RETRIAL QUEUES IN THE PERFORMANCE MODELING OF CELLULAR MOBILE NETWORKS USING MOSEL*

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Abstract: This paper investigates a multiserver infinite-source retrial queueing system for the performance modeling of cellular mobile communication networks.

The objective is to demonstrate how performance tool MOSEL (Modeling, Specification and Evaluation Language) can be efficiently used in the modeling of cell based networks. In our analysis the blocked and dropped users are treated separately, that is they redial with different probabilities and different rates, with reducing the state space by maximizing the number of redialing customers with appropriately large values (i. e. when the ignored probability mass can be neglected). The guard channel scheme is included in the model, too. The novelty of our analysis is that not only the active but also both types of redialing customers are allowed to depart to other cells, which was not the case in the previous works.

The model description is translated step by step into the description language of MOSEL, and then it is automatically converted into the other tool-specific system descriptions and analyzed by the appropriate tools.

As the benefit of the tool the effects of various system parameters on the fresh call blocking probability, on the handoff call dropping probability and on the grade of service are displayed and analyzed graphically.

Keywords: Retrial queues, Cellular systems, Performance tools, MOSEL tool, Grade of service.

1 INTRODUCTION

Queueing network models are widely used in the traffic modeling of cellular mobile systems, such as GSM (Global System for Mobile Communications), GPRS (General Packet Radio Service) and UMTS (Universal Mobile Telecommunication System). Most of the papers consider queueing systems without retrials (see [Litjens and Boucherie, 2003; Dharmaraja et al, 2003] and references therein for some recent results), but after the study of Tran-Gia and Mandjes [Tran-Gia and Mandjes, 1997], which demonstrated in the context of cellular systems that the retrial phenomenon is not neglectable because of the significant negative influence on the system performance measures, authors more and more take it into consideration in their cellular mobile network model.

The main characteristic of retrial queues (or queueing systems with repeated attempts) is that if an arriving customer finds all servers busy, he leaves the service area, but after some random time repeats his demand. For some fundamental results on retrial queues, see for example: [Falin and Templeton, 1997; Artalejo, 1999].

Cellular systems with customer redials are treated in [Marsan et al, 2001], where an approximate technique is proposed for finite and infinite population Markovian models. The authors reduce the state space of the continuous-time Markov chain model by registering only that if there are retrying blocked and dropped customers in the system or not. In the works [Onur et al, 2002; Alfa and Li, 2002], various infinite-source queueing models are studied. In [Onur et al, 2002], not only customer redials, but also automatic retrials by the cellular system are taken into consideration, but the dropped customer redials handled as generating new fresh call attempts in the new cell and in case of blocking the call is treated as a blocked fresh

call. It is probably less realistic, because an interrupted customer may try to reestablish the call with higher probability in shorter time intervals. In [Alfa and Li, 2002], the blocked new and dropped handoff calls are not distinguished, but the involved random variables have general phase type distributions.

In this paper, we discuss an infinite-source retrial queueing model of GSM networks, based upon to the ones that were studied by [Tran-Gia and Mandjes, 1997; Marsan et al, 2001; Onur et al, 2002; Alfa and Li, 2002]. We calculate the main system measures quite easily using the efficient software tool MOSEL, developed at the University of Erlangen, Germany (see [Begain et al, 2001, 2003]). It makes possible to analyze more difficult models, what often not feasible because of the largeness of the state space and the difficulty of the calculations. The blocked and dropped users are treated separately, that is they redial with different probabilities and different rates, like in [Marsan et al, 2001], but we reduce the state space by maximizing the number of redialing customers with appropriately large values (i. e. when the ignored probability mass can be neglected). In [Tran-Gia and Mandjes, 1997; Onur et al, 2002; Alfa and Li, 2002], these two types of redialing customers were not distinguished. Furthermore, in our model we allow not only the active but also both types of redialing customers to depart to other cells, what was not allowed in the previous works. The current study can be considered as an initial step towards the analysis of more complex third generation systems focusing on the quality of service issues.

