Markov-Modulated Finite-Source Queueing Models in Evaluation of Computer and Communication Systems

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Abstract—The present paper deals with a first-come, first-served (FCFS) queueing model to analyze the behaviour of a heterogeneous finite-source system with a single server. The sources and the server are assumed to operate in independent Markovian environments, respectively. Each request is characterized by its own exponentially distributed source and service time with parameter depending on the state of the corresponding environment; that is, the arrival and service rates are subject to random fluctuations. Our aim is to get the usual stationary performance measures of the system, such as utilizations, mean number of requests staying at the server, mean queue lengths, average waiting, and sojourn times.

In the case of fast arrivals or fast service, asymptotic methods can be applied. In the intermediate situations, stochastic simulation is used.

As applications of this model, some problems in the field of telecommunications and reliability theory are treated. © 2003 Elsevier Science Ltd. All rights reserved.

Keywords—Finite-source queueing, Communication system, Markov-modulation, "Fast" arrival, "Fast" service.

1. INTRODUCTION

Performance modeling of recent computer and communication system development has become more complicated as the size and complexity of the system has increased (see [1]). One of the measures of greatest interest is the distribution of the time to the first system failure. It is well known that the majority of the problems can be treated with the help of semi-Markov processes (SMP). Since the failure-free operation time of the system corresponds to sojourn time problems,

we can use the results obtained for SMP. If the exit from a given subset of the state space is a "rare" event, that is, it occurs with a small probability, it is natural to investigate the asymptotic behaviour of the sojourn time in that subspace; see [2,3].

Realistic consideration of certain stochastic systems, however, often requires the introduction of a random environment, sometimes referred as to Markov-modulation, where system parameters are subjected to randomly occurring fluctuations or bursts. This situation may be attributed to certain changes in the physical environment such as weather, or sudden personal changes and work load alterations. Gaver *et al.* [4] proposed an efficient computational approach for the analysis of a generalized structure involving finite-state space birth-and-death processes in a Markovian environment.

This paper deals with a first-come, first-served (FCFS) queueing model to analyze the behaviour of a heterogeneous finite-source system with a single server. The sources and the server are supposed to operate in independent random environments, respectively, allowing the arrival and service processes to be Markov-modulated ones. Each request is characterized by its own exponentially distributed source and service time with parameter depending on the state of the corresponding environment; that is, the arrival and service rates are subject to random fluctuations. Our aim is to get the usual stationary performance measures of the system, such as utilizations, mean number of requests staying at the server, mean queue lengths, average waiting, and sojourn times. The main problem is that the state space of the underlying continuous-time Markov-chain will be very large, so we have the state space explosion problem. To avoid it in the case of "fast" arrivals or "fast" service situations, asymptotic methods can be applied. In the intermediate situations, stochastic simulation is used.

This paper generalizes the results of Sztrik and Rigo [5] where assuming "fast" arrivals the sources are homogeneous and the whole system is governed by two random environments. In the case of "fast" service, it extends the results of Sztrik [6] where the request is heterogeneous and the arrival and service rates depend on the state of two governing Markov-chains, respectively.

The technique used here is similar to the one applied in [5,7], and closely related to the theoretic investigations of Anisimov [8,9].

2. THE QUEUEING MODEL

Consider a finite-source queueing system with N heterogeneous sources and a single server. The sources and the server operate in independent random environments. The environmental changes are reflected in the values of the access and service rates that prevail at any point in time. The main objective is to adapt these parameters to respond to random changes effectively and thus maintain the derived level of system performance.

