

# HETEROGENEOUS FINITE-SOURCE RETRIAL QUEUES WITH SERVER SUBJECT TO BREAKDOWNS AND REPAIRS

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## Abstract

This paper deals with a retrial queueing system with a finite number of heterogeneous sources of calls and a single non-reliable server, which means that the server is subject to random breakdowns depending on whether it is busy or idle. The failure of the server may block or unblock the systems's operations and the service of the interrupted request may be resumed or the call can be transmitted to the orbit. All random variables involved in the model constructions are supposed to be exponentially distributed and independent of each other. However, it should be underlined, that because the sources are heterogeneous they are characterized by different arrival, repeated and service rates of their own.

The novelty of investigations is the variability of the non-reliability of the server. The present paper generalizes the results of the same authors obtained for heterogeneous systems with reliable server. The heterogeneity of calls and the different types of failures of the server makes the system rather complicated. The MOSEL ( Modeling, Specification and Evaluation Language ) tool, developed at the University of Erlangen, Germany, was used to formulate and solve the problem and the main performance and reliability measures were derived and graphically displayed. Several numerical calculations were performed to show the effect of the non-reliability of the server on the mean response times of the calls.

This type of queueing systems can be applied, for instance, to model performance of magnetic disk memory systems, local area networks with CSMA/CD protocols.

**Keywords:** heterogeneous population, retrial queueing systems, finite number of sources, non-reliable server, performance tool, performance and reliability measures

**AMS 2000 subject classification:** 60K25, 68M20, 90B22, 90B25

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# 1 Introduction

Queueing system with repeated attempts have wide practical use in designing telephone switching systems, telecommunication networks, computer networks and computer systems, etc. For a systematic account of the fundamental methods and results, furthermore an accessible classified bibliography on this topic the interested reader is referred to, for example [5], [6], [11], and references therein.

Since in practice some components of the systems are subject to random breakdowns ( see for example [18], ) it is of basic importance to study reliability of retrial queues with server breakdowns and repairs because of limited ability of repairs and heavy influence of the breakdowns on the performance measures of the system. However, so far the repairable retrial queues are analyzed only by queueing theory. For related literature the reader is referred to the works [7], [19], [27] where infinite-source non-reliable retrial queues were treated.

In many practical situations it is important to take into account the fact that the rate of generation of new primary calls decreases as the number of customers in the system increases. This can be done with the help of finite-source, or quasi-random input models. Queueing systems without retrials, that is systems with classical waiting lines and finite population have been review in detail by Takagi [25]. A complete survey on related results can be found in Artalejo [5] for systems of type  $M/G/1//K$  and  $M/M/c//K$  in Kendall's notation. In addition, in the papers Falin and Artalejo [12], Falin [13] not only the outside observer's distributions of the systems in steady state, but also the stationary performance characteristics are considered on more detail. In particular, all main measures were expressed in terms of the server utilization. Arriving customer's distribution of the system state, busy period and waiting time processes ( which is especially complex for retrial queues due to the overtaking ) were investigated, too. Further recent results with finite-source of primary requests can be found in [11], [20], [23].

Retrial queues with quasi-random input are recent interest in modelling magnetic disk memory systems [22], cellular mobile networks [26], computer networks [14], and local-area networks with nonpersistent CSMA/CD protocols [20], with star topology [15, 21], with random access protocols [16], and with multiple-access protocols [17].

In this paper single server finite-source queueing systems with the following assumptions are investigated. The primary calls are generated by  $K$ ,  $1 < K < \infty$  heterogeneous sources. The server can be in operational ( up ) or non-operational ( down ) states. If it is idle and up, it can serve the calls of the sources. Each of the sources can be in three states: generating a primary call ( busy ), sending repeated calls and under service. The  $i$ -th source can generate a primary call during interval  $(t, t + dt)$  with probability  $\lambda_i dt + o(t)$ . If the server is free at the time of arrival of a call then the call starts to be served immediately, the source moves into the under service state and the server moves into busy state. The service is finished during the interval  $(t, t + dt)$  with probability  $\mu_i dt + o(t)$  if the server is available ( up ). If the server is busy, then the source starts generating of a Poisson flow of repeated calls with rate  $\nu_i$  until it finds the server free. After service the source can generate a new primary call, and the server becomes idle so it can serve a new call. The server can fail during the interval  $(t, t + dt)$  with probability  $\delta dt + o(t)$  if it is idle, and with probability  $\gamma dt + o(t)$  if it is busy. If  $\delta = 0, \gamma > 0$

or  $\delta = \gamma > 0$  *active or independent breakdowns* can be discussed, respectively. If the server fails in busy state, it either *continues servicing* the interrupted call after it has been repaired or the interrupted request *returns to the orbit*. The repair time is exponentially distributed with a finite mean  $1/\tau$ . If the server is failed two different cases can be treated. Namely, *blocked sources* case when all the operations are stopped, that is no new primary and repeated calls are generated. In the *non-blocked ( intelligent ) sources* case only service is interrupted but all the other operations are continued ( new and repeated calls can be generated ). All the times involved in the model are assumed to be mutually independent of each other.

As it can be seen this systems is rather complicated since it involves two types of failures, continued or repeated service and blocked or non-blocked operations during breakdowns.

Our objective is to give the main usual stationary performance and reliability measures of the system and to display the effect of different parameters on them. To achieve this goal a tool called MOSEL ( Modeling, Specification and Evaluation Language ) developed at the University of Erlangen, Germany, see [8], is used to formulate and solve the problem. We show how this system can be modelled, and how easily performance measures can graphically be represented using IGL ( Intermediate Graphical Language ).

This paper generalizes the results of [11] where homogeneous systems with reliable multiple servers were dealt with. Similarly, it is another extension of investigations for heterogeneous finite-source queueing systems without retrials but with server's breakdowns which were treated in [24]. Finally, it can be considered as the natural continuation of [4] in which reliable heterogeneous finite-source retrial systems were analyzed.

The paper is organized as follows. In Section 2 the full description of the model by the help of the corresponding multi-component Markov chain is given. Then the main performance and reliability measures of the system are derived that can be obtained using MOSEL tool. In Section 3 several numerical examples are presented and some comments are made. Finally, the paper ends with a Conclusion.

