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Сборник содержит избранные материалы XXI Международной конференции имени А.Ф. Терпугова по следующим направлениям: теория массового обслуживания и ее приложения, интеллектуальный анализ данных и визуализация, информационные технологии и программная инженерия, математическое и компьютерное моделирование технологических процессов. Также в сборник вошли материалы международного симпозиума "Современные стохастические модели и проблемы актуарной математики"(МАМОНТ-2022).

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This volume presents selected papers from the XXI International Conference named after A.F. Terpugov. The papers are devoted to new results in the following areas: queuing theory and its applications, data mining and visualization, information technology and software engineering, mathematical and computer modeling of technological processes. The collection also presents the proceedings of symposium "Modern Stochastic Models and Problem of Actuarial Mathematics"(MAMMOTH-2022).

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SIMULATION OF RETRIAL QUEUEING SYSTEM WITH TWO-WAY COMMUNICATION IN DIFFERENT SCENARIOS

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The aim of this paper is to examine a finite-source retrial queueing system with two-way communication. The customers arrive from a finite-source (primary customers) according to an exponential distribution. The service of these customers starts if the service unit is idle, otherwise, they are forwarded to the orbit, and after a random time, they try to reach the server. The main feature of this system is that when the server becomes idle, an outgoing call (secondary customer) is performed to the orbit and to the source with different parameters. The service time of the primary and secondary customers is exponentially distributed with different rates. The novelty of this investigation is to carry out a sensitivity analysis using different distributions of retrial time of the customers on the main performance measures in two different cases. The different comparisons and the results are presented graphically.
Keywords: *finite-source queuing system, retrial queues, two-way communication, sensitivity analysis, simulation.*

Introduction

The two-way communication scheme is quite a popular topic because those systems can be modeled with the help of retrial queueing systems in many areas of life. One outstanding example is the operation of call centres where agents perform other particular activities during an idle period such as selling, advertising, and promoting products apart from handling the calls of the customers. One of the most important measures is utilization, and how to optimize the efficiency of the service units or agents which is always a key issue, see for example [1], [2], [10]. The characteristic of two-way communication relies on performing calls inside and outside of the system when the server is idle. In our model, it can perform outgoing calls to the source or to the orbit. In the past, researchers investigated infinite source retrial queueing systems with two-way communication, and here are some examples: [4], [9]. Dragieva and Phung-Duc [5] have investigated the scenario when a secondary outgoing call returns to the source after the service. This paper is the natural continuation of [7] where a more realistic scenario was considered. Instead of sending back the secondary outgoing customers to the source, they will be sent back to the orbit where the call has the opportunity to retry his request for servicing the original incoming call. The novelty of this work is to accomplish a sensitivity analysis to check the effect of various distributions of retrial time on several performance measures. The results are generated by our stochastic simulation program using the basics of SimPack ([6]). This is a collection of C/C++ libraries and executable programs for computer simulation to support discrete event simulation, continuous simulation, and combined (multi-model) simulation. It gives the freedom to model any type of queueing system and any type of simulation model calculating any performance measure using arbitrary random number generators for the desired random variable. The table of input parameters and graphical illustrations of the results of the comparison of the operation modes and different distributions are presented.

1. System model

In this section, the considered finite-source retrial queueing model with one server is introduced. Altogether N requests are located in the source, and each of them is capable of generating a primary incoming call toward the server, and the inter-request times are exponentially distributed random variables with parameter λ_1. In the case of an idle server, the service of an incoming customer begins instantaneously that follows an exponential distribution with parameter μ_1. After the successful service, the customers go back to the source. When the incoming customer finds the service unit busy, those customers will not be lost and they are transmitted to the orbit. These will be the secondary incoming jobs from the orbit that may retry to reach the service unit after a random waiting time. The distribution of this period follows gamma, hyper-exponential, Pareto, and lognormal distribution with different parameters but with the same mean value. However, the idle server can make outgoing calls from the source and the orbit as well. We differentiate two types of outgoing calls:

— the service unit may call a job from the source to be served (primary outgoing call) after an exponentially distributed period λ_2,
— the service unit may perform a call from the orbit (secondary outgoing call) after an exponentially distributed period ν_2.

The service time of the outgoing customers is exponentially distributed with parameter μ_2. Two scenarios are distinguished when an outgoing call comes from the orbit:

- Case 1: The call has an unserved incoming request so that call is sent back to the orbit after the outgoing service is finished to have its incoming call be served,
- Case 2: Here, the call has also got an unserved incoming request but after the outgoing service is done the service unit serves the incoming request right away. This will result in a two-phase service, first the outgoing call then the incoming one is executed. The call returns to the source after both service phases are finished.

It is assumed that the arrivals of primary incoming calls, retrial intervals of secondary incoming calls, service times of incoming and outgoing calls, and the time to make outgoing calls are mutually independent.

2. Simulation results

SimPack is used to obtain the results as a basic block of our program and it was extended with the desired features. We used a statistical package that can estimate the desired measures. It utilizes the batch means method which is a quite popular method. Briefly, the running period is divided into batches (altogether T) and in every batch, $s = R - M/T$ are executed. M denotes the warm-up period observations at the beginning of the simulation which are rejected, and R is the length of the simulation. After the initial phase, the sample average of the whole run is calculated. To have appraisable outcome batches should be long enough and the sample averages of the batches should be independent. More detailed information about the used process you can find in these papers: [3], [8].