Source p is assumed to operate in a random environment governed by an ergodic Markov chain $(\xi_p(t),\ t \geq 0)$ with state space $(1, \ldots, r_p)$ and with transition density matrix

$$\left(a_{i_p j_p}^{(p)},\ i_p, j_p = 1, \ldots, r_p,\ a_{i_p i_p}^{(p)} = \sum_{k \neq i_p} a_{i_p k}^{(p)} \right).$$

Whenever the environmental process $\xi_p(t)$ is in state i_p, the probability that the source generates a request in the time interval $(t, t+h)$ is $\lambda_p(i_p, \varepsilon) + o(h)$, $p = 1, \ldots, N$. Each request is transmitted to a server where the service immediately starts if it is idle; otherwise a queueing line is formed. The service discipline is first-come, first-served (FCFS). The server is also supposed to operate in a random environment governed by an ergodic Markov chain $(\xi(t),\ t \geq 0)$ with state space $(1, \ldots, r_{N+1})$ and with transition density matrix

$$\left(a_{i_{N+1} j_{N+1}}^{(N+1)},\ i_{N+1}, j_{N+1} = 1, \ldots, r_{N+1},\ a_{i_{N+1} i_{N+1}}^{(N+1)} = \sum_{k \neq i_{N+1}} a_{i_{N+1} k}^{(N+1)} \right).$$

Whenever the environmental process $\xi_{N+1}(t)$ is in state i_p, the probability that the service of request p is completed in time interval $(t, t+h)$ is $\mu_p(i_{N+1}, \varepsilon)h + o(h)$. If a given source has sent a request, it stays idle and it cannot generate another one. After being serviced, each request immediately returns to its source and the whole procedure starts again. All random variables involved here and the random environments are supposed to be independent of each other.

In practical applications, it is very important to know the distribution of time until the server becomes idle, which is actually the busy period of the processor, or the distribution of time until the number of requests staying at the service facility reaches a certain level. In the following, we will use two special situations, namely, "fast" arrival and "fast" service assumptions to get the above-mentioned distributions, respectively.

2.1. Fast Arrival Case

Let us consider the system under the assumption of "fast" arrivals, i.e., $\lambda_p(i_p, \varepsilon) \to \infty$ as $\varepsilon \to 0$, and $\mu_p(i_{N+1}, \varepsilon) = \mu_p(i_{N+1})$. For simplicity, let $\lambda_p(i_p, \varepsilon) = \lambda_p(i_p)/\varepsilon$, $p = 1, \ldots, N$.

Denote by $Y_\varepsilon(t)$ the number of requests staying in the sources at time t, and let

$$\Omega_\varepsilon(N-1) = \inf\left\{t : t > 0, \ Y_\varepsilon(t) = \frac{N}{Y_\varepsilon(0)} \le N-1\right\},$$

that is, the case when the server becomes idle; i.e., $\Omega_\varepsilon(N-1)$ is the busy period length of the server, or processor.

Denote by $\pi_0(i_1, i_2, \ldots, i_N, i_{N+1} : 0; \ k_1, \ldots, k_N)$ the steady-state probability that random environment $\xi_p(t)$ is in state i_p, $p = 1, \ldots, N+1$, there is no request in the source, and the order of their arrivals to the server is $(k_1, \ldots, k_N)$. Similarly, denote by $\pi_0(i_1, i_2, \ldots, i_N, i_{N+1} : 1; \ k_2, \ldots, k_N)$ the steady-state probability that the pth random environment is in state i_p, $p = 1, \ldots, N+1$, source k_1 is generating a new request, and the other sources sent their requests in the order $(k_2, \ldots, k_N)$. Clearly, $(k_s, \ldots, k_N) \in V_N^{N-s+1}$, $s = 1, 2$ where V_N^{N-s+1} denotes the set of all variations of order $N - s + 1$ of elements $1, \ldots, N$. Now we have the following theorem.

THEOREM 1. *For the system in question under the above assumptions, independently of the initial state, the distribution of the normalized random variable $\varepsilon^{N-1}\Omega_\varepsilon(N-1)$ converges weakly to an exponentially distributed random variable with parameter*

$$\Lambda = \sum_{i_1=1}^{r_1} \cdots \sum_{i_{N+1}=1}^{r_{N+1}} \sum_{(k_1, \ldots, k_N) \in V_N^N} \pi_0(i_1, \ldots, i_{N+1} : 1; \ k_2, \ldots, k_N)$$

