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ON GAUSSIAN APPROXIMATION OF MULTI-CHANNEL NETWORKS WITH INPUT FLOWS OF GENERAL STRUCTURE

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In this paper, a multi-channel queueing network with input flow of a general structure is considered. The multi-dimensional service process is introduced as the number of customers at network nodes. In the heavy-traffic regime, a functional limit theorem of diffusion approximation type is proved under the condition that the input flows converge to their limits in the uniform topology. A limit Gaussian process is constructed and its correlation characteristics are represented explicitly via the network parameters. A network with nonhomogeneous Poisson input flow is studied as a particular case of the general model, and a correspondent Gaussian limit process is built.

1. Introduction

Queueing models are an efficient tool for analytical modeling of information networks (see [3–5, 10], etc.), cellular communication networks (see [19]), and economic models such as models in logistics, insurance, demography, etc. This is particularly important at the stage of networks design when it is necessary to obtain estimates of basic network performance measures.

Most studies in queueing theory are based on the assumption that the input flow is standard Poisson. However, in many situations there are deviations of real flows from the standard Poisson. For example, the rate of the flow may depend on time. Moreover, the input flow may have general distribution, which can depend on time or state of the network. Queueing network of such type have been used in modeling information processing in data networks and mobile communication systems (see [3, 18, 19], etc.), in the study of the parameters of the ionization track in tracking chambers (see [6]), and in pharmacokinetics (see [20]) and other fields.

Models of general form are extremely difficult to study, and their exact calculations usually cannot be carried out. This leads to the nessecity to simplify the models and to develop different methods of their study. There are a number of publications devoted to networks with general input flow (see [1,3,5,10,21], etc.). Other mathematical problems arise in the studies of queueing systems and networks with time-varying parameters. Such dependence is typical in real-life networks. Dependence of the rate on time yields additional difficulties in investigation of the corresponding models. At the same time, no commonly accepted research methodologies exist for such models yet in spite of the fact that a lot of publications contain the studies of them. Thus, there is a need to develop such methods at least for certain classes of systems. In the works (see [3, 10, 15, 17], etc.) some network models with time-dependent Poisson input flow are considered, and different asymptotic approaches in a heavy-traffic regime are used. Networks with time-dependent periodical input flow are investigated in [16]. Sometimes, the rate of the non-Poisson flow may depend on time, but under certain natural conditions the flow remains close to a nonhomogeneous Poisson process. Such a situation occurs, in particular, when customers arrive from various sources, and the total flow is the sum of flows with low rate (see [8]). In the work [22], MAP-flows and MMP-flows are considered, and certain limit conditions for them are presented under which the sequence of such flows is close to a nonhomogeneous Poisson process. An asymptotic approach for studying finite-source networks is used in [2, 23].

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The present work develops the direction connected with functional limit theorems for the class of multi-channel queueing networks. We consider models of multi-channel queueing networks where an input flow at each node is a flow of general structure. Each node operates as a multi-channel queueing system. Once the service is completed at a node, the customer is transferred to another node or leaves the network with the corresponding probabilities. Input flows to the different nodes can be interdependent. Service times are independent random variables with an exponential-type distributions. It is proved that under some heavy-traffic assumptions concerning the network parameters, the multi-channel service process, that is, the number of customers at the nodes, converges to a Gaussian process in the uniform topology. Correlation characteristics of the Gaussian process are written via network parameters in an explicit form.

This paper is organized as follows. In Section 2 we give the main model description, and the conditions of a heavy-traffic regime are formulated as conditions for the model parameters. In Section 3 multi-channel networks in the heavy-traffic regime are considered. The main result concerning the convergence of the service process for the network with a general input flow as well as some auxiliary results are presented. We formulate a functional limit theorem on convergence in the uniform topology and construct the limit Gaussian approximating process with its characteristics written out in explicit form. The scheme of the proof is given. In Section 4 similar result for the model with nonhomogeneous Poisson input flow is obtained as a consequence from the main result for the general model. The conclusions and some suggestions for future research are given in Section 5.

2. Model description and heavy-traffic conditions

The main model under consideration is a stochastic queueing network consisting of r nodes. Preliminarily, no restrictions are imposed on the structure of the input flow. Let customers arrive at the ith node at instants $\tau_k^{(i)}$, $k = 1, 2, \ldots$, and let $\nu_i(t)$ be the total number of customers arriving at the ith node, $i = 1, 2, \ldots, r$, during the time interval $[0, t]$. It is allowed also that the components of the r-dimensional process $\nu(t) = (\nu_1(t), \ldots, \nu_r(t))$ can be interdependent.

