Modeling and Simulation of Markov Modulated Multiprocessor Systems with Petri Nets

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Abstract

Markov modulated processes are very often used to model different arrival processes of multiprocessor systems.

In this paper a heterogeneous multiprocessor system is presented, which was analyzed and validated by many different techniques. The different job arrivals of the system are presented using stochastic Petri nets. The calculated performance measures are validated against system simulation.

The major problem of each Markov model is the size of the state space which grows exponentially in the system complexity.

1 Introduction/Motivation

In the paper of [Kelling93], the determination of an initial checkpoint of the spectral variance analysis is considered and a model to validate the presented algorithm is described. Only some algorithms could be applied to that model because it is a MMPP/D/1/K system (**M**arkov **M**odulated **P**]**oisson Process). The Poisson arrival process of this system represents the traffic conditions in a network. The model is described using a deterministic stochastic Petri net. The notation distinguishes between timed and immediate firing transitions. Deterministic transitions are represented by filled rectangles while exponentially distributed transitions are represented by blank rectangles. Immediate transitions are usually represented by thin lines.**

Figure 1 shows a DSPN of the M/D/1/K - queueing system.

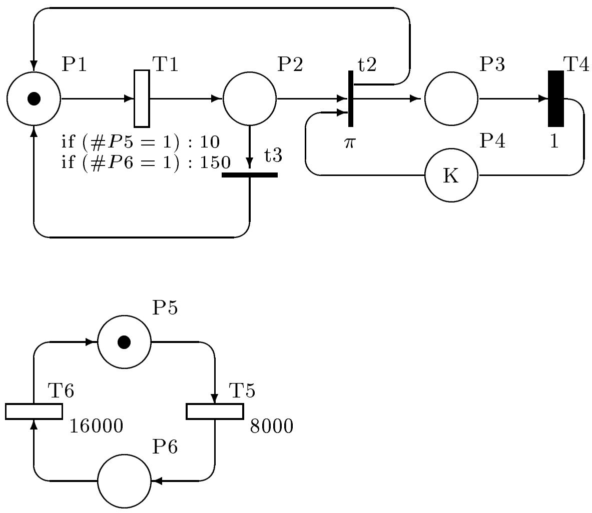

Figure 1: DSPN of the M/D/1/K-Queueing-System

The first subnet, consisting of the places P5 and P6 and the transitions T5 and T6 represents the two different states of the Markov modulated arrival process. The second subnet, consisting of P1, P2, T1, t2 and t3, models the Poisson distributed arrival process. The transition rate of T1 depends on the marking of the places P5 and P6 giving the possibility to describe different traffic intensities because the rates of the transitions T5 and T6 characterize the duration of the different phases of the Markov chain.

The server is shown in Figure 1 and consists of the places P3, P4 and transition T4. As long as P4 has at least one mark which means that there are still resources available, an arriving request can be served.

Transition t2 has higher priority than t3. Therefore t3 can only fire if t2 is disabled (i.e. there is no token in P4 and/or all marks are in P3 and no service station is available).

The subnet consisting of T5, T6, P5, P6 models the Markov modulation by activating T1 for the next arriving job.

The traffic intensity of the network is indicated by two different values which are exponentially distributed with mean times 10 and 150 respectively.

The variable of interest in this system is the mean number of jobs waiting for service. This measure is given by the number of marks in P3.

2 Analysis of Multiprocessor Systems

Now we will present a multiprocessor system with a common storage, N different processors and only one bus. The real traffic intensities in this model will be simulated by different Markov modulated processes, using a stochastic Petri subnet as modulator.

A simple system specification looks as follows:

Each of the N processors, which are connected with one common bus, sends requests to the common memory using the bus.

Figure 2 shows a simple model of this multiprocessor system.

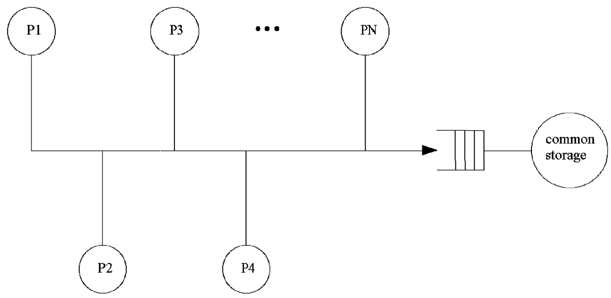

Figure 2: Multiprocessorsystem with common storage

The incoming requests are processed according to FCFS. Therefore the protocol assigns that processor to the bus, which sends the first request. There is at most one request per processor in the system. The utilization of the processors is close to 1.

For a better overview a queueing model of the system is presented as shown in Figure 3.

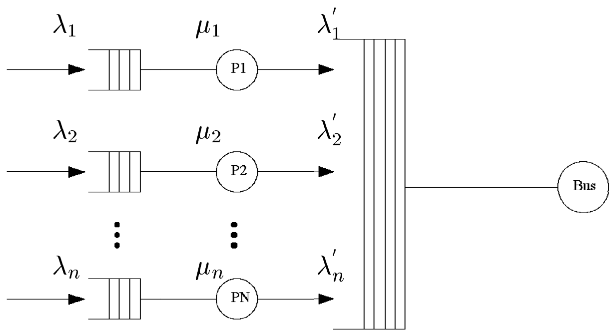

Figure 3: Queueing - model for n processors

The request generation time is assumed to be exponentially distributed as well as the bus allocation time.