$$\times \frac{\mu_{k_2}(i_{N+1})}{\lambda_{k_1}(i_{k_1})} \times \frac{\mu_{k_3}(i_{N+1})}{\lambda_{k_1}(i_{k_1}) + \lambda_{k_2}(i_{k_2})} \times \cdots \times \frac{\mu_{k_N}(i_{N+1})}{\lambda_{k_1}(i_{k_1}) + \cdots + \lambda_{k_{N-1}}(i_{k_{N-1}})} \frac{1}{D},$$

where

$$D = \sum_{p=1}^{N+1} \sum_{j_p=1}^{r_p} \sum_{\substack{i_p=1 \\ i_p \ne j_p}}^{r_p} \sum_{(k_1, \ldots, k_n) \in V_N^N} \pi_0(i_1, \ldots, i_{N+1} : 0; \ k_1, \ldots, k_N) \times \frac{a_{i_p j_p}^{(p)}}{\left(\sum_{q=1}^{N+1} a_{i_q i_q}^{(q)} + \mu_{k_1}(i_{N+1})\right)^2}.$$

PROOF. It is similar to the one used in [5,7] and due to the limited page numbers, only the main parts are discussed.

Let us introduce the following stochastic process:

$$Z_\varepsilon(t) = \left(\xi_1(t), \ldots, \xi_{N+1}(t) : Y_\varepsilon(t); \ \beta_1(t), \ldots, \beta_{Y_\varepsilon(t)}\right),$$

where $\beta_1(t), \ldots, \beta_{Y_\varepsilon(t)}$ denote the indices of the messages in order of their arrival at the processor. It is easy to see that $(Z_\varepsilon(t), t \geq 0)$ is a multidimensional Markov chain with rather complex state space

$$E = \left((i_1, \ldots, i_{N+1} : s; \ k_1, \ldots, k_{N-s}), \ i_p = 1, \ldots, r_p, \ p = 1, \ldots, N+1, \right.$$
$$\left. (k_1, \ldots, k_{N-s}) \in V_N^{N-s}, \ s = 0, \ldots, N \right),$$

where $k_0 = \{0\}$ by definition. Furthermore, let

$$\langle \alpha_m \rangle = \left((i_1, \ldots, i_{N+1} : s; \ k_1, \ldots, k_{N-s}), \ i_p = 1, \ldots, r_p, \ p = 1, \ldots, N+1, \right.$$
$$\left. (k_1, \ldots, k_{N-s}) \in V_N^{N-s}, \ s = 0, \ldots, m \right)$$

be a subset of the states.

Let us denote by $g_\varepsilon(\langle \alpha_m \rangle)$ the steady-state probability of exit from $\langle \alpha_m \rangle$; that is,

$$g_\varepsilon(\langle \alpha_m \rangle) = \sum_{i^{(m)} \in X_m} \pi_\varepsilon\left(i^{(m)}\right) \sum_{j^{(m+1)} \in X_{m+1}} p_\varepsilon\left(i^{(m)}, j^{(m+1)}\right).$$

Hence, our aim is to determine the distribution of the first exit time of $Z_\varepsilon(t)$ from $\langle \alpha_m \rangle$, provided that $Z_\varepsilon(t) \in \langle \alpha_m \rangle$.

It can easily be verified that the transition probabilities for the embedded Markov chain are

$$p_\varepsilon[(i_1, \ldots, i_{N+1} : s; \ k_1, \ldots, k_{N-s}), (i_1, \ldots, j_p, \ldots, i_{N+1} : s; \ k_1, \ldots, k_{N-s})]$$
$$= \frac{a_{i_p j_p}^{(p)}}{\gamma_\varepsilon(i_1, \ldots, i_{N+1} : s; \ k_1, \ldots, k_{N-s})}, \qquad \text{for } s = 0, \ldots, N, \quad p = 1, \ldots, N+1,$$

$$p_\varepsilon[(i_1, \ldots, i_{N+1} : s; \ k_2, \ldots, k_{N-s}), (i_1, \ldots, i_{N+1} : s+1; \ k_1, \ldots, k_{N-s+1})]$$
$$= \frac{\mu_{k_1}(i_{N+1})}{\gamma_\varepsilon(i_1, \ldots, i_{N+1} : s; \ k_1, \ldots, k_{N-s})}, \qquad \text{for } s = 0, \ldots, N-1,$$