## 2 The $\vec{M}/\vec{M}/1//K$ retrial queueing model

### 2.1 The underlying Markov chain

Because of the exponentiality of the involved random variables the following process will be a Markov chain. The state of the system at time  $t$  can be described by the process

$$X(t) = (Y(t); (\alpha_{C(t)}; \beta_1, \dots, \beta_{N(t)}), t \geq 0)$$

where  $Y(t) = 0$  if the server is up,  $Y(t) = 1$  if the server is down,  $C(t) = 0$  if the server is idle,  $C(t) = 1$  if the server is busy, and  $\alpha_{C(t)}$  is the index of the request under service at time  $t$  if the server is busy. Let  $N(t)$  be the number of sources of repeated calls at time  $t$ , and because of the heterogeneity of the sources we need to identify their indices that are denoted by  $\beta_j$ ,  $j = 1, \dots, N(t)$  if there is a customer in the orbit, otherwise the third component is 0.

Since its state space is finite the process  $(X(t), t \geq 0)$  is ergodic for all values of the rate of generation of new primary calls, and from now on we assume that the system is in the steady state.

We define the stationary probabilities

$$P(q; 0; 0) = \lim_{t \rightarrow \infty} P(Y(t) = q; C(t) = 0; N(t) = 0), \quad q = 0, 1$$

$$P(q; j; 0) = \lim_{t \rightarrow \infty} P(Y(t) = q; \alpha_1 = j; N(t) = 0), \\ q = 0, 1, \quad j = 1, \dots, K,$$

$$P(q; 0; i_1, \dots, i_k) = \lim_{t \rightarrow \infty} P(Y(t) = q; C(t) = 0; \beta_1 = i_1, \dots, \beta_k = i_k), \\ q = 0, 1, \quad k = 1, \dots, K^*,$$

$$P(q; j; i_1, \dots, i_k) = \lim_{t \rightarrow \infty} P(Y(t) = q; \alpha_1 = j; \beta_1 = i_1, \dots, \beta_k = i_k), \\ q = 0, 1, \quad k = 1, \dots, K - 1.$$

where

$$K^* = \begin{cases} K - 1 & \text{for blocked case,} \\ K & \text{for non-blocked case.} \end{cases}$$

The traditional way is to derive the related Kolmogorov equations for these probabilities and using the norming condition somehow we have to solve the set of equations. Usually it is not so easy, but in our case these two steps are performed by the help of the tool demonstrated in the next subsection.

Once we have obtained these limiting probabilities the **main system performance measures** can be derived in the following way.

1. *The server utilization with respect to source  $j$*

$$U_j = P(\text{ the server is up and busy with source } j)$$

that is, we have to summarize all the probabilities where the first component is 0 and the second component is  $j$ . Formally,

$$U_j = \sum_{k=0}^{K-1} \sum_{i_1, \dots, i_k \neq j} P(0; j; i_1, \dots, i_k).$$

Hence the *server utilization*

$$U_S = E[Y(t) = 0; C(t)] = \sum_{j=1}^K U_j.$$

2. *Utilization of source  $i$*

$$U^{(i)} = P(\text{ source } i \text{ generates a new primary call}).$$

It should be mentioned that in the blocked case the server have to be up, but in the non-blocked case the request generation is independent of the server's state.

### 3. Utilization of the repairman

$$U_R = E[Y(t)] = \sum_{j=0}^K \sum_{k=0}^{K^*} \sum_{i_1, \dots, i_k \neq j} P(1; j; i_1, \dots, i_k).$$

### 4. Availability of the server

$$A_S = 1 - U_R.$$

Let us denote by  $P_O^{(i)}$  the steady state probability that request  $i$  is staying in the orbit. It is easy to see that

$$P_O^{(i)} = \sum_{q=0}^1 \sum_{j=0, j \neq i}^K \sum_{k=1}^{K^*} \sum_{i \in (i_1, \dots, i_k)} P(q; j; i_1, \dots, i_k).$$

Similarly, it can easily be seen, that the steady state probability  $P_S^{(i)}$  that request  $i$  is at the server is given by

$$P_S^{(i)} = \sum_{q=0}^1 \sum_{k=0}^{K^*} \sum_{i \neq i_1, \dots, i_k} P(q; i; i_1, \dots, i_k).$$

Hence, the probability  $P^{(i)}$  that request  $i$  is at the service facility can be obtained by

$$P^{(i)} = P_S^{(i)} + P_O^{(i)}.$$

### 5. Mean response time of source $i$

The derivation of the following formulae are based on [1, 25]. Let us denote by  $E[T_i]$  the mean response time of customer  $i$ , and by  $\gamma_i$  the *throughput* of request  $i$ , that is, the mean number of times that request  $i$  is served per unit time. These are related by

$$\gamma_i = \frac{1}{E[T_i] + E[S_i]} = \lambda_i U^{(i)} = \mu_i U_i, \quad i = 1, \dots, K, \quad (1)$$

where  $E[S_i]$  denotes the mean sojourn time of request  $i$  in the source. Since the server is subject to random breakdowns which may stop the operations of the sources, it is clear that  $E[S_i] = E[D_i] + 1/\lambda_i \geq 1/\lambda_i$ , where  $E[D_i]$  denotes the mean delay time due to the failure of the server.

Hence, with the aid of (1) for  $U^{(i)}$  we get

$$U^{(i)} = \frac{1/\lambda_i}{E[T_i] + E[S_i]} = \frac{\mu_i U_i}{\lambda_i} \leq 1 - P^{(i)}, \quad i = 1, \dots, K,$$

and for  $P^{(i)}$  we have

$$P^{(i)} = \frac{E[T_i]}{E[T_i] + E[S_i]} = \gamma_i E[T_i] = \lambda_i U^{(i)} E[T_i], \quad i = 1, \dots, K, \quad (2)$$

which represents **Little's theorem** for request  $i$  at the service facility.