In cellular networks, the most important quality of service measures are the following:

- the fresh call blocking probability (P_f), i. e. the fraction of new call requests in the cell that cannot be served due to the lack of free channels, and

- the handoff call dropping probability (P_h), that is the average fraction of incoming handoff calls that are terminated because of the lack of free channels.

The grade of service (GoS) is generally defined as the combination of these two probabilities, for example as

$$GoS = \frac{P_f + 10P_h}{11}.$$

Because of the fact, that the handoff call dropping probability has more significant impact on the grade of service, it is important to reduce it even at the expense of increased fresh call blocking probability. In order to prioritize handoff calls, several channel allocation schemes are utilized. One of the most popular policies is the guard channel scheme [Dharmaraja et al, 2003; Tran-Gia and Mandjes, 1997; Marsan et al,

2001; Alfa and Li, 2002], where some channels are reserved for the calls that move across the cell boundary, that is if there are g reserved channels in the cell, a new fresh call is only accepted if there are at least $g + 1$ available channels. A handoff call is rejected only if all the channels in the cell are occupied.

The paper is organized as follows. In Section 2, the accurate description of the cellular model is given, and in Section 3 it is shown how it can be translated into the description language of MOSEL. Section 4 is devoted to some numerical examples, where the analytical results of the calculations are displayed graphically to demonstrate the effect of the changing of various system parameters on the quality of service measures and on the grade of service. Conclusions and directives for the future work are given in Section 5.

2 MODEL DESCRIPTION

In this section we consider the following cell model (illustrated by Figure 1) in a cellular mobile network.

In our cellular network model we treat only one cell. The cells are considered identical and to have the same traffic parameters, so it is enough to investigate one cell, and the handoff effect from the adjacent cells to this cell and from this cell to adjacent cells is described by handoff processes. Instead of the frequently used single arrival stream model we distinguish the fresh call and handoff call arrivals, what is gainful if we investigate complex call handling policies.

We assume, that the number of channels in the cell is C, and the number of guard channels is g, where $g < C$.

The arrival process of the fresh calls is a Poisson process with rate λ_f. If the number of the active users is smaller than $C - g$, the incoming call starts to be served. Otherwise it is blocked and it starts generation of a Poisson flow of repeated calls (redialing) with probability Θ_1 or leaves the system with probability $1 - \Theta_1$. A blocked customer repeats his call after a random time which is exponentially distributed with mean $1/\nu_{bl}$, and it can be served or blocked again like the fresh calls. The call duration time is exponentially distributed with mean $1/\mu$.

The arrival process of the handoff calls is a Poisson process with rate λ_h. If the number of active users is smaller than C, the incoming call starts to be served. Otherwise it is dropped (handoff failure) and it starts generation of a Poisson flow of repeated calls with probability Θ_2 or leaves the system with probability $1 - \Theta_2$. A dropped customer tries to repeat his call after a random time which is exponentially distributed with mean $1/\nu_{dr}$. If it is blocked it continues redialing with probability Θ_2. The call duration time for handoff calls is also exponentially distributed with

mean $1/\mu$.

The active, redialing blocked and dropped customers leave the cell after an exponentially distributed time with mean $1/\mu_a$, $1/\mu_b$ and $1/\mu_d$, respectively.

The number of redialing users because of blocking

and dropping is limited to an appropriately large values of N_{bl} and N_{dr} to make the state space finite in order to make the calculations possible by the tools in the steady state.
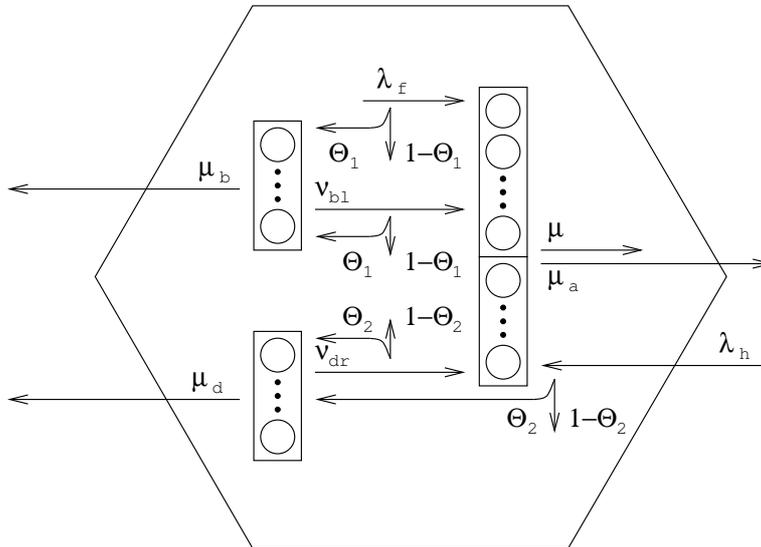

