A Survey of Recent Results in Finite-Source Retrial Queues with Collisions

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Abstract. The aim of the present paper is to give a review of recent results on single server finite-source retrial queuing systems with collision of the customers. There are investigations when the server is reliable and there are models when the server is subject to random breakdowns and repairs depending on whether it is idle or busy. Tool supported, numerical, simulation and asymptotic methods are considered under the condition of unlimited growing number of sources. Several cases and examples are treated and the results of different approaches are compared to each other showing the advantages and disadvantages of the given method. In general we could prove that the steady-state distribution of the number of customers in the service facility can be approximated by a normal distribution with given mean and variance. Using asymptotic methods under certain conditions in steady-state the distribution of the sojourn time in the orbit and in the system can be approximated by a generalized exponential one. Furthermore, it is proved that the distribution of the number of retrials until the successful service in the limit is geometrically distributed. By the help of stochastic simulation several systems are analyzed showing directions for further analytic investigations. Tables and Figures are collected to illustrate some special features of these systems.

Keywords: Finite-source queuing system · Retrial queues
Collisions · Server breakdowns and repairs · Analytic results
Algorithmic approach · Stochastic simulation · Asymptotic analysis

1 Introduction

Finite-source retrial queues are very useful and effective stochastic systems to model several problems arising in telephone switching systems, telecommunication networks, computer networks and computer systems, call centers, wireless communication systems, etc. To see their importance the interested reader is referred to the following works and references cited in them, for example [3,9,15,19]. Searching the scientific databases we have noticed that relatively

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just a small number of papers have been devoted to systems when the arriving calls (primary or secondary) causes collisions to the request under service and both go to the orbit, see for example [1,7,18,24,40].

Nazarov and his research group developed a very effective asymptotic method [39] by the help of which various systems have been investigated. Concerning to finite-source retrial systems with collision we should mention the following papers [25–28,35].

Sztrik and his research group have been dealing with systems with unreliable server/s as can be seen, for example in [2,44,45,51] and that is why it was understandable that the two research groups started cooperation in 2017.

Our investigations have been based on the analytical, numerical, simulation and asymptotic approached as treated in, for example [3,5,6,10,16,20,23,29,30, 34,39,42,43,50,52].

The primary aim of the present paper is to give a survey on the results obtained in this field in the near past by means of different methods. Doing so we have tried to unify the notation appeared in different publications and to use the standard notation of Western-style papers which is many times differs from the Russian-style ones.

The rest of the paper is organized as follows. In Sect. 2 description of the model is given, the corresponding multi-dimensional non-Markov process is defined. In Sects. 3 and 4 systems with a reliable and an unreliable server are treated, respectively. In the subsections models with exponentially and generally distributed service times are investigated, and then analyzed by means of tool supported, algorithmic, simulation and asymptotic methods, respectively. The main results of the papers are collected and several Figures illustrate the most interesting features of the given system. Finally, the paper ends with a Conclusion and some future plans are highlighted.

2 Model Description and Notations

In the following we introduce the model in the most general form as it was treated by the help of numerical and asymptotic methods.

Let us consider a retrial queuing system of type M/GI/1//N with collision of the customers and an unreliable server (Fig. 1). The number of sources is N and each of them can generate a primary request during an exponentially distributed time with rate λ/N. A source cannot generate a new call until the end of the successful service of this customer.

If a primary request finds the server idle, he enters into service immediately, in which the required service time has a probability distribution function $B(x)$. Let us denote its service rate function by $\mu(y) = B^{'}(y)(1 - B(y))^{-1}$ and its Laplace -Stieltjes transform by $B^{*}(y)$, respectively. If the server is busy, an arriving (primary or repeated) customer involves into collision with customer under service and they both move into the orbit. The inter-retrial times of customers are supposed to be exponentially distributed with rate σ/N. We assume that the server is unreliable, that is its lifetime is supposed to be exponentially