Each of r nodes is a multi-channel stochastic queueing system. When a customer comes into such a system, its service starts immediately. The service times are supposed to be exponentially distributed random variables. The service rate μ_i depends on the node number i, $i = 1, 2, \ldots, r$. Once the service is completed at the ith node, the customer is transferred for service to the jth node with probability p_{ij}, or it leaves the network with probability $p_{ir+1} = \sum_{j=1}^r p_{ij}$. Denote the switching matrix of the network by $P = \|p_{ij}\|_{i,j=1}^r$. An additional node numbered by $r+1$ is interpreted as the "exit" from the network. In accordance with the common notation in queueing theory, we will denote the above model by $[G|M|\infty]^r$.

For a $[G|M|\infty]^r$-type network, we define the service process as an r-dimensional stochastic process

$$Q(t) = (Q_1(t), \ldots, Q_r(t))', \quad t \geqslant 0,$$

where $Q_i(t)$, $i = 1, 2, \ldots, r$, is the number of customers in the ith node at the instant $t \geqslant 0$.

Our main goal is to study the $[G|M|\infty]^r$ network operating in a heavy-traffic regime.

The heavy-traffic regime means that the input flow and service rates at the network nodes depend on the series number n:

$$\nu_i(t) = \nu_i^{(n)}(t), \quad \mu_i(t) = \mu_i^{(n)}(t), \quad i = 1, 2, \ldots, r,$$

and the following conditions take place.

Condition 1. There exists a sequence of nondecreasing functions $a_i^{(n)}(t)$, $t \geqslant 0$, $i = 1, 2, \ldots, r$, such that on any finite interval $[0, T]$:

a) $n^{-1} a_i^{(n)}(nt) \overset{U}{\Rightarrow}_{n\to\infty} a_i^{(0)}(t) \in C[0, T], \quad i = 1, 2, \ldots, r;$

b) $n^{-1/2} \left(\nu_1^{(n)}(nt) - a_1^{(n)}(nt), \ldots, \nu_r^{(n)}(nt) - a_r^{(n)}(nt) \right) \overset{U}{\Rightarrow}_{n\to\infty} W'(t) = (W_1(t), \ldots, W_r(t)),$

where $C[0,T]$ is the set of continuous functions defined on the interval $[0,T]$, and $W(t)$ is an r-dimensional time-inhomogeneous Wiener process with zero mean vector $\mathsf{E}W'(t) = \overline{0}' = (0,\dots,0)$ and correlation matrix $\mathsf{E}W(t)W'(t) = B(t) = \|b_{ij}(t)\|_{i,j=1}^{r}$ that is continuous and nondecreasing for $t \geqslant 0$ (the matrix $B(t) - B(s)$ is a positive semi-definite matrix for $s < t$). The symbol $\overset{U}{\Rightarrow}$ means convergence in the uniform topology for both nonrandom functions and stochastic processes.

This condition means that a version of the functional central limit theorem for a multi-dimensional input flow holds.

Condition 2. $\lim_{n\to\infty} n\mu_i^{(n)} = \mu_i \neq 0$, $i = 1,2,\dots,r$.

Obviously, if this condition is satisfied, then customers at the network nodes are served "asymptotically slowly."

Under Conditions 1 and 2 let us consider a sequence of the following stochastic processes:

$$\xi^{(n)}(t) = n^{-1/2}\left(Q^{(n)}(nt) - \int_0^{nt}\left[da^{(n)}(\tau)\right]' P^{(n)}(nt-\tau)\right), \quad t \geqslant 0,$$

where $\left[da^{(n)}(\tau)\right]' = \left(da_1^{(n)}(\tau),\dots,da_r^{(n)}(\tau)\right)$,

$$P^{(n)}(\tau) = \left\|p_{ij}^{(n)}(\tau)\right\|_{i,j=1}^{r} = \exp\left\{\Delta(\mu^{(n)})(P-I)t\right\},$$

$\Delta(\mu^{(n)}) = \left\|\mu_i^{(n)}\delta_{ij}\right\|_{i,j=1}^{r}$ is a diagonal matrix, $\mu^{(n)} = \left(\mu_1^{(n)},\dots,\mu_r^{(n)}\right)'$, and $I = \|\delta_{ij}\|_{i,j=1}^{r}$ is an identity matrix.