Figure 1: Retrial queueing model of a cell

2.1 The Underlying Markov Chain

The state of the system can be described with a stochastic process $X(t) = (C(t); N(t); M(t))$, where $C(t)$ is the number of active customers (i. e. the number of busy channels), $N(t)$ is the number of blocked new customers who are sending repeated calls and $M(t)$ is the number of dropped customers at handoff who are trying to redial at time t.

Because of the exponentiality of the involved random variables the describing process is a Markov chain with a finite state space $S = \{0, ..., C\} \times \{0, ..., N_{bl}\} \times \{0, ..., N_{dr}\}$. Since its state space is finite, the process is ergodic for all values of the rate of the arrival of new and handoff calls, and we can investigate it in the steady state.

We define the stationary probabilities:

$$P(i; j; k) = \lim_{t \to \infty} P(C(t) = i, N(t) = j,$$
$$M(t) = k),$$
$$i = 0, ..., C, \quad j = 0, ..., N_{bl}, \quad k = 0, ..., N_{dr}.$$

Because of the fact that the state space of $(X(t), t \geq 0)$ with sufficiently large N_{bl} and N_{dr} is very large and the functioning of the system is complex, it is very difficult to calculate the steady state probabilities. To simplify these calculations and to make our study more usable in practice, we use the software tool MOSEL to formulate the model and to calculate

these probabilities and the system measures. MOSEL has already been used, and it has proved its applicability for the modeling of several computer and communication systems. For some examples about computer systems see [Almási et al, 2001] (which results are identical to the calculated results of [Almási, 1999]), [Zreikat et al, 2003; Almási et al, 2004] and in the context of cellular systems [Begain et al, 2003, 1999; Li et al, 2003]. The MOSEL description can be translated automatically into the language of various performance tools and then analyzed by the appropriate tools (at present SPNP – Stochastic Petri Net Package and TimeNET are supported and suitable for this model) to get these measures.

Knowing the steady state probabilities the system performance and the quality of service measures can be obtained as follows.

- *The mean number of active customers*

$$N_c = \sum_{i=0}^{C} \sum_{j=0}^{N_{bl}} \sum_{k=0}^{N_{dr}} iP(i, j, k).$$

- *The mean number of sources of repeated calls because of the blocking of fresh calls*

$$N_{bl} = \sum_{i=0}^{C} \sum_{j=0}^{N_{bl}} \sum_{k=0}^{N_{dr}} jP(i, j, k).$$

- *The mean number of sources of repeated calls because of the dropping of handoff calls*

$$N_{dr} = \sum_{i=0}^{C} \sum_{j=0}^{N_{bl}} \sum_{k=0}^{N_{dr}} k P(i,j,k).$$

- *Fresh call blocking probability*

$$P_f = \sum_{i=0}^{g} \sum_{j=0}^{N_{bl}} \sum_{k=0}^{N_{dr}} P(C-i,j,k).$$

- *Handoff call dropping probability*

$$P_h = \sum_{j=0}^{N_{bl}} \sum_{k=0}^{N_{dr}} P(C,j,k).$$

3 MODEL CONVERSION TO MOSEL

In this section we discuss the translation of the model into the language of the MOSEL tool. The full MOSEL program can be assembled from the following program parts among the model description in the order of the part numbers.

The number of channels in the cell is C, which is denoted as *N_CHS* in the program, and the number of guard channels is g, which is denoted as *N_G_CHS*.
In the first part of the MOSEL description, we have to define some other system parameters too, these will be introduced at the appropriate program parts.