$$p_\varepsilon[(i_1, \ldots, i_{N+1} : s; \ k_1, \ldots, k_{N-s}), (i_1, \ldots, i_{N+1} : s-1; \ k_1, \ldots, k_{N-s-1})]$$
$$= \frac{\lambda_{k_{N-s}}(i_{k_{N-s}})/\varepsilon}{\gamma_\varepsilon(i_1, \ldots, i_{N+1} : s; \ k_1, \ldots, k_{N-s})}, \qquad \text{for } s = 1, \ldots, N,$$

as $\varepsilon \to 0$ implies

$$p_\varepsilon[(i_1, \ldots, i_{N+1} : 0; \ k_1, \ldots, k_N), (i_1, \ldots, j_p, \ldots, i_{N+1} : 0; \ k_1, \ldots, k_N)]$$
$$= \frac{a_{i_p j_p}^{(p)}}{a_{i_1 i_1}^{(1)} + \cdots + a_{i_{N+1} i_{N+1}}^{(N+1)} + \mu_{k_1}(i_{N+1})}, \qquad p = 1, \ldots, N+1,$$

$$p_\varepsilon[(i_1, \ldots, i_{N+1} : s; \ k_1, \ldots, k_{N-s}), (i_1, \ldots, j_p, \ldots, i_{N+1} : s; \ k_1, \ldots, k_{N-s})] = o(1),$$
$$\text{for } s = 1, \ldots, N, \quad p = 1, \ldots, N+1,$$

$$p_\varepsilon[(i_1, \ldots, i_{N+1} : 0; \ k_1, \ldots, k_N), (i_1, \ldots, i_{N+1} : 1; \ k_2, \ldots, k_N)]$$
$$= \frac{\mu_{k_1}(i_{N+1})}{a_{i_1 i_1}^{(1)} + \cdots + a_{i_{N+1} i_{N+1}}^{(N+1)} + \mu_{k_1}(i_{N+1})},$$

$$p_\varepsilon[(i_1, \ldots, i_{N+1} : s; \ k_1, \ldots, k_{N-s}), (i_1, \ldots, i_{N+1} : s+1; \ k_2, \ldots, k_{N-s})]$$
$$= \frac{\mu_{k_1}(i_{N+1})\varepsilon}{\sum\limits_{p \neq k_1, \ldots, k_{N-s}} \lambda_p(i_p)} (1 + o(1)), \qquad s = 1, \ldots, N-1.$$

The probabilities

$$\pi_0(i_1,\ldots,i_{N+1}:0;\ k_1,\ldots,k_N), \qquad \pi_0(i_1,\ldots,i_{N+1}:1;\ k_1,\ldots,k_{N-1}),$$

$i_p = 1,\ldots,r_p,\ p = 1,\ldots,N+1,\ (k_1,\ldots,k_{N-s}) \in V_N^{N-s},\ s = 0,1$ satisfy the following system of equations:

$$\pi_0(j_1,j_2,\ldots,j_{N+1}:0;\ k_1,\ldots,k_N) = \sum_{i_1 \neq j_1} \pi_0(i_1,j_2,\ldots,j_{N+1}:0;\ k_1,\ldots,k_N)$$

$$\times a_{i_1 j_1}^{(1)} \left(a_{i_1 i_1}^{(1)} + a_{j_2 j_2}^{(2)} + \cdots + a_{j_{N+1} j_{N+1}}^{(N+1)} + \mu_{k_1}(j_{N+1}) \right)^{-1}$$

$$+ \sum_{i_2 \neq j_2} \pi_0(j_1,j_2,\ldots,j_{N+1}:0;\ k_1,\ldots,k_N)$$

$$\times a_{i_2 j_2}^{(2)} \left(a_{j_1 j_1}^{(1)} + a_{i_2 i_2}^{(2)} + \cdots + a_{j_{N+1} j_{N+1}}^{(N+1)} + \mu_{k_1}(j_{N+1}) \right)^{-1} + \cdots \qquad (1)$$