By the help of (2) we can express the mean response time  $E[T_i]$  for request  $i$  as

$$E[T_i] = \frac{P^{(i)}}{\lambda_i U^{(i)}} = \frac{P^{(i)}}{\mu_i U_i}, \quad i = 1, \dots, K.$$

6. Mean waiting time of source  $i$

The mean waiting time  $E[W_i]$  of request  $i$  is due to the time spent in the orbit ( irrespective of whether the server is up or down ), and the delay time because of the server's failure. It is easy to see that  $E[W_i]$  is given by

$$E[W_i] = E[T_i] - 1/\mu_i = \frac{P^{(i)} - U_i}{\mu_i U_i}, \quad i = 1, \dots, K.$$

7. Mean number of sources of repeated calls

$$N = E[N(t)] = \sum_{i=1}^K P_O^{(i)}.$$

8. Mean number of calls staying at the service facility

$$M = E[C(t) + N(t)] = \sum_{i=1}^K P^{(i)} = \sum_{i=1}^K (P_S^{(i)} + P_O^{(i)}) = \sum_{i=1}^K P_S^{(i)} + \sum_{i=1}^K P_O^{(i)}.$$

9. Mean rate of generation of primary calls

$$\bar{\lambda} = \sum_{i=1}^K \gamma_i = \sum_{i=1}^K \lambda_i U^{(i)} = \sum_{i=1}^K \mu_i U_i.$$

10. Blocking probability of primary call  $i$

$$B_i = \begin{cases} \frac{\lambda_i \sum_{j=1, j \neq i}^K \sum_{k=0}^{K-1} \sum_{i_1, \dots, i_k} P(0; j; i_1, \dots, i_k)}{\bar{\lambda}} & \text{for blocked case,} \\ \frac{\lambda_i \sum_{j=1, j \neq i}^K \sum_{k=0}^{K-1} \sum_{i_1, \dots, i_k} (P(0; j; i_1, \dots, i_k) + P(1; j; i_1, \dots, i_k))}{\bar{\lambda}} & \text{for non-blocked case.} \end{cases}$$

Hence blocking probability of primary calls

$$B = \sum_{i=1}^K B_i$$

which is the fraction of primary calls which were blocked ( i.e. met the server busy ).

It is easy to see that in the case of non-blocked operations ( intelligent sources ) with non-reliable server,  $U^{(i)} = P^{(i)}, i = 1, \dots, K$ , and we get the same formulae derived in [4] that is, most performance measures can be expressed in the terms of the corresponding utilizations  $U_i$  as it was stated in [12].

## 2.2 The MOSEL implementation

We used the software tool MOSEL (Modeling, Specification and Evaluation Language) ( see [8] ) to formulate the model and to calculate and display the main performance measures. Because of page limitation only some interesting figures are displayed, the technical details of programming are omitted here, but can be found in [9].

## 3 Numerical examples

In this section we consider some sample numerical results to illustrate the influence of the non-reliable server on the mean response time  $E[T_i]$  and the mean number of request M staying at the service facility ( in the orbit and at the server ). In the cases when the server's failure rate is very small and the repair rate is large the non-reliable model should be very close to the reliable system. The results in the homogeneous case were validated by the Pascal program given in [11]. For the heterogeneous case the calculations were checked by the numerical results of [4]. In the case of homogeneous sources but with server's breakdowns the program was tested by the examples of [3].

### 3.1 Validation of results

In Table 1 some test results are collected when the server's failure rate is quite small and the request's retrial rates are quite large. Hopefully the corresponding performance measures should be very close to each other in the case of continued, restarted ( abbreviated by orbit ) service after repair and FIFO disciplines. As we can see, the results confirm our expectation.

	non-rel. retrial(cont.)	non-rel. retrial(orbit)	non-rel. FIFO
Number of sources:	3	3	3
Request's generation rate:	0.2, 0.3, 0.5	0.2, 0.3, 0.5	0.2, 0.3, 0.5
Service rate:	1, 1.2, 1.1	1, 1.2, 1.1	1, 1.2, 1.1
Retrial rate:	1e+20	1e+20	-
Server's failure rate:	0.002	0.002	0.002
Server's repair rate:	0.04	0.04	0.04
Utilization of the server:	0.578593008176	0.578593460071	0.578595143583
Mean response time			
Source 1:	1.61016598407	1.61027143737	1.6109393482
Source 2:	1.41365083148	1.41357129589	1.41287007613
Source 3:	1.35362123345	1.35362137877	1.35372999206

Table 1: Validations

### 3.2 Further examples

In this part some additional sample results are displayed showing the effect of different parameters on mean response times of calls and the mean number of requests staying

at the service facility. The input parameters are collected in Table 2. In each cases independent breakdowns were treated. In homogeneous cases the parameters are the arithmetic means of the corresponding values.

In Figures 1 - 6 the mean response time  $E[T_i]$  is displayed in continuous and non-continuous service after repair in the case of blocked, non-blocked ( intelligent sources ) operations, as the function of primary request generation, retrial and service rates, respectively.

In Figures 7 - 8 also  $E[T_i]$  is pictured in the case of blocked operations with continuous and non-continuous service after repair as the function of the server's failure rate in busy state, respectively.

Finally, in Figures 9 - 10 the mean number of request  $M$  staying at the service facility can be seen in reliable and non-reliable case with homogeneous, heterogeneous sources under blocked, non-blocked operations combined with continuous and non-continuous service after repair as the function of the retrial rate of repeated calls.

	NT	$\lambda$	$\mu$	$\nu$	$\delta$	$\gamma$	$\tau$
Fig. 1,2	5	x axis	4.1,4.3,4.5,4.7,4.9	0.35,0.4,0.45,0.6,0.7	0.05	0.05	0.1
Fig. 3,4	5	2.5,3,4,6.5,9	6,7,8,13,16	x axis	0.05	0.05	0.1
Fig. 5,6	5	0.04,0.06,0.1,0.14,0.16	x axis	0.2,0.25,0.3,0.55,0.7	0.05	0.05	0.1
Fig. 7,8	5	0.6,0.7,0.8,0.9,1	4.1,4.3,4.5,4.7,4.9	0.35,0.4,0.45,0.6,0.7	0.05	x axis	0.1
Fig. 9	5	0.1	0.5	x axis	0.05	0.05	0.1
Fig. 10	5	0.06,0.08,0.1,0.12,0.14	0.3,0.4,0.5,0.6,0.7	x axis	0.05	0.05	0.1

Table 2: Input parameters

### 3.3 Comments

- In Table 1 we can see that there is a slight difference between continuous, restarted service and the FIFO disciplines which can be expected.
- In Figures 1-2 we can see the mean response time  $E[T_i]$  for the non-reliable system with continuous, non-continuous service after repair, with blocked and non-blocked operations during service failure when the primary request generation rate increases. Figure 1 demonstrates a surprising phenomenon of retrial queues having a maximum of  $E[T_i]$  which was noticed for homogeneous sources treated in [12], too. The difference between continuous, non-continuous service, moreover blocked, non-blocked ( intelligent ) systems's operations is clearly shown. The non-continuous case always have longer response times. Similarly, the intelligent sources suffers from longer times, too. However, their curves are very interesting, since for some sources at the beginning they decrease then increase, finally decrease again. This shows that the systems is very complex and at different parameter setup we can see different effects.
- In Figures 3,4, and similarly in Figures 5, 6 the effect of retrial rate, service rate is demonstrated on  $E[T_i]$ , respectively. Again the The non-continuous case always have longer response times, and the intelligent sources suffers from longer times, too. However, the difference between the continuous and non-continuous case decreases as the corresponding rate increase. Furthermore, in each case  $E[T_i]$  decreases as we expected.