(1) CONST N_CHS := 15;
     CONST N_G_CHS := 1;
     CONST MAX_BL_USERS := 25;
     CONST MAX_DR_USERS := 25;
     CONST call_arrive := 1.5;
     CONST call_retry_bl := 5;
     CONST call_retry_dr := 6;
     CONST call_duration := 0.05;
     CONST handoff_arrive := 0.4;
     CONST handoff_dep_ac := 1/3;
     CONST handoff_dep_bl := 1/3;
     CONST handoff_dep_dr := 1/3;
     CONST p_retry_bl := 0.7;
     CONST p_retry_dr := 0.9;

The state of the system is described by the number of active users, the number of blocked users who redial after some random time, and the number of users whose calls are dropped at handoff and who are redialing. It can be wrote down in MOSEL as defining the nodes of the system. The number of active users is denoted by *active_users*. Its maximum value is the number of channels, and it is 0 at the starting time. The number of redialing users because of blocking and dropping is limited to *MAX_BL_USERS* and *MAX_DR_USERS*, which are defined in (1).

(2) NODE active_users[N_CHS] := 0;
     NODE redialing_users_bl[MAX_BL_USERS]
                                    := 0;
     NODE redialing_users_dr[MAX_DR_USERS]
                                    := 0;

The arrival process of the fresh calls is a Poisson process with rate λ_f, that is denoted in the program as *call_arrive*, and defined in (1) like the other parameters. If the number of active users is smaller than $C - g$, the incoming call starts to be served. Otherwise it is blocked and it starts generation of a Poisson flow of repeated calls (redialing) with probability Θ_1 (denoted by p_retry_bl) or leaves the system with probability $1 - \Theta_1$.

(3) IF active_users < N_CHS-N_G_CHS
       FROM EXTERN TO active_users
       RATE call_arrive;
     IF active_users >= N_CHS-N_G_CHS
       FROM EXTERN RATE call_arrive THEN {
       TO redialing_users_bl
                  WEIGHT p_retry_bl;
       TO EXTERN WEIGHT 1 - p_retry_bl;
     }

The blocked user redials can be handled similar to the fresh call arrivals. If a user is blocked, he repeats his call after a random time which is exponentially distributed with mean $1/\nu_{bl}$. ν_{bl} is denoted as *call_retry_bl*. It can be served or blocked as the fresh calls in the previous part.

(4) IF active_users < N_CHS-N_G_CHS
       FROM redialing_users_bl TO active_users
       RATE call_retry_bl*redialing_users_bl;
     IF active_users >= N_CHS-N_G_CHS
       FROM redialing_users_bl
       RATE call_retry_bl*redialing_users_bl
         THEN {
       TO redialing_users_bl
                  WEIGHT p_retry_bl;
       TO EXTERN WEIGHT 1 - p_retry_bl;
     }

The call duration time is exponentially distributed with mean $1/\mu$. μ is denoted as *call_duration*.

(5) FROM active_users TO EXTERN
       RATE call_duration*active_users;

The arrival process of the handoff calls is a Poisson process with rate λ_h. λ_h is denoted in the program as *handoff_arrive*. If the number of active users is smaller than C, the incoming call starts to be served. Otherwise it is dropped and it starts generation of a Poisson flow of repeated calls with probability Θ_2 (denoted by p_retry_dr) or leaves the system with probability $1 - \Theta_2$.

(6) IF active_users < N_CHS
    FROM EXTERN TO active_users
    RATE handoff_arrive;
  IF active_users = N_CHS
    FROM EXTERN RATE handoff_arrive
      THEN {
    TO redialing_users_dr
         WEIGHT p_retry_dr;
    TO EXTERN WEIGHT 1 - p_retry_dr;
  }

The dropped user redials can be handled like the blocked fresh call redials. The customer repeats his call after a random time which is exponentially distributed with mean $1/\nu_{dr}$. ν_{dr} is denoted as *call_retry_dr*. If it is blocked it continues retrying with probability Θ_2 (p_retry_dr).