In order to describe the limit of the sequence of stochastic processes $\xi^{(n)}(t)$ as $n\to\infty$, we introduce two independent Gaussian processes $\xi^{(i)}(t) = \left(\xi_1^{(i)}(t),\dots,\xi_r^{(i)}(t)\right)'$, $i = 1,2$.

The process $\xi^{(1)}(t)$ is determined by the mean values

$$\mathsf{E}\xi^{(1)}(t) = \overline{0}, \tag{1}$$

and by the correlation matrices

$$R^{(1)}(t) = \mathsf{E}\xi^{(1)}(t)\xi^{(1)'}(t) - \mathsf{E}\xi^{(1)}(t)\mathsf{E}\xi^{(1)'}(t) = \int_0^t P'(t-\tau)dB(\tau)P(t-\tau), \tag{2}$$

$$R^{(1)}(s,t) = \mathsf{E}\xi^{(1)}(s)\xi^{(1)'}(t) - \mathsf{E}\xi^{(1)}(s)\mathsf{E}\xi^{(1)'}(t) = R^{(1)}(s)P(t-s), \quad s < t, \tag{3}$$

where $P(\tau) = \exp\{\Delta(\mu)(P-I)\tau\}$, $\Delta(\mu) = \|\mu_i\delta_{ij}\|_{i,j=1}^{r}$, $\mu = (\mu_1,\dots,\mu_r)'$. Hereafter, for any vector $x = (x_1,\dots,x_r)$, we will denote by $\Delta(x)$ a diagonal matrix $\|x_i\delta_{ij}\|_{i,j=1}^{r}$ with the vector x on the principal diagonal.

For the process $\xi^{(2)}(t)$ we have

$$\mathsf{E}\xi^{(2)}(t) = \overline{0}, \tag{4}$$

$$R^{(2)}(t) = \int_0^t \left[\Delta\left[(da^{(0)}(\tau))'P(t-\tau)\right] - P'(t-\tau)\Delta\left[da^{(0)}(\tau)\right]P(t-\tau)\right], \tag{5}$$

$$R^{(2)}(s,t) = R^{(2)}(s)P(t-s), \quad s < t. \tag{6}$$

3. Main result for the network with the input flow of a general structure

The following result is valid for the normalized service process $\xi^{(n)}(t)$, $t \geqslant 0$.

Theorem 1. *Let for the $[G|M|\infty]^r$–network Conditions 1, 2 be satisfied, and at the initial instant $t = 0$ the network be empty: $Q^{(n)}(0) = \overline{0}$. Then on any finite interval $[0, T]$, the sequence of stochastic processes $\xi^{(n)}(t)$ converges as $n \to \infty$ to the process $\xi^{(1)}(t) + \xi^{(2)}(t)$ in the uniform topology.*

The proof of the theorem requires some auxiliary results.

Lemma 1. *The finite-dimensional distributions of $\int\limits_0^t dW'(u)P(t-u)$, $t \geqslant 0$, coincide with those of the Gaussian process $\xi^{(1)}(t)$.*

Proof. Note that the functions $b_{ii}(t) \geqslant 0$, $i = 1, 2, \ldots, r$, are nondecreasing. This follows from that the matrix $B(t) - B(s)$, $s < t$, is positive semi-definite. From this condition we obtain (see [9, p. 473])

$$(b_{ij}(t) - b_{ij}(s))^2 \leqslant [b_{ii}(t) - b_{ii}(s)]\,[b_{jj}(t) - b_{jj}(s)]\,,\ i, j = 1, 2, \ldots, r.$$

Let us arbitrarily partition the interval $[0, t]$ by partitioning points

$$0 = t_0 < t_1 < t_2 < \ldots < t_{n-1} < t_n = t,$$

and let $\Delta b_{ij}(t_k) = b_{ij}(t_{k+1}) - b_{ij}(t_k)$, $k = 0, 1, \ldots, n-1$.

Then

$$\sum_k |\Delta b_{ij}(t_k)| \leqslant \sum_k \sqrt{\Delta b_{ii}(t_k)} \cdot \sqrt{\Delta b_{jj}(t_k)} \leqslant \left(\sum_k \Delta b_{ii}(t_k)\right)^{1/2} \left(\sum_k \Delta b_{jj}(t_k)\right)^{1/2}.$$

This yields that both functions $b_{ii}(t)$ and $b_{ij}(t)$, $i \neq j$, are functions with bounded variation on any finite interval $[0, t]$. Therefore, there exists an integral defining $R^{(1)}(t)$.