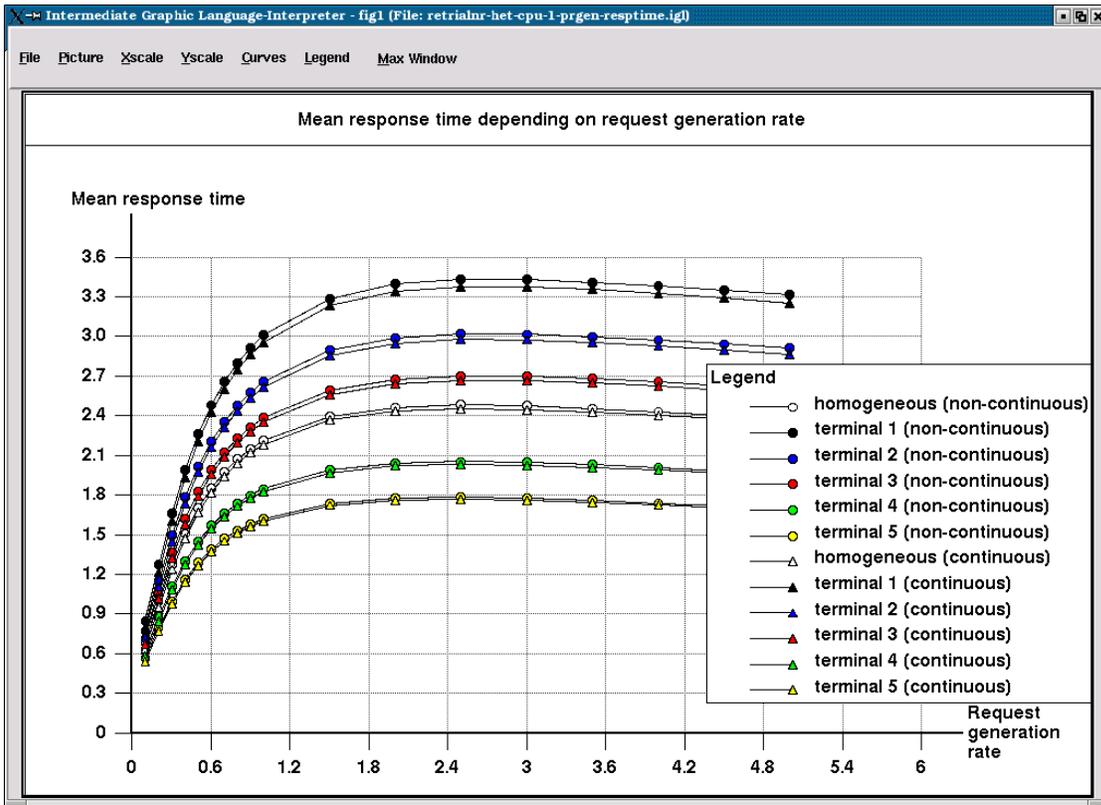


Figure 1:  $E[T]$  versus primary request generation rate

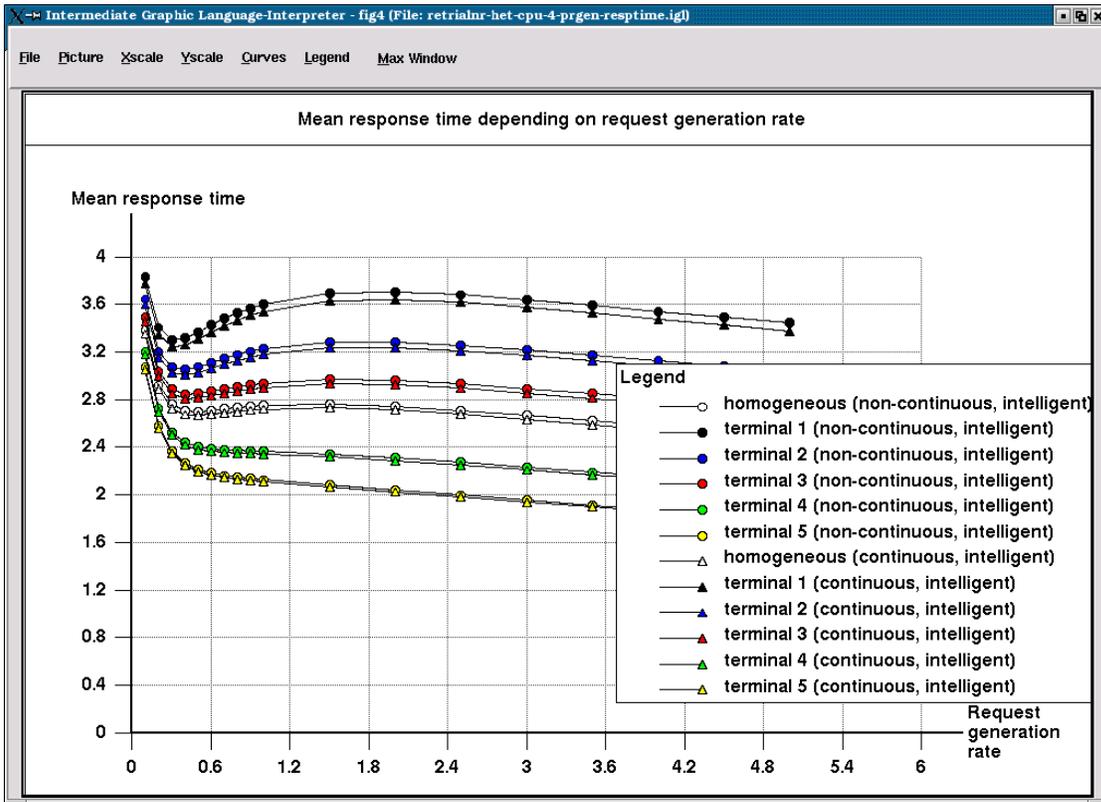


Figure 2:  $E[T]$  versus primary request generation rate

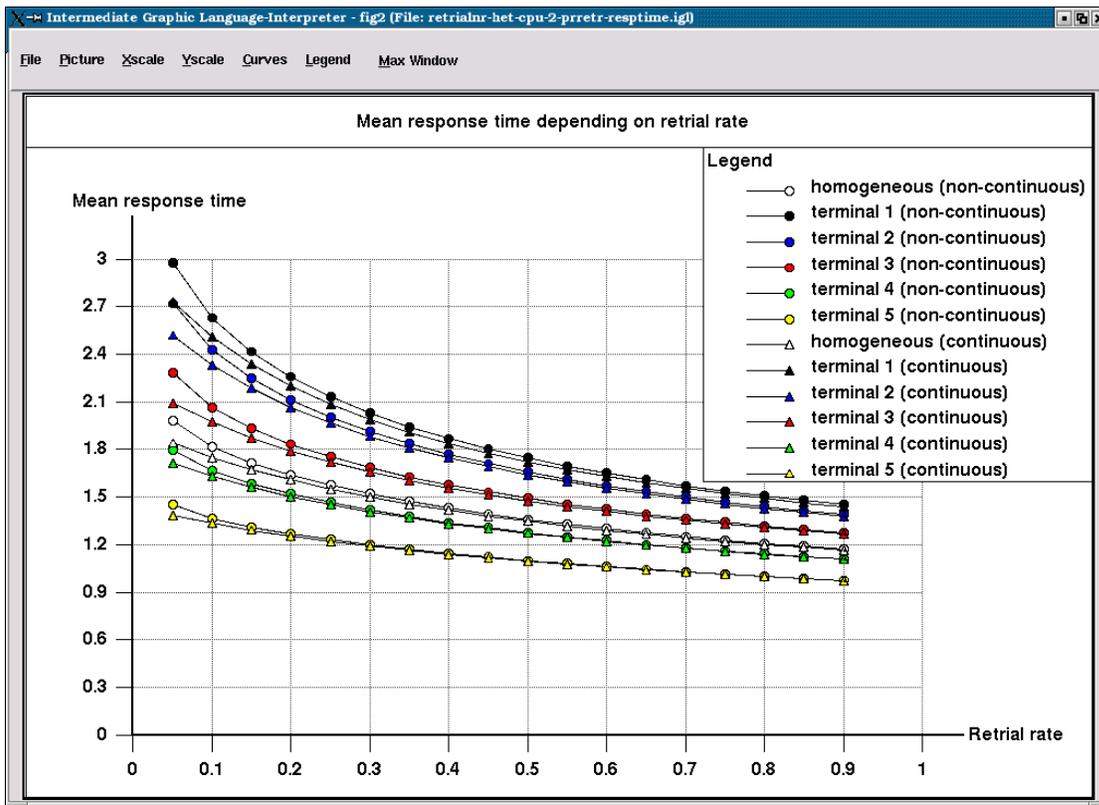


Figure 3:  $E[T]$  versus retrial rate

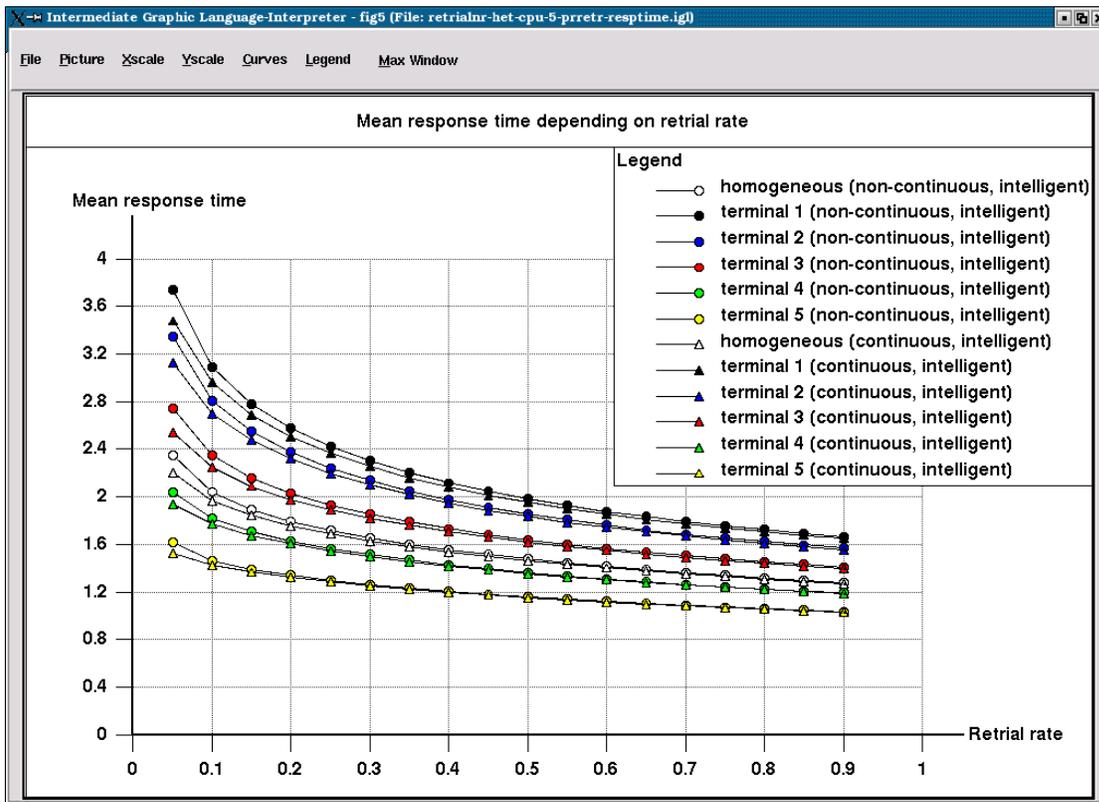


Figure 4:  $E[T]$  versus retrial rate

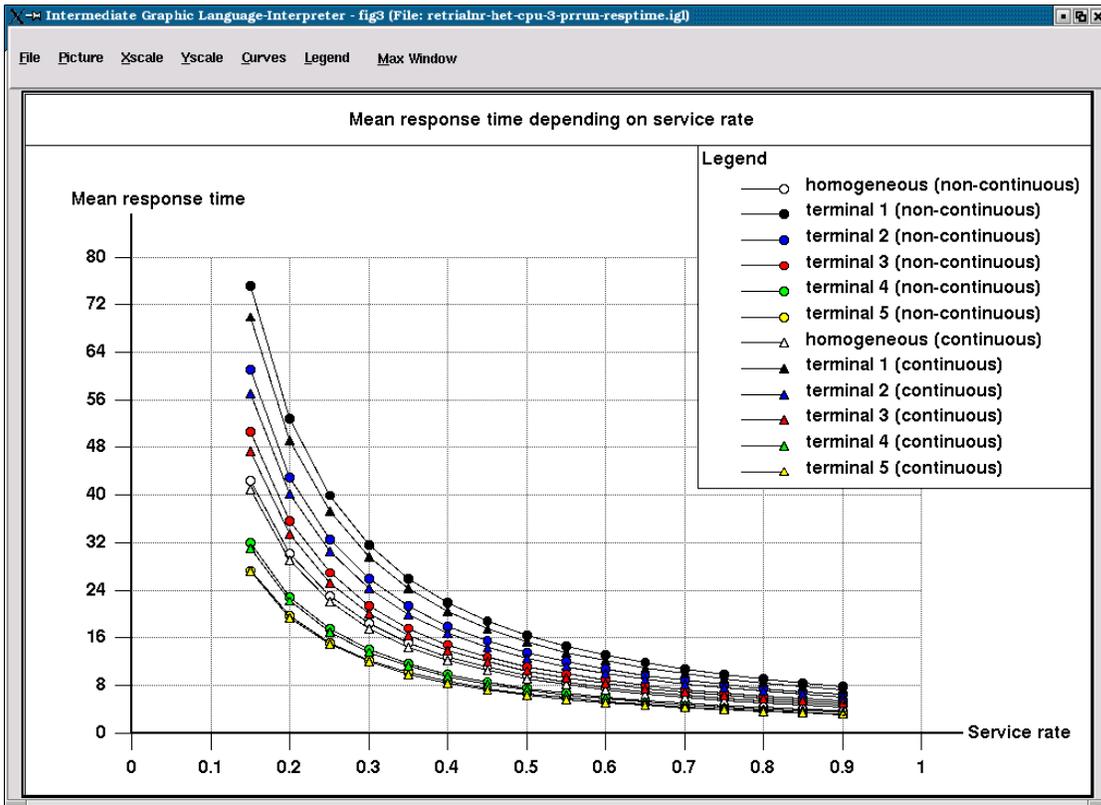


Figure 5:  $E[T]$  versus service rate

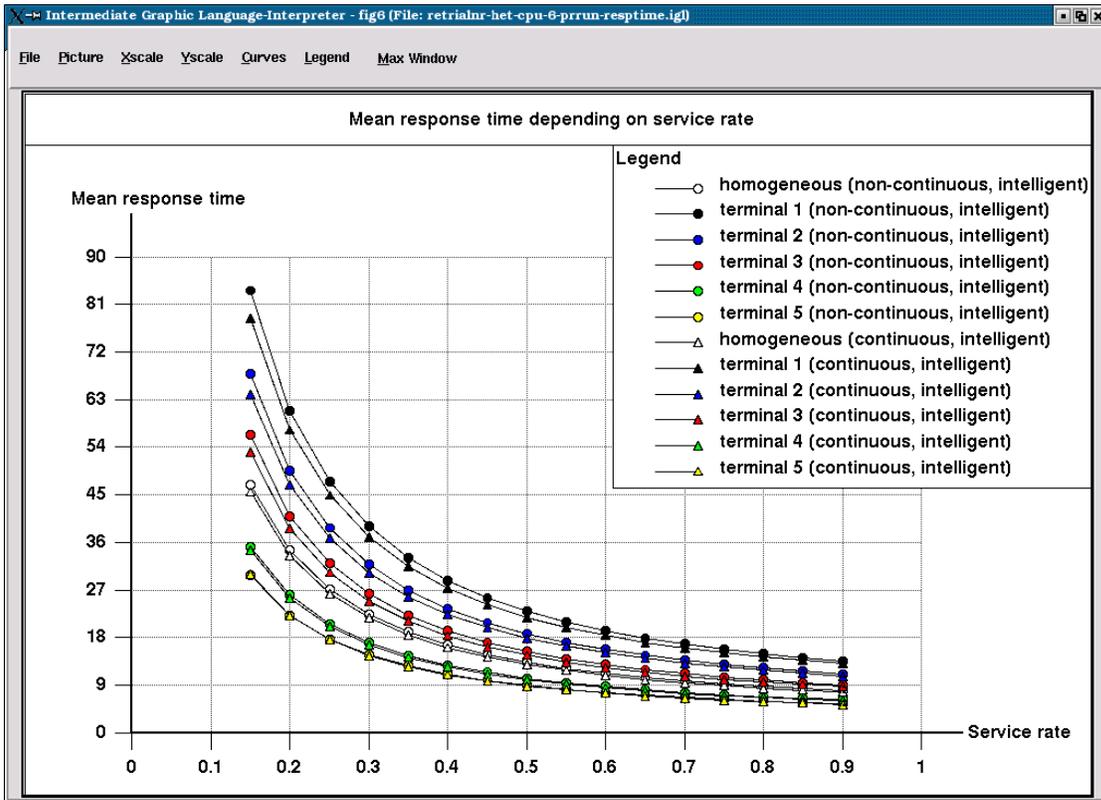


Figure 6:  $E[T]$  versus service rate