$$+ \sum_{i_{N+1} \neq j_{N+1}} \pi_0(j_1,j_2,\ldots,i_{N+1}:0;\ k_1,\ldots,k_N)$$

$$\times a_{i_{N+1} j_{N+1}}^{(N+1)} \left(a_{j_1 j_1}^{(1)} + a_{j_2 j_2}^{(2)} + \cdots + a_{i_{N+1} i_{N+1}}^{(N+1)} + \mu_{k_1}(i_{N+1}) \right)^{-1}$$

$$+ \pi_0(j_1,\ldots,j_{N+1}:1;\ k_1,\ldots,k_{N-1}),$$

$$\pi_0(j_1,\ldots,j_{N+1}:1;\ k_1,\ldots,k_{N-1}) = \pi_0(j_1,\ldots,j_{N+1}:0;\ k_N,k_1,\ldots,k_{N-1})$$

$$\times \mu_{k_N}(j_{N+1}) \left(a_{j_1 j_1}^{(1)} + \cdots + a_{j_{N+1} j_{N+1}}^{(N+1)} + \mu_{k_N}(j_{N+1}) \right)^{-1}. \qquad (2)$$

To apply the results of [8,9], we need the solution of (1),(2) with normalizing condition

$$\sum_{p=1}^{N+1} \sum_{i_p=1}^{r_p} \sum_{(k_1,\ldots,k_n)} \{ \pi_0(i_1,\ldots,i_{N+1}:0;\ k_1,\ldots,k_N) \qquad (3)$$
$$+ \pi_0(i_1,\ldots,i_{N+1}:1;\ k_1,\ldots,k_{N-1}) \} = 1.$$

Suppose that we have this solution. Then, we get

$$g(\langle \alpha_m \rangle) = \varepsilon^m \sum_{i_1=1}^{r_1} \cdots \sum_{i_{N+1}=1}^{r_{N+1}} \sum_{(k_1,\ldots,k_N) \in V_N^N} \pi_0(i_1,\ldots,i_{N+1}:1;\ k_2,\ldots,k_N)$$

$$\times \frac{\mu_{k_2}(i_{N+1})}{\lambda_{k_1}(i_{k_1})} \times \frac{\mu_{k_3}(i_{N+1})}{\lambda_{k_1}(i_{k_1}) + \lambda_{k_2}(i_{k_2})} \times \cdots \times \frac{\mu_{k_{m+1}}(i_{N+1})}{\lambda_{k_1}(i_{k_1}) + \cdots + \lambda_{k_m}(i_{k_m})} (1+o(1)).$$

Taking into account the exponentiality of $\tau_\varepsilon(i_1,\ldots,i_{N+1}:s;\ k_1,\ldots,k_{N-s})$ for fixed Θ, we have

$$\mathbb{E} \exp\{i\varepsilon^m \Theta \tau_\varepsilon(i_1,\ldots,i_{N+1}:0;\ k_1,\ldots,k_{N-s})\}$$

$$= 1 + \varepsilon^m \frac{i\Theta}{a_{i_1 i_1}^{(1)} + \cdots + a_{i_{N+1} i_{N+1}}^{(N+1)} + \mu_{k_1}(i_{N+1})} (1+o(1)).$$

Notice that $\beta_\varepsilon = \varepsilon^m$, and therefore, we immediately get that $\varepsilon^m \Omega_\varepsilon(m)$ converges weakly to an exponentially distributed random variable with parameter

$$\Lambda = \sum_{i_1=1}^{r_1} \cdots \sum_{i_{N+1}=1}^{r_{N+1}} \sum_{(k_1,\ldots,k_N) \in V_N^N} \pi_0(i_1,\ldots,i_{N+1}:1;\ k_2,\ldots,k_N)$$

$$\times \frac{\mu_{k_2}(i_{N+1})}{\lambda_{k_1}(i_{k_1})} \times \frac{\mu_{k_3}(i_{N+1})}{\lambda_{k_1}(i_{k_1}) + \lambda_{k_2}(i_{k_2})} \times \cdots \times \frac{\mu_{k_{m+1}}(i_{N+1})}{\lambda_{k_1}(i_{k_1}) + \cdots + \lambda_{k_m}(i_{k_m})} \frac{1}{D}. \qquad (4)$$