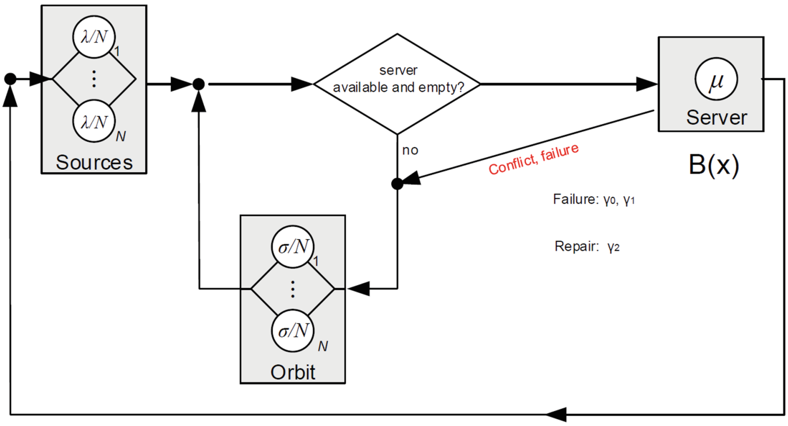

Fig. 1. Retrial queueing system of type M/GI/1//N with collisions of the customers and an unreliable server

distributed with failure rate γ_0 if the server is idle and with rate γ_1 if it is busy. When the server breaks down, it is immediately sent for repair and the repair time is assumed to be exponentially distributed with rate γ_2. We deal with the case when the server is down all sources continue generation of customers and send it to the orbit, similarly customers may retry from the orbit to the server but all arriving customers immediately go into the orbit. Furthermore, in this unreliable model we suppose that the interrupted request goes to the orbit immediately and its next service is independent of the interrupted one. Of course in the case of reliable server $\gamma_0 = \gamma_1 = 0$. All random variables involved in the model construction are assumed to be independent of each other.

Let $J(t)$ be the number of customers in the system at time t, that is, the total number of customers in the orbit and in service. Similarly, let $K(t)$ be the server state at time t, that is

$$K(t) = \begin{cases} 0, & \text{if the server is idle,} \\ 1, & \text{if the server is busy,} \\ 2, & \text{if the server is down (under repair).} \end{cases}$$

Thus, we will investigate the process $\{K(t), J(t)\}$, which is not a Markov-process unless the service time is exponentially distributed. To be a Markov one we will use the method of supplementary variables, namely, we will consider two variants: the residual service time method and the elapsed service time method depending on what is the aim of the investigation.

Let us denote by $Y(t)$, and $Z(t)$, the supplementary random process equal to the elapsed service time of the customer till the moment t and by $Z(t)$ the residual service time, that is time interval from the moment t until the end of successful service of the customer, respectively. It is obvious that $\{K(t), J(t), Y(t)\}$ and $\{K(t), J(t), Z(t)\}$ are Markov processes. Let us note, that $Y(t)$ and $Z(t)$ are defined only in those moments when the server is busy, that is, when $K(t) = 1$.

Let us define the stationary probabilities as follows:

$$
\begin{aligned}
P_0(j) &= P\{K=0, J=j\},\\
P_1(j,y) &= P\{K=1, J=j, Y<y\},\\
P_1(j,z) &= P\{K=1, J=j, Z<z\},\\
P_2(j) &= P\{K=2, J=j\}.
\end{aligned}
$$

Of course in the case of exponentially distributed service time the steady-state probabilities are denoted as follows:

$$P_k(j) = P\{K=k, J=j\}, \quad k=0,1,2, \quad j=0,...,N.$$

The steady-state distribution of the server's state is denoted by

$$R_k = P(K=k), k=0,1,2$$

and the distribution of number of customers in the system is designated by

$$P(j) = P(J=j), j=0,...,N.$$

It is clear that in the case of reliable server all the probabilities where $K=2$ are 0.

The main aim of the investigations is to get these distributions and other performance measures of the systems, such as the distribution of the sojourn time in the system, distribution of the total service time, distribution of the number of retrials. These are very complicated problems and to the best knowledge of the authors there are no exact analytical formulas to the solutions. That is the reason we have tried to obtain the characteristics of different systems by the help of tool supported, algorithmic, stochastic simulation and asymptotic methods.

3 Systems with a Reliable Server

3.1 M/M/1 Systems

Algorithmic Approach. In papers [26,35] the steady-state Kolmogorov equations were derived and the distribution of the system's state were obtained by an algorithmic approach. Then the distribution of the number of customers in the system were calculated and used to validate the asymptotic results.

Asymptotic Approach. The main contribution of paper [35] is that the in steady-state the prelimit distribution of the number of customers in the system can be approximated by a normal distribution with given mean and variance.
In paper [35] 2nd and 3rd order approximations of the prelimit distribution were compared to the exact distribution obtained by the algorithmic method.

In different parameter setup and for different N the applicability of the asymptotic method was validated and some conclusions were drawn.

