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SIMULATING RETRIAL QUEUES WITH A FINITE SOURCE, TWO-WAY COMMUNICATION TO THE ORBIT, THE INCLUSION OF A BACKUP SERVER AND IMPATIENT CUSTOMERS

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This paper explores a retrial queuing system with two-way communication and an unreliable server that may encounter random breakdowns. The system is of the finite-source M/M/1//N type, where the idle server can initiate calls to customers in the orbit, termed as secondary customers. Both primary and secondary customer service times are characterized by independent exponential distributions, with rates denoted as μ_1 and μ_2, respectively. The novelty of this study lies in its investigation of various failure time distributions and their impact on critical performance metrics, such as the mean response time of a random customer, while utilizing a backup server with impatient customers. The backup server can be likened to a primary server operating at a reduced rate during maintenance intervals. To ensure a valid comparison, a fitting process equalizes the mean and variance across all distributions. The outcomes are visually depicted through the utilization of our self-made simulation program. **Keywords:** *finite-source queuing system, retrial queues, two-way communication, sensitivity analysis, simulation, impatience.*

Introduction

In these days, the analysis of telecommunication systems and the creation of optimal designs for these schemes have become formidable endeavors due to the immense traffic and escalating number of users. Information exchange pervades every facet of contemporary life, underscoring the need to develop mathematical and simulation models for telecommunication systems or adapt existing ones to keep pace with these dynamic changes. Retrial queues stand out as potent and fitting tools for modeling real-world challenges that arise in telecommunications, networks, mobile networks, call centers, and similar systems. A plethora of literature, exemplified by works like [2] and [4], delves into the examination of various retrial queuing systems characterized by recurring calls.

We are currently exploring a retrial queuing system endowed with two-way communication capabilities, a research area that has gained substantial prominence owing to its striking resemblance to certain real-world systems. This correspondence is particularly pronounced in the context of call centers, where service units often performs multitasking, engaging in activities such as sales, promotions, and product advertising alongside handling incoming calls. In our investigation, the primary server, following a random idle interval, calls customers in from the orbit, called secondary customers. The system's utilization of the service unit is under scrutiny and has undergone extensive examination in prior works, exemplified by studies like [3] and [6].

In various research scenarios, some assume that service units remain continuously available, but real-world events like failures or unexpected incidents can occur during their operation, resulting in the rejection of incoming customers. Devices used across different industries are prone to breakdowns, and relying on their uninterrupted operation is often overly optimistic and unrealistic.

In the service sector, it's not uncommon for service providers to encounter disruptions for various reasons, including difficulties in accessing their databases to address customer requests. When such disruptions transpire, service providers frequently employ alternative measures, such as resorting to backup systems or gathering additional information from customers to meet their needs.

The primary aim of this investigation is to assess how the system's unreliable operation affects performance measures, such as the mean response time of a customer or service unit utilization, by comparing various failure time distributions while the customers may depart after a random long enough waiting.

To acquire the desired performance metrics, we developed a simulation model utilizing SimPack [5], which encompasses a collection of C/C++ libraries and executable programs tailored for computer simulation. Simulation serves as an excellent alternative for approximating performance metrics when deriving precise formulas proves problematic or nearly impossible. This paper introduces a sensitivity analysis of different failure time distributions' impact on key performance measures. We elucidate these findings by means of graphical representations that highlight intriguing facets of sensitivity-related issues.

1. System model

The system is a retrial queuing system characterized by an unreliable server and a finite source of customers. Within the source, there exist N customers, each generating primary requests at an exponential rate denoted by λ. Consequently, the inter-arrival times adhere to an exponential distribution parameterized by λ. Notably, our model does not contain waiting queues; thus, incoming customers can only occupy the server when it is available and idle. The service time for primary customers follows an exponential distribution with a parameter of μ_1. Following the successful completion of a service, the customer returns to the source. However, if an incoming customer (whether from the source or orbit) encounters a server in a busy or failed state, its request is redirected to the orbit. While within the orbit, a customer may attempt to fulfill its service requirement after an exponentially distributed random time with a parameter of σ.

The system assumes the presence of an unreliable server prone to failures, which can occur according to different distributions—such as gamma, hypo-exponential, hyper-exponential, Pareto, and lognormal. Each distribution comes with distinct parameters while sharing the same mean value. The repair process initiates immediately upon the server's failure, with the repair time following an exponential distribution characterized by parameter γ_2. If the server is busy and subsequently fails, the customer is promptly transferred to the orbit. Regardless of the service unit's availability, all customers within the source can generate requests. However, these requests are directed to the backup server, which operates at a reduced rate—an exponentially distributed random variable with parameter μ_3—when the primary server is unavailable. Importantly, the backup server is assumed to be reliable and functions solely during periods of primary server unavailability. In cases where the backup server is busy, incoming requests are placed into the orbit. Yet, during idle periods, the main server can initiate outgoing calls to customers within the orbit after a random time interval, characterized by an exponential distribution with a rate of τ. The service time for these secondary customers follows an exponential distribution with parameters μ_2. Customers in the orbit, after waiting an exponentially distributed time with parameter τ, may choose to leave the system without getting their service.