Now consider the characteristic function of the one-dimensional distribution of the process $\eta(t) = \int\limits_0^t dW'(u)P(t-u)$:

$$\varphi(t) = \mathsf{E}\exp\left\{i\eta'(t)s\right\} = \mathsf{E}\exp\left\{i\int\limits_0^t dW'(u)P(t-u)s\right\},\ \ s' = (s_1, \ldots, s_r) \in \mathbb{R}^r.$$

Choose a sequence of partitions of interval $[0, t]$ by partitioning points

$$0 = t_{n_0} < t_{n_1} < \ldots < t_{n_{m_n-1}} < t_{n_{m_n}} = t$$

such that $\max\limits_{0 \leqslant k \leqslant m_n-1} \Delta t_{n_k} \to 0$, $\Delta t_{n_k} = t_{n_{k+1}} - t_{n_k}$.

Then in accordance with the properties of the stochastic integral $\eta(t)$ (see [7, p. 280–292]), we find

$$\varphi(t) = \lim_{n\to\infty} \prod_{k=0}^{m_n-1} \mathsf{E}\exp\left\{i\Delta W'(t_{n_k})P(t-t_{n_k})s\right\} =$$
$$= \lim_{n\to\infty} \prod_{k=0}^{m_n-1} \exp\left\{-\frac{1}{2}\left[P(t-t_{n_k})s\right]'\Delta B(t_{n_k})P(t-t_{n_k})s\right\} =$$
$$= \lim_{n\to\infty} \exp\left\{-\frac{1}{2}\sum_{k=0}^{m_n-1} s'P'(t-t_{n_k})\Delta B(t_{n_k})P(t-t_{n_k})s\right\} =$$
$$= \exp\left\{-\frac{1}{2}s'\int\limits_0^t P'(t-u)dB(u)P(t-u)s\right\}, \qquad (7)$$

where $\Delta W(t_{n_k}) = W(t_{n_{k+1}}) - W(t_{n_k})$, $\Delta B(t_{n_k}) = B(t_{n_{k+1}}) - B(t_{n_k})$, $k = 0, 1, \dots, m_n - 1$.

Now we find the characteristic function of the two-dimensional distribution of $\eta(t)$. Let $0 < t_1 < t_2$ be any two instants and $s'(1) = (s_1(1), \dots, s_r(1))$, $s'(2) = (s_1(2), \dots, s_r(2)) \in \mathbb{R}^r$. Then

$$\varphi(t_1, t_2) = \mathsf{E} \exp\left\{ i \int_0^{t_1} dW'(u) P(t_1 - u) s(1) + i \int_0^{t_2} dW'(u) P(t_2 - u) s(2) \right\} =$$

$$= \mathsf{E} \exp\left\{ i \int_0^{t_1} dW'(u) P(t_1 - u) \left[s(1) + P(t_2 - t_1) s(2) \right] \right\} \times$$

$$\times \mathsf{E} \exp\left\{ \int_{t_1}^{t_2} dW'(u) P(t_2 - u) s(2) \right\} =$$

$$= \exp\left\{ -\frac{1}{2} \left[s(1) + P(t_2 - t_1) s(2) \right]' \int_0^{t_1} P'(t_1 - u) dB(u) P(t_1 - u) \left[s(1) + P(t_2 - t_1) s(2) \right] \right\} \times$$

$$\times \exp\left\{ -\frac{1}{2} s'(2) \int_{t_1}^{t_2} P'(t_2 - u) dB(u) P(t_2 - u) s(2) \right\} =$$

$$= \exp\left\{ -\frac{1}{2} s'(1) \int_0^{t_1} P'(t_1 - u) dB(u) P(t_1 - u) s(1) - \right.$$

$$- \frac{1}{2} s'(2) \int_0^{t_2} P'(t_2 - u) dB(u) P(t_2 - u) s(2) -$$

$$\left. - s'(1) \int_0^{t_1} P'(t_1 - u) dB(u) P(t_1 - u) P(t_2 - t_1) s(2) \right\}.$$

From the form of $\varphi(t_1, t_2)$ it follows that the joint distribution of $\eta(t_1)$, $\eta(t_2)$ is Gaussian.

Now, for $0 < t_1 < \ldots < t_n$, $n > 2$, the proof of the lemma can be completed by induction using the fact that the stochastic integrals $\int_0^{t_{n-1}} dW'(u) P(t_n - u)$ and $\int_{t_{n-1}}^{t_n} dW'(u) P(t_n - u)$ are independent.

The lemma is proved.