A more complicated problem, namely the distribution of the sojourn time in the service facility was investigated in [25] by the help of asymptotic methods as N tends to infinity. It was proved that the characteristic function of the sojourn time T of a customer spends in the service facility can be approximated by

$$\mathsf{E}\exp\{iuT\} \approx q + (1-q)\frac{\sigma q/N}{\sigma q/N - iu}, \quad q = \frac{\mu R_0}{\delta + \mu}.$$

3.2 M/GI/1 System

This section deals with the results when the required service times are generally distributed but in the examples the gamma distribution is used due to its useful properties. Namely, it is easy to see that its squared coefficient of variation can be less, equal or greater than 1 depending on the values of the shape and scale parameters.

Algorithmic Approach. Paper [27] deals with the algorithmic approach how to get the steady-state distribution of the system. The method of supplementary variable technique with residual service time were applied and several numerical examples were treated with gamma distributed service time. The results helped the validation of asymptotic results for the same model.

Stochastic Simulation. Papers [37,38] are devoted to the asymptotic analysis of the mean total service time, distribution of the sojourn time in the system and the distribution of number of retrials. It must be noted that the results have not been validated by simulation, yet. Meanwhile simulations have been carried out the estimations for the mean and variance of the sojourn time have been obtained, and the distribution of the number of retrials also has been determined. The simulation analysis will be published in the near future.

Asymptotic Approach. In this part the asymptotic results published in [37,38] are summarized. Before doing that we need some notations, namely

$$B^*(\alpha) = \int_0^\infty e^{-\alpha x} dB(x), \quad \delta(\kappa_1) = \lambda + (\sigma - \lambda)\kappa_1.$$

Then κ_1 can be obtained from

$$\kappa_1 = 1 - \frac{\delta(\kappa_1)}{\lambda} \cdot \frac{B^*(\delta(\kappa_1))}{2 - B^*(\delta(\kappa_1))}, \tag{1}$$

and the distribution of the server's state can be determined by

$$R_0 = \frac{1}{2 - B^*(\delta)}, \quad R_1 = \frac{1 - B^*(\delta)}{2 - B^*(\delta)}.$$

Introducing the notations

$$A_1 = \lambda(1-\kappa_1), \qquad R_1^*(\alpha) = -\delta R_0\left[B^*(\alpha)\right],$$

we obtain

$$\kappa_2 = \frac{A_1\Big(R_0 \cdot B^*(\delta)\left[\delta + A_1\right] - (\delta + A_1 R_0)\Big)}{A_1(\sigma-\lambda)\Big(R_1^*(\delta)-R_1-R_0(B^*(\delta)-1)\Big)+\delta\Big((\sigma-\lambda)\Big(R_1^*(\delta)-R_0B^*(\delta)\Big)-\lambda\Big)}.$$

Consequently the steady-state prelimit distribution of the number of customers in the system can be approximated by a normal distribution with mean $N\kappa_1$ and variance $N\kappa_2$.

For the distribution of the number of retrials/transitions of the tagged customer into the orbit we have the following results.
Let ν be the number of transitions of the tagged customer into the orbit, then

$$\lim_{N\to\infty} \mathsf{E}\, z^{\nu} = \frac{q}{1-(1-q)z},$$

where value of parameter q has a form

$$q = R_0 B^*(\delta).$$

From the proved theorem it is obviously follows that the probability distribution $P\left\{\nu = n\right\},\ n = \overline{0,\infty}$ of the number of transitions of the tagged customer into the orbit is geometric and

$$P\left\{\nu = n\right\} = q(1-q)^n, \quad n = \overline{0,\infty}.$$

Consequently, by using the law of total probability for the characteristic function of the sojourn/waiting time W of the tagged customer in the orbit we get

$$\mathsf{E}e^{iuW} \approx q + (1-q)\frac{\sigma q}{\sigma q - iuN}.$$

In the case of $N \to \infty$ the limiting probability distributions of the sojourn time of the customer in the system T and the sojourn time of the customer in the orbit W coincide, namely

$$\lim_{N\to\infty} \mathsf{E}\exp\left\{iu\frac{T}{N}\right\} = \lim_{N\to\infty} \mathsf{E}\exp\left\{iu\frac{W}{N}\right\} = q + (1-q)\frac{\sigma q}{\sigma q - iu}.$$

4 Systems with an Unreliable Server

In many practical situations the server is not reliable and after a random time it can fail and needs repair which also takes a random duration. To deal with these service interruptions several papers have been published, see for example [2,8,11,12,14,21,41,45,48,49,53]. In the following parts we summarize our results obtained by different methods.

