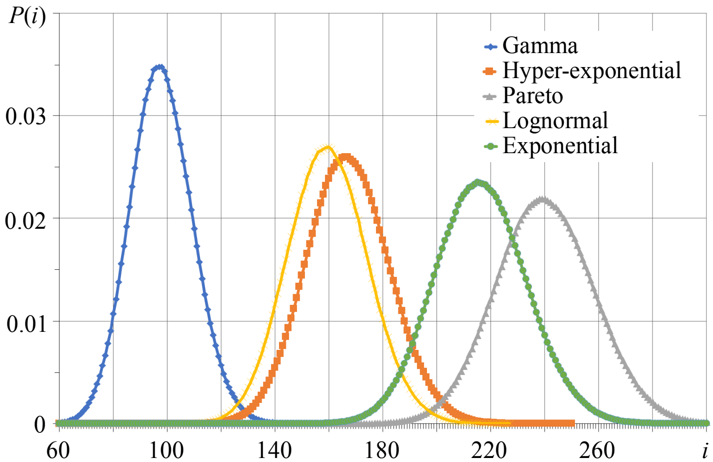

Fig. 3. Distribution of the number of customers in the orbit using various distributions, $\sigma = 0.001$, $\tau = 0.002$, $\lambda = 0.7$

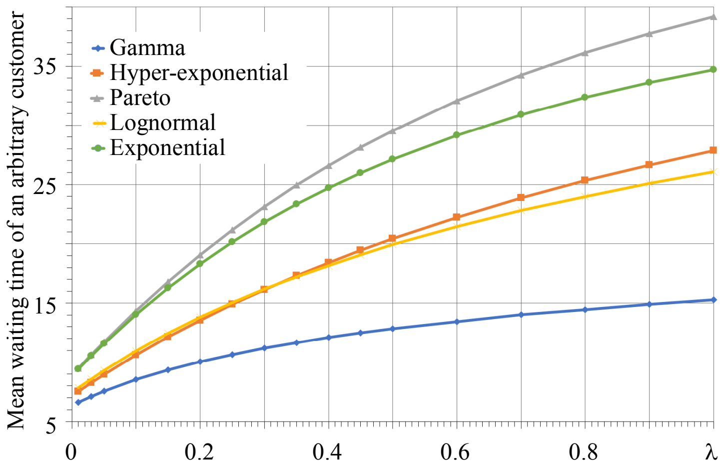

Fig. 4. Mean waiting time of an arbitrary customer vs arrival intensity using various distributions, $\sigma = 0.01$, $\tau = 0.02$

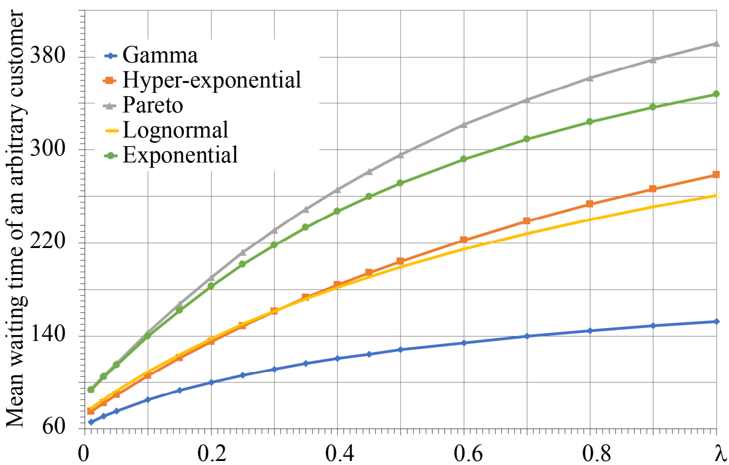

Fig. 5. Mean waiting time of an arbitrary customer vs arrival intensity using various distributions, $\sigma = 0.001$, $\tau = 0.002$

Conclusions

We studied the development of performance measures like the mean number of customers in the orbit or the mean waiting time of an arbitrary customer in a retrial queueing system of type M/G/1 with a non-reliable server and impatient customers in the orbit. Simulation has been carried out, the obtained results demonstrate that the number of customers in the orbit corresponds to the normal distribution in the case of every applied distribution. It is also displayed how the different distributions affect the performance measures despite the equality of mean value and variance when the squared coefficient of variation is more than one.

REFERENCES

1. *Danilyuk E.Yu.*, *Moiseeva S.P.*, *Sztrik J.* Asymptotic analysis of retrial queueing system M/M/1 with impatient customers, collisions and unreliable serv // J. Sib. Fed. Univ. Math. Phys. 2020. V. 13. No. 2. P. 218–230.
2. *Fiems D.*, *Phung-Duc T.* Light-traffic analysis of random access systems without collisions // Ann. Oper. Res. 2019. V. 277. P. 311–327.
3. *Kim J.*, *Kim B.* A survey of retrial queueing systems // Ann. Oper. Res. 2016. V. 247. P. 3–36.
4. *Tóth Á.*, *Sztrik J.* Simulation of finite-source retrial queuing systems with collisions, non-reliable server and impatient customers in the orbit // Proceedings of 11th International Conference on Applied Informatics. 2020. V. 2650. P. 408–419.
5. *Jouini Q.*, *Koole G.*, *Roubos A.* Performance indicators for call centers with impatient customers // IIE Transactions. 2013. V. 45. No. 3. P. 341–354.
6. *Kumar S.*, *Sharma S.* Transient performance analysis of a single server queuing model with retention of reneging customers // Yugoslav Journal of Operations Research. 2018. V. 28. No. 3. P. 315–331.
7. *Kim C.S.*, *Dudin S.*, *Dudin A.*, *Samouylov K.* Analysis of a semi-open queuing network with a state dependent marked Markovian arrival process, customers retrials and impatience // Mathematics. 2019. V. 7. P. 715–734.
8. *Francini A.*, *Neri. F.* A comparison of methodologies for the stationary analysis of data gathered in the simulation of telecommunication networks // Proceedings of MASCOTS ’96 – 4th International Workshop on Modeling, Analysis and Simulation of Computer and Telecommunication Systems. 1996. P. 116–122.
9. *Chen E.*, *Kelton D.* A procedure for generating batch-means confidence intervals for simulation: Checking independence and normality // Simulation. 1996. V. 83. P. 683–694.
10. *Sztrik J.*, *Tóth Á.*, *Pintér Á.*, *Bács Z.* Simulation of finite-source retrial queues with two-way communications to the orbit // Dudin A., Nazarov A., Moiseev A. (eds) Information Technologies and Mathematical Modelling. Queueing Theory and Applications. ITMM 2019. Communications in Computer and Information Science 2019. V. 1109. P. 270–284.