The service of an individual customer at the nodes of the $[G|M|\infty]^r$ network is independent of the service of other customers. In order to constructively define the service process of the network, we consider a Markov chain $x(t)$, $t \geqslant 0$, in the set of states $\{1, 2, \dots, r, r+1\}$, given by the following infinitesimal transition rates:

$$\alpha_{ij} = \begin{cases} -\mu_i (1 - p_{ii}), & i = j = 1, 2, \dots, r, \\ \mu_i p_{ij}, & i \neq j, \ i = 1, 2, \dots, r, \ j = 1, 2, \dots, r, r+1, \\ 0, & i = r+1, \ j = 1, 2, \dots, r, r+1; \end{cases}$$

and the initial distribution $p(0) = (p_1(0), \dots, p_{r+1}(0))'$.

If $p_i(0) = 1$, then the corresponding chain is denoted by $x^{(i)}(t)$. The state numbered $r+1$ is absorbing for the chain $x(t)$. The transition probabilities $\mathsf{P}\{x(t) = j / x(0) = i\} = \mathsf{P}\left\{x^{(i)}(t) = j\right\}$, $i, j = 1, \dots, r$, are the same as $p_{ij}(t)$ defined above.

The path of a customer from the moment of arrival at the network through the ith node until exit can be described by the chain $x^{(i)}(t)$, $t \geqslant 0$, with absorption in the state $r+1$ interpreted as the customer exit from the network.

The Markov chain $x^{(i)}(t)$, $i = 1, \ldots, r$, $t \geqslant 0$, defines the following r-dimensional stochastic process pertaining to indicator:

$$\chi^i(t) = \left(\chi_1^i(t), \chi_2^i(t), \ldots, \chi_r^i(t)\right)', \;\; t \geqslant 0, \;\; i = 1, \ldots, r,$$

$$\chi^i(t) = \begin{cases} e_j, & \text{if } x^{(i)}(t) = j, \; j = 1, \ldots, r, \\ e_0, & \text{if } x^{(i)}(t) = r+1, \end{cases}$$

where e_j is an r-dimensional vector whose jth component is equal to 1 while the rest are zeroes, and e_0 is the r-dimensional vector with null entries. Then the position of 1 in the vector $\chi^i(t)$ points out clearly the node where the customer will be situated at the instant $t > 0$ provided that it is at the ith node at the initial instant $t = 0$. If $\chi^i(t) = e_0$, this means that the customer starting its service at the ith node completes its service and leaves the network until the instant $t > 0$.

For an arbitrary natural N and $z(i) = (z_1(i), \ldots, z_r(i))'$, $i = 1, 2, \ldots, N$, $|z(i)| \leqslant 1$, we denote by $\Phi^{(m)} = \Phi^{(m)}(z(1), \ldots, z(N), t_1, \ldots, t_N)$, $m = 1, 2, \ldots, r$, the joint generating function of the vectors $\chi^m(t_1), \ldots, \chi^m(t_N)$, $0 < t_1 < \ldots < t_N$, $\Phi' = \left(\Phi^{(1)}, \ldots, \Phi^{(m)}\right)$.

Hereinafter we will need the following vector-matrix form of Φ.

Lemma 2. *For any $N = 1, 2, \ldots$ and $0 < t_1 < \ldots < t_N$ the joint generating function Φ can be written as*

$$\Phi = \bar{1} + \sum_{i=1}^{N} P(\Delta t_1)\Delta[z(1)] \ldots P(\Delta t_{i-1})\Delta[z(i-1)]P(\Delta t_i)\Delta[z(i) - \bar{1}], \tag{8}$$

where $\bar{1}$ is the r-dimensional vector with unit entries, $\Delta t_i = t_i - t_{i-1}$ $(t_0 = 0)$, $i = 1, 2, \ldots, N$.

The proof of Eq. (2) can be obtained by induction over N.

Based on Lemmas 1 and 2, we can derive Theorem 1 in the part of convergence of finite-dimensional distributions by the method of characteristic functions. Moreover, we can strengthen the convergence of finite-dimensional distributions to convergence of $\xi^{(n)}(t)$ to $\xi^{(1)}(t) + \xi^{(2)}(t)$ in the uniform topology in the same way as in [11].

4. Limit process for the network with nonhomogeneous Poisson input flow

Let us apply Theorem 1 to $\left[\overline{M_t}|M|\infty\right]^r$-type networks. In such networks a separate nonhomogeneous Poisson flow of customers $\nu_i(t)$ arrives from outside into each of the r nodes. Let $\nu_i(t)$ be the Poisson flow with a leading function $\Lambda_i(t)$ for the node numbered i, $i = 1, 2, \ldots, r$. The stochastic processes $\nu_i(t)$, $i = 1, 2, \ldots, r$, are assumed to be independent.