4.1 M/M/1 System

Tool Supported Approach by MOSEL. Because of the fact, that in many practical situations the state space of the describing Markov chain is very large, it is rather difficult to calculate the system measures in the traditional way of writing down and solving the underlying steady-state equations. To simplify this procedure several software packages have been developed and effectively used for performance evaluation of complex systems, see for example [11–14,17]. In our investigations a similar software tool called MOSEL (Modeling, Specification and Evaluation Language) has been used to formulate the model and to obtain the performance measures. Paper [4] deals with the model formulation, derivation of several performance measures and generation of illustrative examples showing an interesting phenomenon of finite-source retrial queues, that is under specific parameter setup the mean waiting/ sojourn time has a maximum as the arrival intensity is increasing.

Stochastic Simulation. To validate the applicability of the asymptotic approach we need either numerical or simulation results. The correct operation of the simulation software was tested by the numerical sample examples. The investigations carried out by the simulation and asymptotic methods have been submitted for publication, see [31,32].

Asymptotic Approach. First we deal with the distribution of the number customers in the system as it has been published in [31]. The first order asymptotic results are the following

$$\lim_{N\to\infty} E\exp\left\{iw\frac{J}{N}\right\} = \exp\left\{iw\kappa_1\right\},$$

where κ_1 is the positive solution of the equation

$$(1-\kappa_1)\,\lambda - \mu R_1(\kappa_1) = 0,$$

where the stationary distributions of probabilities $R_k(\kappa_1)$ of the server state $k = 0, 1, 2$ are obtained as follows

$$R_0(\kappa_1) = \left\{\frac{\gamma_0+\gamma_2}{\gamma_2} + \frac{\gamma_1+\gamma_2}{\gamma_2}\cdot\frac{a\left(\kappa_1\right)}{a\left(\kappa_1\right)+\gamma_1+\mu}\right\}^{-1},$$

$$R_1(\kappa_1) = \frac{a\left(\kappa_1\right)}{a\left(\kappa_1\right)+\gamma_1+\mu}\cdot R_0(\kappa_1),$$

$$R_2(\kappa_1) = \frac{1}{\gamma_2}\left[\gamma_0 R_0(\kappa_1) + \gamma_1 R_1(\kappa_1)\right],$$

here $a\left(\kappa_1\right)$ is

$$a\left(\kappa_1\right) = \left(1-\kappa_1\right)\lambda + \sigma\kappa_1.$$

The second order asymptotic results are

$$\lim_{N\to\infty} E\exp\left\{iw\frac{J-\kappa_1 N}{\sqrt{N}}\right\} = \exp\left\{\frac{(iw)^2}{2}\kappa_2\right\},$$

where κ_2 is

$$\kappa_2 = \frac{\gamma_2\mu(R_1-b_1)+(1-\kappa_1)\lambda\left\{(\gamma_1+\gamma_2)\,b_1+(1-\kappa_1)\,\lambda R_2\right\}}{(\lambda+\mu b_2)\,\gamma_2-(1-\kappa_1)\,\lambda\,(\gamma_1+\gamma_2)\,b_2},$$

and

$$b_1 = \frac{(1-\kappa_1)\,\lambda}{a+\gamma_1+\mu}R_0, \qquad b_2 = \frac{(\sigma-\lambda)(R_0-R_1)}{a+\gamma_1+\mu}.$$

Consequently the prelimit distribution of the number of customers in the system can be approximated by a normal distribution with mean $N\kappa_1$ and variance $N\kappa_2$.

One of the main contributions of paper [32] is that for the limit of the characteristic function of the normalized sojourn time we have

$$\lim_{N\to\infty} \mathsf{E}\exp\left\{iw\frac{T}{N}\right\} = q+(1-q)\frac{\sigma q}{\sigma q-iw},$$

where q is

$$q = \frac{(1-\kappa_1)\lambda}{(1-\kappa_1)\lambda+\sigma\kappa_1}.$$

Consequently the characteristic function of the sojourn time of the customer in the system in the prelimit situation of finite N can be approximated by

$$\mathsf{E}\,e^{iuT} \approx q+(1-q)\frac{\sigma q}{\sigma q-iuN}. \tag{2}$$

For the distribution of the number of transitions/retrials of the tagged customer into the orbit we got the following results.
Let ν be the number of transitions of the tagged customer into the orbit, then

$$\lim_{N\to\infty} \mathsf{E}\,z^{\nu} = \frac{q}{1-(1-q)z},$$

resulting that the probability distribution $P\left\{\nu=n\right\},\ n=\overline{0,\infty}$ of the number of transitions of the tagged customer into the orbit is geometric and has the form