(7) IF active_users < N_CHS-N_G_CHS
    FROM redialing_users_dr TO active_users
    RATE call_retry_dr*redialing_users_dr;
  IF active_users >= N_CHS-N_G_CHS
    FROM redialing_users_dr
    RATE call_retry_dr*redialing_users_dr
      THEN {
    TO redialing_users_dr
         WEIGHT p_retry_dr;
    TO EXTERN WEIGHT 1 - p_retry_dr;
  }

The active and redialing customers leave the cell after an exponentially distributed time with parameter μ_a, μ_b and μ_d, denoted as *handoff_dep_ac*, *handoff_dep_bl* and *handoff_dep_dr*, respectively.

(8) FROM active_users TO EXTERN
    RATE handoff_dep_ac*active_users;
  FROM redialing_users_bl TO EXTERN
    RATE handoff_dep_bl*redialing_users_bl;
  FROM redialing_users_dr TO EXTERN
    RATE handoff_dep_dr*redialing_users_dr;

After describing the system functioning, we can define the system measures we would like to calculate, such as the mean number of active and redialing customers because of blocking and handoff failure, the fresh call blocking and the handoff call dropping probabilities.

(9) PRINT mean_active_users =
         MEAN(active_users);
  PRINT mn_redialing_users_bl =
         MEAN(redialing_users_bl);
  PRINT mn_redialing_users_dr =
         MEAN(redialing_users_dr);
  PRINT call_blocking_prob =
    PROB(active_users >= N_CHS-N_G_CHS);
  PRINT handoff_call_dropping_prob =
         PROB(active_users = N_CHS);

Finally, we define two pictures that show the changing of the blocking and dropping probabilities depending

on the number of channels. If we use *N_CHS* as parameter, we have to define it in (1) as follows: *PARAMETER N_CHS := 6, 7, 8, 9, 10;*

(10) PICTURE "Blocking probability vs N_CHS"
    PARAMETER N_CHS
    CURVE call_blocking_prob;
  PICTURE "Dropping probability vs N_CHS"
    PARAMETER N_CHS
    CURVE handoff_call_dropping_prob;

4 NUMERICAL EXAMPLES

In this section we consider some sample numerical results to illustrate graphically how the system measures depend on variable system parameters.

In Figures 2 and 3 the fresh call blocking and handoff call dropping probabilities are displayed versus the number of channels with and without user redials. The system parameters belonging to the curves without redials are the same as in [Dharmaraja et al, 2003], where a similar model is studied without customer redials ($g = 3$, $\lambda_f = 0.5$, $\mu = 0.05$, $\mu_a = \mu_b = \mu_d = 1/3$, $\lambda_h = 0.4$, $\nu_{bl} = \nu_{dr} = 10^6$, $\Theta_1 = \Theta_2 = 10^{-6}$ and for the other curve $\nu_{bl} = \nu_{dr} = 6$, $\Theta_1 = 0.8$, $\Theta_2 = 0.9$, furthermore the maximum number of redialing customers is 25, respectively). These results are in agreement with theirs in the exponential case.

In Figures 4 and 5 the fresh call blocking and handoff call dropping probabilities are displayed versus the mean handoff call arrival rate. The system parameters are the same as in Figures 2 and 3, except of that $C = 8$, and λ_h is on the x axis, like in [Dharmaraja et al, 2003].

The negative influence of the retrial phenomenon is shown in each figures, and we can see that it increases as the handoff call arrival rate increases.

In Figure 6 we can see the fresh call blocking probability, the handoff call dropping probability and the grade of service as the mean fresh call arrival rate increases. The following system parameters were used: $C = 7$, $g = 1$, $\mu = 0.05$, $\mu_a = \mu_b = \mu_d = 1/3$, $\lambda_h = 0.4$, $\nu_{bl} = 6$, $\nu_{dr} = 7$, $\Theta_1 = 0.8$ and $\Theta_2 = 0.9$.