Throughout the model's creation, the fundamental assumption is maintained that all random variables remain entirely independent of each other.

2. Simulation results

We employed a statistical module class that incorporates a statistical analysis tool, enabling us to quantitatively estimate both the mean and variance values of observed variables via the batch mean method. This method involves aggregating n consecutive observations from a steady-state simulation to generate a sequence of independent samples. The batch mean method is a widely utilized technique for establishing confidence intervals

Table 1

Used numerical values of model parameters

N	λ	γ_2	σ	μ_1	μ_2	ν	μ_3	τ
100	0.01	1	0.01	1	1.2	0.02	0.1	0.001

Table 2

Parameters of failure time

Distribution	**Gamma**	**Hyper-exponential**	**Pareto**	**Lognormal**
Parameters	$\alpha = 0.6$ $\beta = 0.5$	$p = 0.25$ $\lambda_1 = 0.41667$ $\lambda_2 = 1.25$	$\alpha = 2.2649$ $k = 0.67018$	$m = -0.3081$ $\sigma = 0.99037$
Mean	1.2			
Variance	2.4			
Squared coefficient of variation	1.6666666667			

concerning the steady-state mean of a process. It is important to note that, in order to ensure that the sample averages exhibit approximate independence, the use of sizable batches is imperative. Further details on the batch mean method can be found in [1].

The steady-state distribution, corresponding to different failure time distributions, is visually represented in Figure 2. In this graph, the X-axis is labeled as i, which denotes the number of customers present in the system, while the Y-axis is labeled as $P(i)$, indicating the probability of precisely i customers being in the system. A closer examination of the curves reveals that all of them closely resemble the normal distribution. Notably, the Pareto distribution seems to exhibit a lower number of customers in the system. Nevertheless, when comparing the different distributions examined in our study, no significant disparities emerge.

Figure 2 provides an illustration of the correlation between the mean response time of customers and the arrival intensity. On the contrary to the patterns observed in Figure 2, the highest mean response time is associated with the Pareto distribution. However, the distinctions among the other distribution types become more pronounced. Remarkably, the gamma distribution stands out by yielding the lowest mean response time. A noteworthy phenomenon is that, as the arrival intensity increases, the mean response time initially experiences an uptrend, but subsequently, it starts to decrease after reaching a specific threshold. This behavior is a distinctive characteristic of retrial queuing systems with a finite source, and it tends to manifest when appropriate parameter configurations are applied. In the expanded version, we intend to display outcomes with an alternative parameter configuration when the squared coefficient of variation is below one.

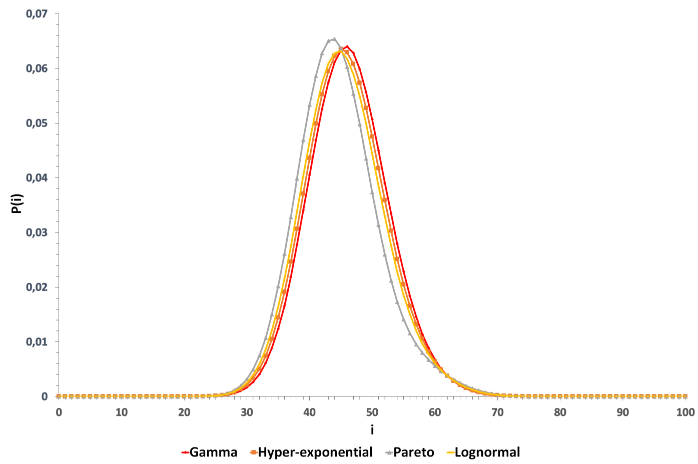

Figure 1. Comparison of steady-state distributions

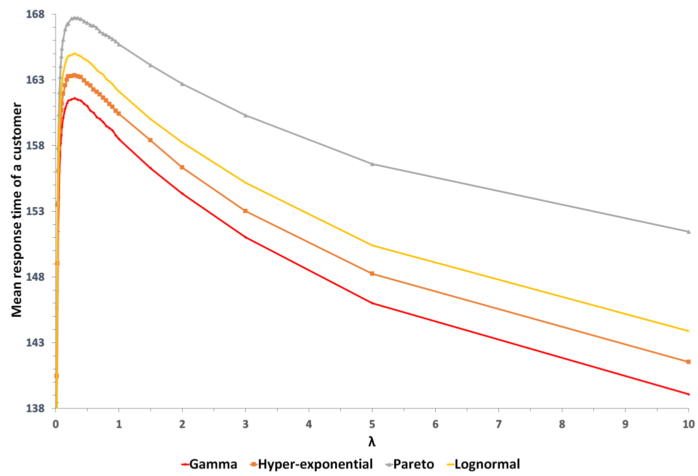

Figure 2. Mean response time vs. arrival intensity

3. Conclusion

We introduced a retrial queuing system characterized by a finite source and two-way communication with impatient customers. Within this system, a primary server exhibits unreliability, and during periods of malfunction, a secondary service unit takes over. Furthermore, we conducted a sensitivity analysis utilizing a range of random number generators to investigate how different distributions of failure time impact performance metrics, such as the mean response time of any given customer. It's worth noting that when the squared coefficient of variation exceeds one, we observed variations in the mean response time among the values.

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