$$P\left\{\nu=n\right\} = q(1-q)^n,\quad n=\overline{0,\infty}.$$

Consequently the prelimit characteristic function of the sojourn/waiting time W of the tagged customer in an orbit can be approximated as

$$\mathsf{E}e^{iuW} \approx q+(1-q)\frac{\sigma q}{\sigma q-iuN}.$$

In the case of $N \to \infty$ the limiting probability distributions of the sojourn time of the customer in the system T and the sojourn time of the customer in an orbit W coincide, namely

$$\lim_{N\to\infty} \mathsf{E}\exp\left\{iu\frac{T}{N}\right\} = \lim_{N\to\infty} \mathsf{E}\exp\left\{iu\frac{W}{N}\right\} = q + (1-q)\frac{\sigma q}{\sigma q - iu}.$$

4.2 M/GI/1 System

Stochastic Simulation. In paper [47] the required service time is supposed to be gamma distributed and the input parameters of the system are collected in Table 1.

Table 1. Numerical values of model parameters

Case	N	λ/N	γ_0	γ_1	γ_2	σ/N	α	β
1	100	0.01	0.1	0.1	1	0.01	0.5	0.5
2	100	0.01	0.1	0.1	1	0.01	1	1
3	100	0.01	0.1	0.1	1	0.01	2	2

Figure 2 shows the steady-state distribution of the three investigated cases. It is observed the mean number of customers increases as α and β are getting larger. *Case 2* is a special case because when $\alpha = 1$ it represents the exponential distribution. From the shape of the curves it is clearly visible that the

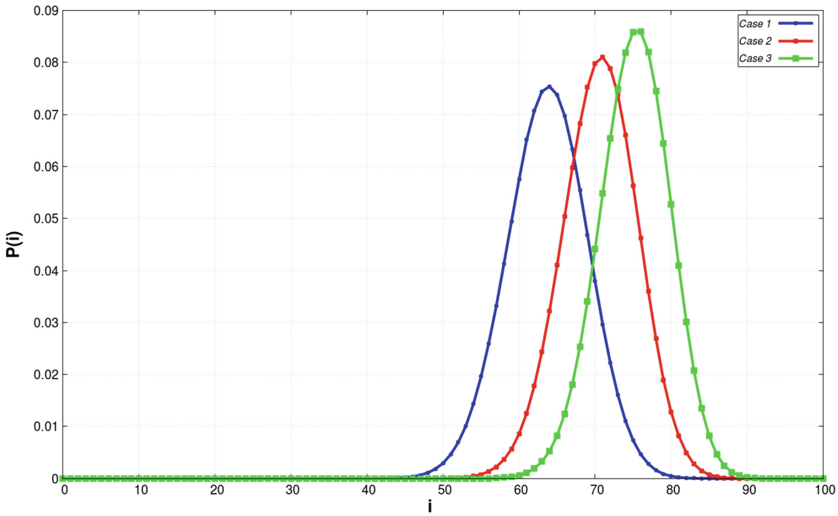

Fig. 2. Comparison of steady-state distributions

steady-state distribution of the cases are normally distributed. The next table presents the considered performance measures in relation with the different cases (see Table 2).

In Table 2 the notations mean the followings: $E(J)$ and $Var(J)$ - mean number and variance of customers in the system, $E(T)$ and $Var(T)$ - mean and variance of response time, $E(W)$ and $Var(W)$ - mean and variance of waiting time, $E(S)$ and $Var(S)$ - mean and variance of successful service time, $E(IS)$ - mean interrupted service time.

Table 2. Simulation results

Case	E(J)	$Var(J)$	E(T)	$Var(T)$	E(W)	$Var(W)$	E(S)	$Var(S)$	E(IS)
1	63.6842	27.9734	175.3073	65657.3454	174.5884	65434.6696	0.3147	0.1979	0.4041
2	70.5912	24.3012	239.9734	105273.4267	238.9734	104918.6389	0.4784	0.2289	0.5217
3	75.1825	21.2439	302.8106	151781.1411	301.5377	151277.6006	0.6472	0.2095	0.6257

Figure 3 represents the confirmation of mean waiting time. The same parameters are (see Table 2) used as in case of Fig. 2 but here the running parameter is λ/N. As it is expected with the increment of λ/N mean waiting time increases as well but an interesting phenomenon is noticeable namely after λ/N is greater than 0.1 mean waiting time starts to decrease.