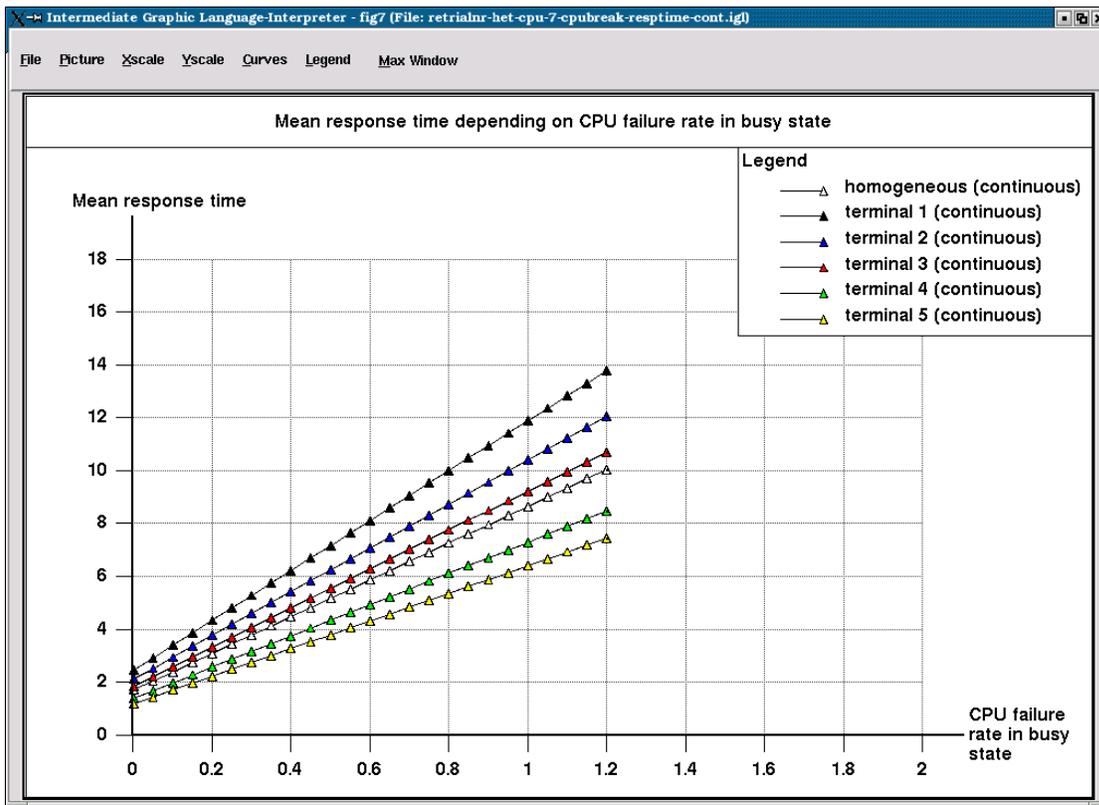


Figure 7:  $E[T]$  versus CPU failure rate in busy state

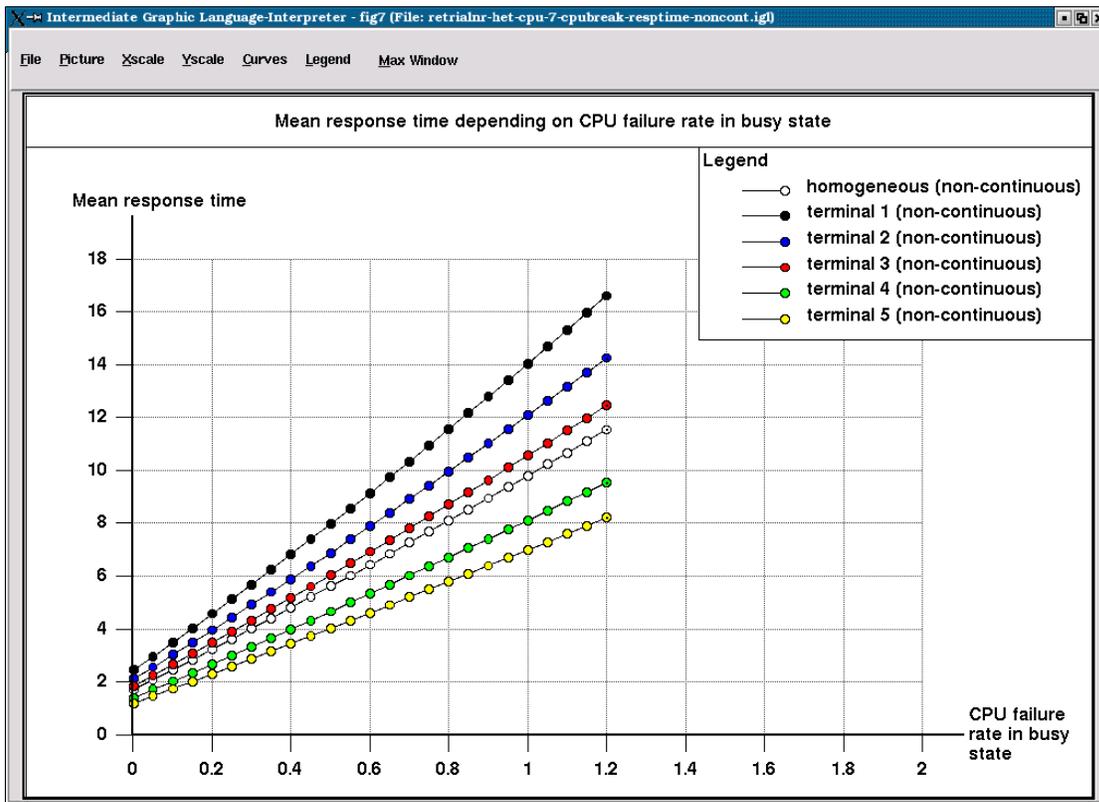


Figure 8:  $E[T]$  versus CPU failure rate in busy state

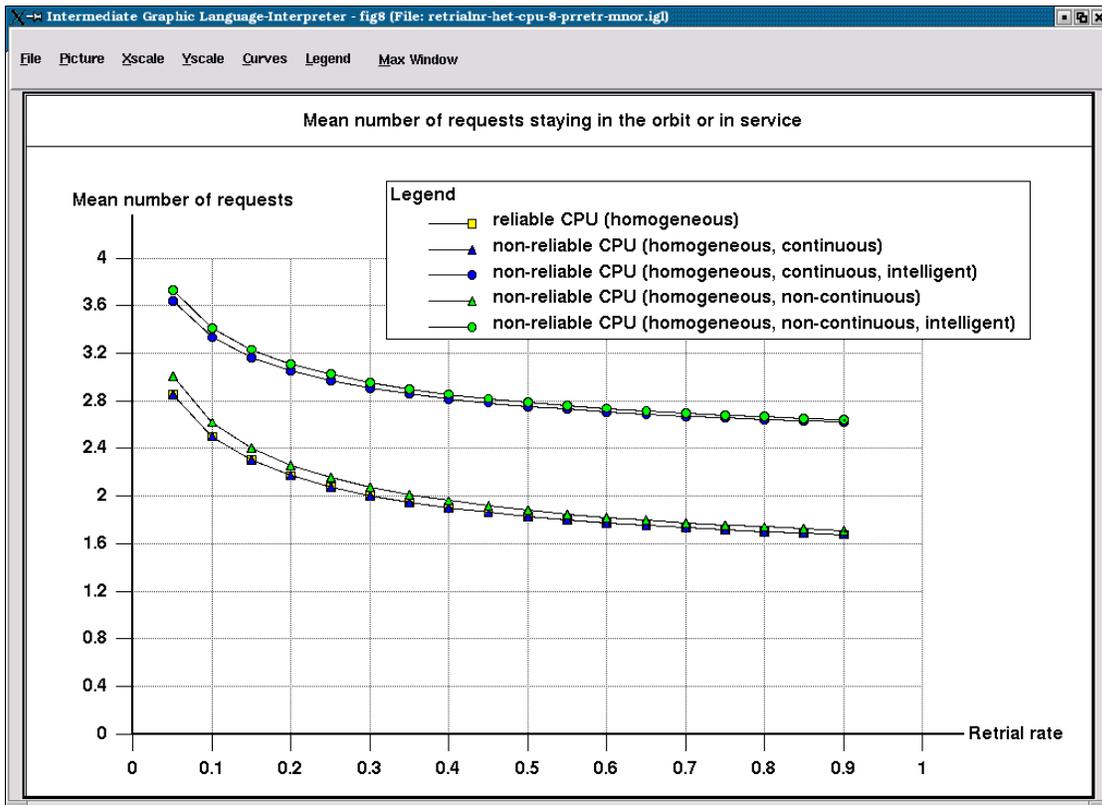


Figure 9:  $M$  versus retrial rate

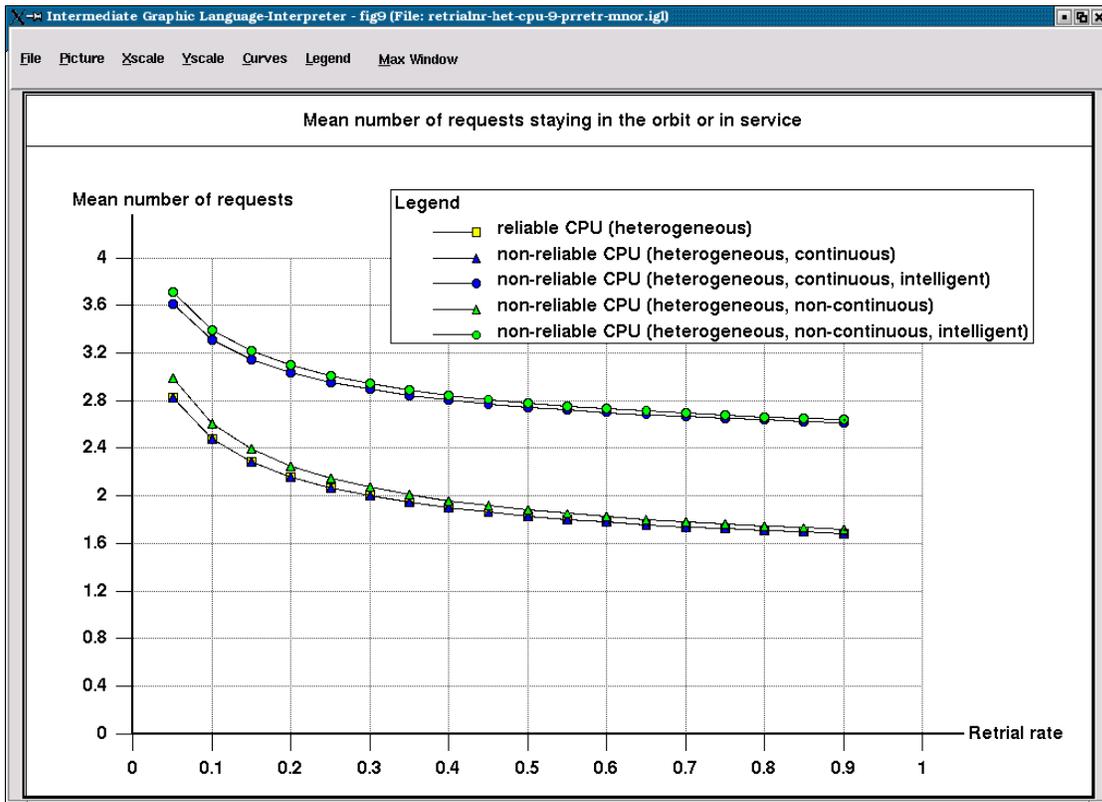


Figure 10:  $M$  versus retrial rate

- In Figure 7, 8 the effect of server's failure is displayed in the blocked operations case. Again the non-continuous case have always longer response times, which is clear. Moreover, for reliable busy server we have the same results which was expected, too. The almost linear increase in  $E[T_i]$  in each case can be explained as follows. Since the failure of the server blocks all the operations and the response time is the sum of the down time of the server, service and repeated call generation time of the request ( which do not change during the failure ) thus the failure has a linear effect on this measure.
- In Figure 9, 10 the effect of the retrial rate on the mean number of request  $M$  staying at the service facility is pictured. The curves confirm our expectation with respect to the service after repair and blocked operations during the failure.  $M$  decreases as the retrial rate increases, which is clear. It is also worth pointing out that the values for the reliable case and non-reliable blocked case coincide. However, it is not so surprising since during the failure the number of requests remain the same.

## 4 Conclusions

In this paper a heterogeneous finite-source homogeneous retrial queueing system with non-reliable server is studied. The novelty of the investigation is this non-reliability of the server and the heterogeneity of sources which makes the system rather complicated. The MOSEL tool was used to formulate and analyze the problem. The main performance measures were derived and graphically displayed. Several numerical calculations were performed to show the effect of the non-reliability of the server on the mean response times of the calls and the mean number of requests staying at the service facility.

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