We will assume that the input flow $\nu_i(t)$ depends on a series number n, and the following condition holds.

Condition 3. On any finite interval $[0, T]$ we have

$$n^{-1}\Lambda_i^{(n)}(nt) \overset{U}{\Rightarrow}_{n\to\infty} \Lambda_i(t) \in C[0, T], \;\; i = 1, 2, \ldots, r.$$

Since under Condition 3 on any finite interval $[0, T]$ the sequence of stochastic processes

$$W_i^{(n)}(nt) = n^{-1/2}\left(\nu_i^{(n)}(nt) - \Lambda_i^{(n)}(nt)\right), \;\; i = 1, 2, \ldots, r,$$

converges in the uniform topology as $n \to \infty$ to the Wiener process $W_i^{(0)}(t)$ with $\mathsf{E}W_i^{(0)}(t) = 0$ and $\mathsf{D}W_i^{(0)}(t) = \Lambda_i^{(0)}(t)$ (see [3]), then Condition 1 for multi-dimensional input flow is satisfied with $a_i^{(n)}(nt) = \Lambda_i^{(n)}(nt)$, $a_i^{(0)}(nt) = \Lambda_i^{(0)}(nt)$, and with diagonal matrix $B(t) = \left\|\delta_{ij}\Lambda_i^{(0)}(t)\right\|_{i,j=1}^{r}$.

Now, relying on Theorem 1, we can obtain the following result.

Corollary 1. Let Conditions 2, 3 hold for the $[M_t|M|\infty]^r$ network, and at the initial instant $t = 0$ the network be empty, $Q^{(n)}(0) = \bar{0}$. Then on any finite interval $[0, T]$, the sequence of stochastic processes

$$\xi^{(n)'}(t) = n^{-1/2}\left(Q^{(n)'}(nt) - \int_0^{nt}\left[d\Lambda^{(n)}(\tau)\right]' P^{(n)}(nt-\tau)\right), \quad t \geqslant 0,$$

$$\left[d\Lambda^{(n)}(\tau)\right]' = \left(d\Lambda_1^{(n)}(\tau), \ldots, d\Lambda_r^{(n)}(\tau)\right),$$

converges as $n \to \infty$ in the uniform topology to the sum of independent Gaussian processes $\xi^{(1)}(t)+\xi^{(2)}(t)$ defined by (1)–(6) with $a^{(0)}(\tau)' = \left(\Lambda_1^{(0)}(\tau), \ldots, \Lambda_r^{(0)}(\tau)\right)$, $B(\tau) = \left\|\delta_{ij}\Lambda_i^{(0)}(\tau)\right\|_{i,j=1}^{r}$.

It is possible to verify that in the case where for each $i = 1, 2, \ldots, r$ $\Lambda_i^{(0)}(t) = \int_0^t \lambda_i^{(0)}(u)du$ and $\lambda_i^{(0)}(\cdot) \in C[0, T]$, i.e., when the leading function is absolutely continuous, the limit Gaussian process is a diffusion process (see [15]).

5. Conclusions

In this paper multi-channel stochastic networks with a general input flow are considered. For such networks in a heavy-traffic regime, we develop an asymptotic approach based on the approximation of jump-type service processes of customers by continuous Gaussian processes. We prove the convergence of the corresponding service processes in the uniform topology that can be used for the calculation of the quality functionals of network operation and for solving optimization problems (see [12]). The limit process in both cases (general input flow and nonhomogeneous Poisson input flow) is a Markov Gaussian process. Under some additional conditions for a network with nonhomogeneous Poisson input flow, it can be proved in a way similar to [15] that the limit process is a diffusion process with certain local characteristics.

Note that similar models of stochastic networks with nonidentically distributed service times and nonhomogeneous Poisson input flow are considered in [14].

The results can be applied to models of multi-channel networks with input flows of various structure. General input flow can be replaced by various types of flows, for example, by a doubly stochastic Poisson flow (see [13]). On the other hand, a multi-channel network can be considered as a closed finite-source network (see [2, 24]). Each separate source generates a request and sends it to the input of the network. The source that sent the request for service is in the standby mode and does not generate new requests until its request service is completed. For such a network a similar result concerning the convergence can be formulated and proved under the condition that the number of sources increases asymptotically.

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