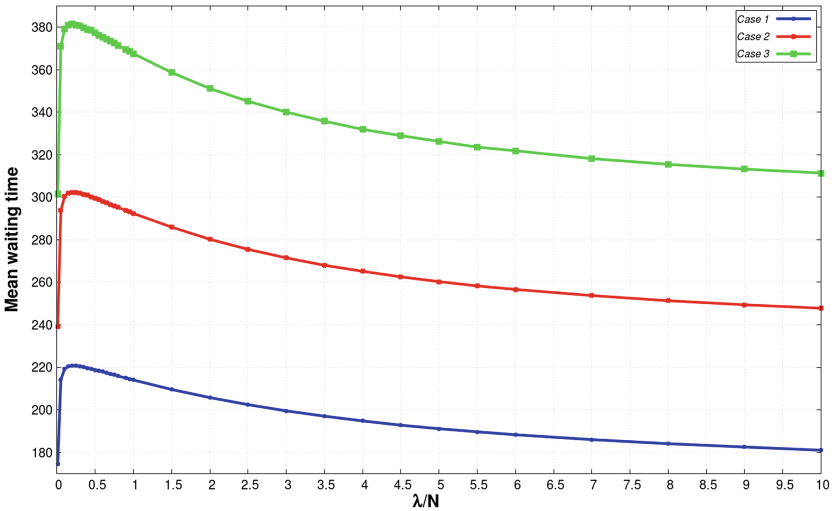

Fig. 3. Mean waiting time vs. intensity of incoming customers

Asymptotic Approach. These results have been published in [36] using supplementary variable technique. The limit of the characteristic function of the scaled number of customers in the systems can be written in the following form

$$\lim_{N\to\infty} \mathsf{E}\exp\left\{iw\frac{J}{N}\right\} = \exp\left\{iw\kappa_1\right\},$$

where κ_1 is the positive solution of the equation

$$(1-\kappa_1)\,\lambda - \delta(\kappa_1)\left[R_0(\kappa_1) - R_1(\kappa_1)\right] + \gamma_1 R_1(\kappa_1) = 0,$$

here $\delta\left(\kappa_1\right)$ is

$$\delta\left(\kappa_1\right) = \left(1-\kappa_1\right)\lambda + \sigma\kappa_1,$$

and the stationary distributions of probabilities $R_k(\kappa_1)$ of the server's state $k = 0, 1, 2$ are determined as follows

$$R_0(\kappa_1) = \left\{\frac{\gamma_0+\gamma_2}{\gamma_2} + \frac{\gamma_1+\gamma_2}{\gamma_2}\cdot\frac{\delta\left(\kappa_1\right)}{\delta\left(\kappa_1\right)+\gamma_1}\left[1 - B^*(\delta(\kappa_1)+\gamma_1)\right]\right\}^{-1},$$

$$R_1(\kappa_1) = R_0(\kappa_1)\frac{\delta\left(\kappa_1\right)}{\delta\left(\kappa_1\right)+\gamma_1}\cdot\left[1 - B^*(\delta(\kappa_1)+\gamma_1)\right],$$

$$R_2(\kappa_1) = \frac{1}{\gamma_2}\left[\gamma_0 R_0(\kappa_1) + \gamma_1 R_1(\kappa_1)\right].$$

4.3 Stochastic Simulation of Special Systems

In paper [47] systems with not only gamma distributed service times but also gamma distributed inter-arrival and gamma distributed retrial times have been investigated.

The Effect of Breakdowns Disciplines. In paper [46] the $M/G/1//N$ and $G/M/1//N$ systems were investigated with exponentially distributed operating and repair times. In case of a server failure two operation modes are considered:

- The interrupted request gets into the orbit instantaneously.
- The service of the interrupted request is suspended and it continues after repairing the server.

As it was expected the second operation mode results lower mean sojourn times and higher mean successful service times. The Figures are similar to the cases treated earlier that is why they are omitted.

5 Conclusion

In this paper tool supported, numerical, simulation and asymptotic methods were considered under the condition of unlimited growing number of sources in a finite-source retrial queue with collisions of customers and an unreliable server.

During the survey several cases and examples were treated and the results of different approaches were compared to each other showing the advantages and disadvantages of the given method. Tables and Figures were collected to illustrate some special features of these systems. In the near future the two research groups would like to continue their investigations in this direction including systems with impatient customers, systems embedded in a random environment, systems with two-way communications, just to mention some alternative generalizations.

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