Let $m = N - 1$, from which our statement immediately follows. ∎

It should be noted that this asymptotic approach considerably reduces the calculations since to solve the system of equations (1),(2) is much easier than to get the stationary distribution of $(Z_\varepsilon(t), t \geq 0)$ and system of stochastic relations concerning its sojourn time in $\langle \alpha_m \rangle$. Notice if $\mu_p(i_{N+1}) = \mu(i_{N+1})$, $p = 1, \ldots, N$, a closed form solution can be obtained.

Denote by $(\pi_{i_p}^{(p)}, i_p = 1, \ldots, r_p)$ the steady-state distribution of the governing Markov chains $(\xi_p(t), t \geq 0)$, $p = 1, \ldots, N+1$, respectively. It can be verified that the solution of (1),(2) is

$$\pi_0(j_1, j_2, \ldots, j_{N+1} : 0;\ k_1, \ldots, k_N) = B\left(\pi_{j_1}^{(1)} \cdots \pi_{j_{N+1}}^{(N+1)}\right)\left(a_{j_1 j_1}^{(1)} + \cdots + a_{j_{N+1} j_{N+1}}^{(N+1)} + \mu(j_{N+1})\right),$$

$$\pi_0(j_1, \ldots, j_{N+1} : 1;\ k_1, \ldots, k_{N-1}) = B\left(\pi_{j_1}^{(1)} \cdots \pi_{j_{N+1}}^{(N+1)}\right)\mu(j_{N+1}),$$

where B is the normalizing constant; i.e.,

$$\frac{1}{B} = N! \sum_{i_1=1}^{r_1} \cdots \sum_{i_{N+1}=1}^{r_{N+1}} \left(\pi_{i_1}^{(1)} \cdots \pi_{i_{N+1}}^{(N+1)}\right)\left(a_{i_1 i_1}^{(1)} + \cdots + a_{i_{N+1} i_{N+1}}^{(N+1)} + 2\mu(i_{N+1})\right).$$

Thus, from (4), it follows that $\varepsilon^{N-1}\Omega(N-1)$ converges weakly to an exponentially distributed random variable with parameter

$$\Lambda = \frac{1}{N!} \sum_{i_1=1}^{r_1} \cdots \sum_{i_{N+1}=1}^{r_{N+1}} \sum_{(k_1,\ldots,k_N) \in V_N^N} \left(\pi_{i_1}^{(1)} \cdots \pi_{i_{N+1}}^{(N+1)}\right)\mu(i_{N+1})$$

$$\times \frac{\mu(i_{N+1})}{\lambda_{k_1}(i_{k_1})} \times \frac{\mu(i_{N+1})}{\lambda_{k_1}(i_{k_1}) + \lambda_{k_2}(i_{k_2})} \times \cdots \times \frac{\mu(i_{N+1})}{\lambda_{k_1}(i_{k_1}) + \cdots + \lambda_{k_{N-1}}(i_{k_{N-1}})}. \tag{5}$$

Thus, from (5), we get that the busy period length of the server is asymptotically an exponentially distributed random variable with parameter

$$\varepsilon^{N-1}\Lambda = \varepsilon^{N-1}\frac{1}{N!} \sum_{i_1=1}^{r_1} \cdots \sum_{i_{N+1}=1}^{r_{N+1}} \sum_{(k_1,\ldots,k_N) \in V_N^N} \left(\pi_{i_1}^{(1)} \cdots \pi_{i_{N+1}}^{(N+1)}\right)\mu(i_{N+1})$$

$$\times \frac{\mu(i_{N+1})}{\lambda_{k_1}(i_{k_1})} \times \frac{\mu(i_{N+1})}{\lambda_{k_1}(i_{k_1}) + \lambda_{k_2}(i_{k_2})} \times \cdots \times \frac{\mu(i_{N+1})}{\lambda_{k_1}(i_{k_1}) + \cdots + \lambda_{k_{N-1}}(i_{k_{N-1}})}. \tag{6}$$