In Figure 7 the fresh call blocking and handoff dropping probabilities and the GoS are displayed versus the number of guard channels. We can see that a very few number of guard channels can improve the grade of service significantly, but then only very small handoff dropping advance can be achieved on the great expense of fresh call blocking probability, and the GoS declines. The system parameters are the following: $C = 15$, $\lambda_f = 3$, $\mu = 0.05$, $\mu_a = \mu_b = \mu_d = 1/3$, $\lambda_h = 0.4$, $\nu_{bl} = 6$, $\nu_{dr} = 7$, $\Theta_1 = 0.8$ and $\Theta_2 = 0.9$.
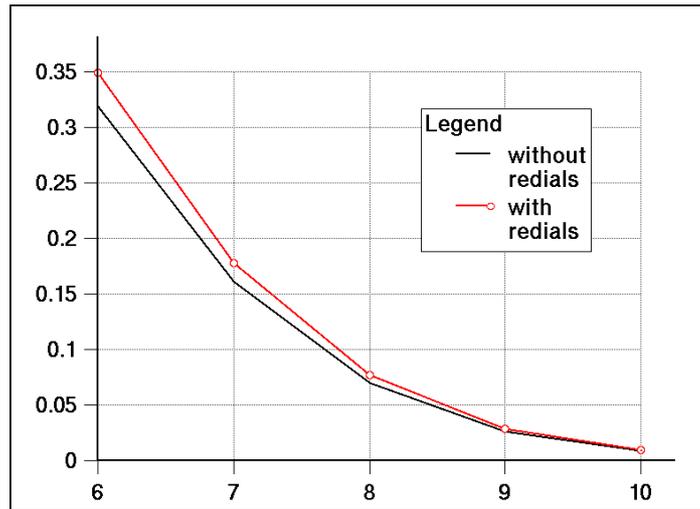
Figure 2: Fresh call blocking probability versus number of channels
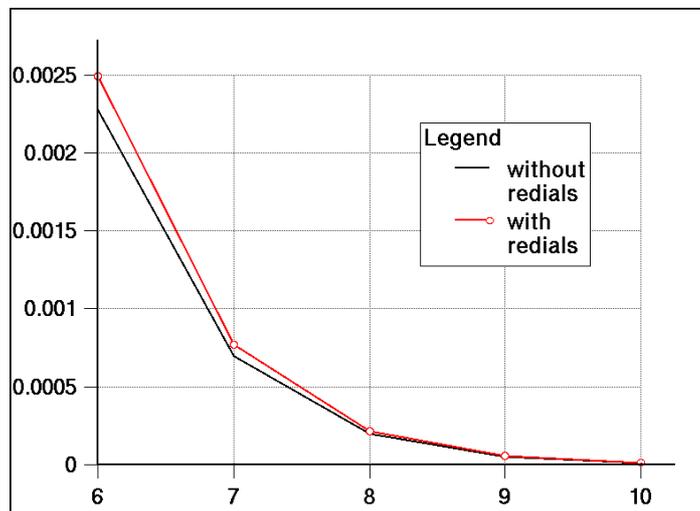

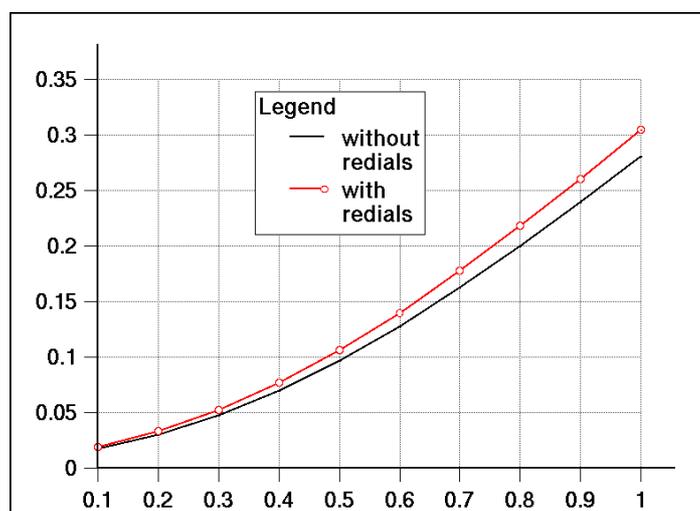Figure 3: Handoff call dropping probability versus number of channels