The confidence level of 99.9% is employed throughout the simulations, and 0.00001 is the amount of the relative half-width of the confidence interval to pause the actual simulation sequence. The size of a batch in the initial transient period can not be too small, therefore, its value is set to 1000.

In Table 1 the used values of input parameters are presented.

Table 1

Numerical values of model parameters

N	$\boldsymbol{\mu_1}$	$\boldsymbol{\mu_2}$	$\boldsymbol{\lambda_2}$	$\boldsymbol{\nu_2}$
10	1	1	0.2	0.2

The next table (Table 2) contains the parameters of the retrial time of the customers, to achieve a valid comparison parameters are chosen according to have the same mean and variance value. The simulation program was tested by many parameter values, and in this paper, the most interesting ones will be revealed. As seen in the table the squared coefficient of variation is more than one in this scenario to check the influence of peculiar random variables. In the extended version, we plan to show results under a different parameter setting when the squared coefficient of variation is less than one.

Table 2

Parameters of the retrial time of the customers

Distribution	**Gamma**	**Hyper-exponential**	**Pareto**	**Lognormal**
Parameters	$\alpha = 0.02$ $\beta = 0.2$	$p = 0.489$ $\lambda_1 = 9.798$ $\lambda_2 = 10.202$	$\alpha = 2.01$ $k = 0.05$	$m = -4.258$ $\sigma = 1.978$
Mean	0.1			
Variance	0.49			
Squared coefficient of variation	49			

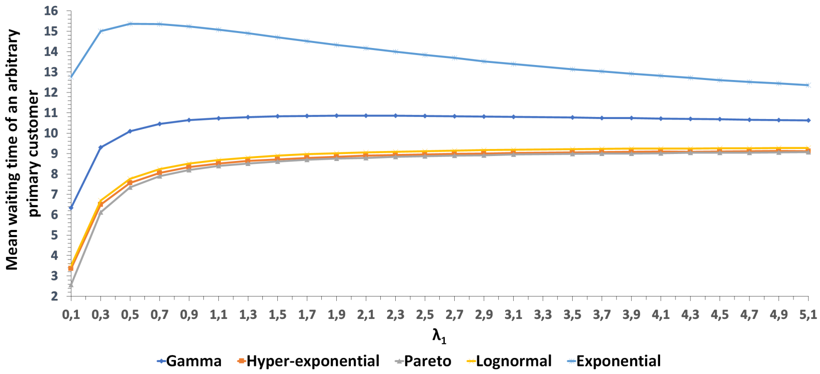

Figure 1. Mean waiting time of an arbitrary primary customer vs. arrival intensity

On Figs. 1 and 2 the mean waiting time of the calls is represented in the function of the incoming generation rate for Case 2 and comparing the different cases. Fig. 1 demonstrates five cases, the four different distributions and the exponential case. In the case of the exponential distribution, the maximum feature can be observed which is a general characteristic of the retrial queues under a suitable parameter setting. Greater mean waiting time appears in the cases of gamma and exponential distribution among the applied ones.

On Figure 2 the comparison of the scenarios is shown using gamma distributed retrial time. There is a “No outgoing” label that means, that there are only incoming calls in the system representing a common finite source retrial system. This figure reflects and ensures the expected behaviour of Case 1 and 2, finding lower mean waiting time in Case 2, but the lowest values are experienced when there are no outgoing calls at all. However, the utilization of the service unit will be higher when outgoing calls are produced as can be seen in the extended version of the paper.

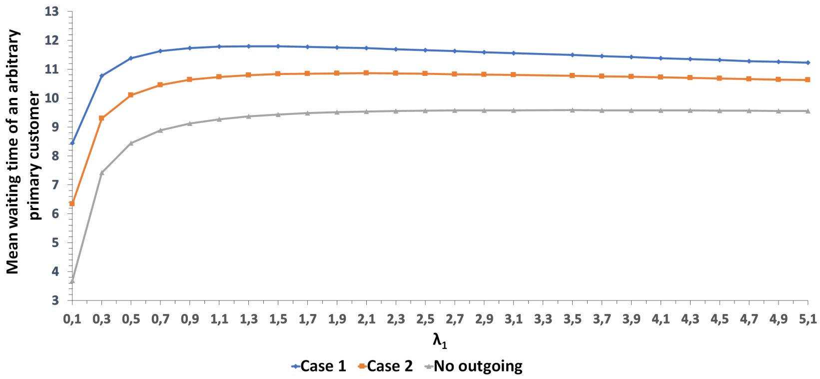

Figure 2. Comparison of the mean waiting times of the different scenarios

3. Conclusion

A finite-source retrial queueing system is introduced with a two-way communication scheme applying different distributions of retrial times. We investigated several scenarios where different parameters are used to carry out a sensitivity analysis to figure out focusing on the mean waiting time of the customers and the utilization of the service unit. The results are gathered by using our simulation program, and several graphical figures demonstrate the effect of using various distributions of retrial time on the operation of the system. In our figures, slight differences are observed among the values of several performance measures when the squared coefficient of variation is greater than one showing how pivotal applying a distribution can be. The curves also reveal the impact of outgoing calls and in Case 2 we obtain better values in the most important performance measures (waiting time, utilization) than in Case 1. In the future, we plan to continue our research work, examining other types of finite-source retrial queuing systems with two-way communication or adding another service unit for backup purposes.

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