In the case when there are no random environments, that is $\mu(i_{N+1}) = \mu$, $\lambda_p(i_p) = \lambda_p$, $i_p = 1, \ldots, r_p$, $p = 1, \ldots, N$, from (6), we get

$$\varepsilon^{N-1}\Lambda = \varepsilon^{N-1}\frac{1}{N!} \sum_{(k_1,\ldots,k_N) \in V_N^N} \mu \frac{\mu}{\lambda_{k_1}} \frac{\mu}{\lambda_{k_1} + \lambda_{k_2}} \times \cdots \times \frac{\mu}{\lambda_{k_1} + \cdots + \lambda_{k_{N-1}}}. \tag{7}$$

Finally, for the totally homogeneous case from (7), we obtain

$$\varepsilon^{N-1}\Lambda = \frac{1}{(N-1)!} \frac{\mu^N}{(\lambda/\varepsilon)^{N-1}}.$$

The utilization U_r of the receiver, which is the long-run fraction of time during which it is busy, can be given by

$$U_r = \frac{1}{\varepsilon^{N-1}\Lambda}\left(\frac{1}{\varepsilon^{N-1}\Lambda} + I\right)^{-1},$$

where I denotes its idle period length.

2.2. Fast Service Case

In this section, let us assume that the service is "fast", that is, $\mu_p(i_{N+1}, \varepsilon) \to \infty$ as $\varepsilon \to 0$, and $\lambda_p(i_p, \varepsilon) = \lambda_p(i_p)$. For simplicity, let $\mu_p(i_{N+1}, \varepsilon) = \mu_p(i_{N+1})/\varepsilon$, $p = 1, \ldots, N$. Our aim is to get the distribution of time until the number of requests staying at the service facility reaches a certain level.

In this case, denote by $Y_\varepsilon(t)$ the number of requests staying at the service facility at time t, and let

$$\Omega_\varepsilon(m) = \inf \left\{ t : t > 0, \ Y_\varepsilon(t) = m + \frac{1}{Y_\varepsilon(0)} \le m \right\}.$$

The following theorem has a practical importance in the field of reliability theory where $Y_\varepsilon(t)$ can be interpreted as the failure-free operational time of the system.

THEOREM 2. *For the system in question under the above assumptions, independently of the initial state, the distribution of the normalized random variable $\varepsilon^m \Omega_\varepsilon(m)$ converges weakly to an exponentially distributed random variable with parameter*

$$\Lambda = \sum_{i_1=1}^{r_1} \cdots \sum_{i_{N+1}=1}^{r_{N+1}} \sum_{(k_1,\ldots,k_{m+1}) \in V_N^{m+1}} \left(\prod_{p=1}^{N+1} \pi_{i_p}^{(p)} \right) \frac{\prod\limits_{s=1}^{m+1} \lambda_{k_s}(i_{k_s})}{\prod\limits_{s=1}^{m} \mu_{k_1}(i_{N+1})}.$$

PROOF. It is similar to the one used in [6] and the above analysis, and thus it is omitted. ∎

As a consequence of this theorem

$$p(\Omega_\epsilon(m) > t) = P\left(\epsilon^m \Omega(\epsilon(m) > \epsilon^m t\right) \approx \exp\left(-\epsilon^m \Lambda t\right);$$

that is, $\Omega_\epsilon(m)$ is asymptotically exponentially distributed with parameter $\epsilon^m \Lambda$.

2.3. Stochastic Simulation

As it can be seen, the above-discussed two special cases result in rare events, since in both situations we get events of very small probabilities. It is also well known that simulation of rare events is quite complicated and time consuming. Even so, the validation of the asymptotic results by simulation is in progress.

In the intermediate situations, that is when the arrival and service rates are in the same scales for moderate source size under different service disciplines, a tool has recently been developed (see [10]), by the help of which not only exponentially distributed source and service times, but also Coxian ones can be investigated. By using the Law-Carson algorithm (see [11]), confidence intervals for the main performance measures are obtained, such as utilizations, mean number of requests staying at the server, mean queue lengths, average waiting, and sojourn times.

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