Figure 4: Fresh call blocking probability versus mean handoff call arrival rate
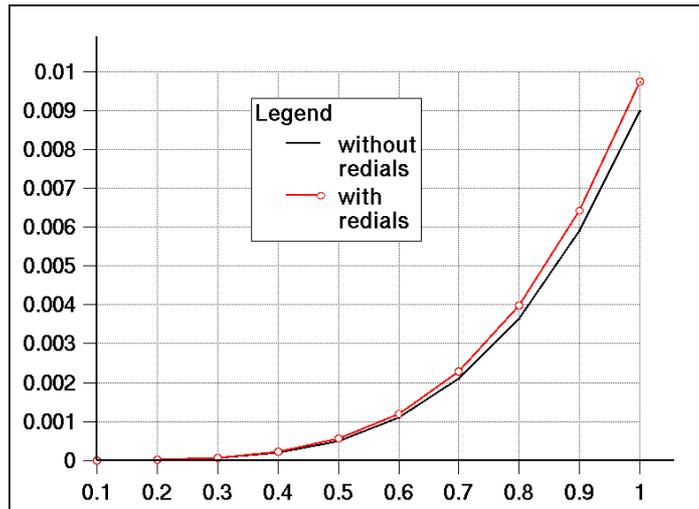
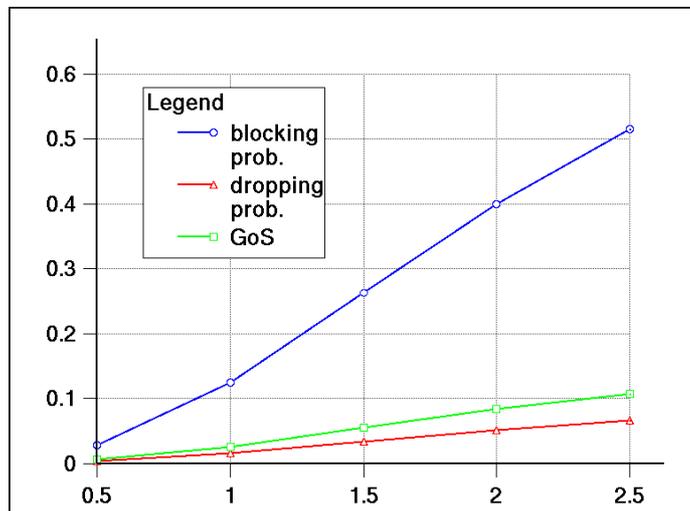Figure 5: Handoff call dropping probability versus mean handoff call arrival rate

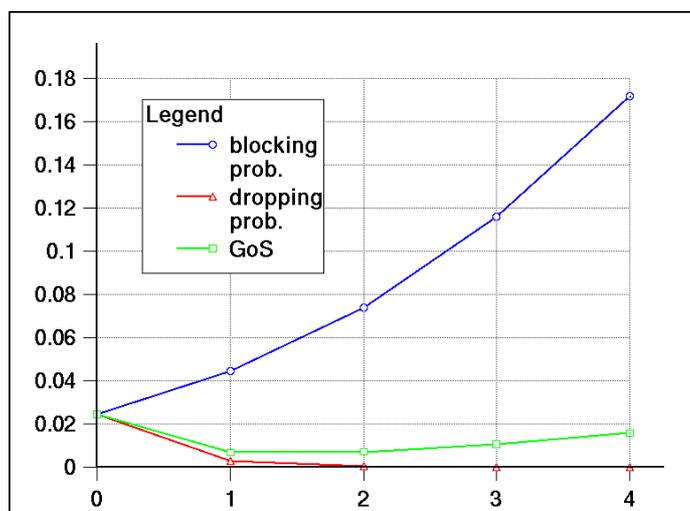Figure 6: System measures versus mean fresh call arrival rate

Figure 7: System measures versus number of guard channels

5   CONCLUSION AND FUTURE WORK

In this paper a multiserver infinite-source retrial queueing system is studied for the performance modeling of GSM networks. It is shown how easily and efficiently the tool MOSEL can be used, and some numerical examples are presented to show the the impact of the retrial phenomenon and some system parameters on the quality of service measures and on the grade of service.

The current study is an initial step towards the analysis of more complex third generation cellular systems. These hierarchical systems may consist two or more layers, and various dynamic channel allocation schemes can be utilized and analyzed. Furthermore, other than exponential distributions can be treated that are supported by both MOSEL and the applied tools.

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