All random variables are assumed to be istochastically independent and each processor is characterized by its own arrival- and service rate.

Examples for exponentially distributed times that are used to model Markov modulated processes are:

1. The mean bus service time.

2. The mean time between processor requests.

The restriction of the model is that only one bus request per processor can appear at one time. Furthermore the bus selection rule is FIFO.

3 Analysis with SPN

From the above mentioned conditions a Petri net can be given as shown in this section. We assume that each processor contains exactly one job.

Because of the complexity of the stochastic Petri net for n processors, we choose a hierarchical structure.

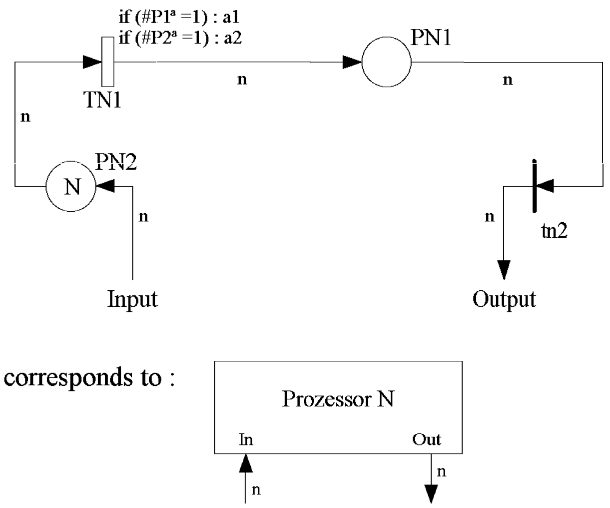

Figure 4: Hierarchical structure of the SPN for n processors

In Figure 5 the stochastic Petri net and in Figure 6 the subnets, which influences the firing rates of some transitions, are presented.

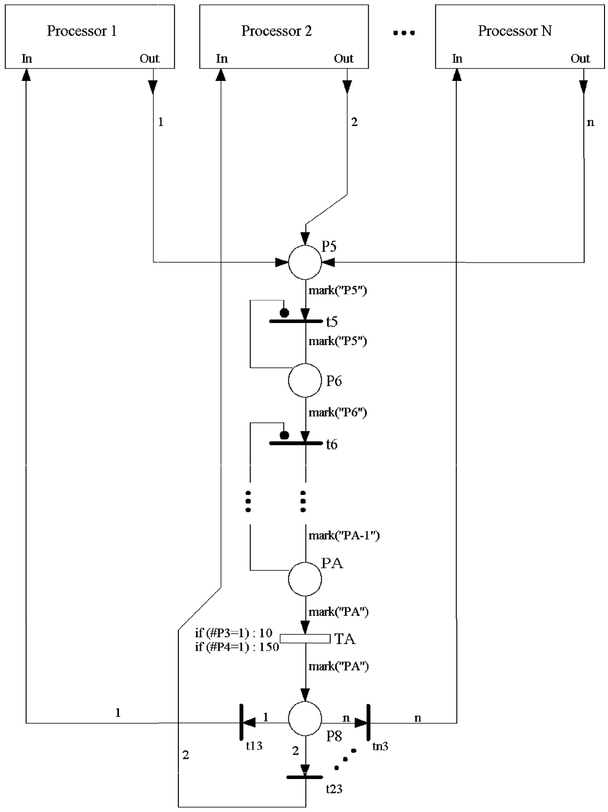

Figure 5: DSPN for n processors

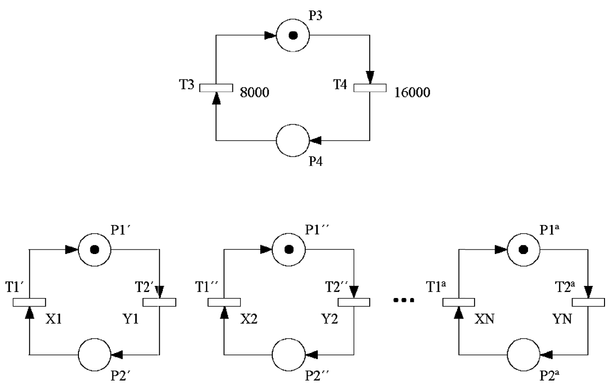

Figure 6: Subnets of the DSPN for n processors

The subnets, consisting of the states PN1 and PN2 and the transition TN1 and tn2 respectively, represent one processor.

The number of tokens in PN2 represent the number of processor. The subnet of the first processor initially contains one mark in PN2, the subnet for the second processor two marks and so on.

The token (one or several, depending on the processor-subnet) in PN2 cause the activation of transition TN1. This corresponds to the arrival of a request and the generation of a request by the processor.

The transition TN1 is modeled by the corresponding subnet, existing of $P1^a, P2^a$ and $T1^a, T2^a$. If the token in this subnet is in the place $P1^a$ the transition fires with the rate $1/a1$ and in the other case with the rate $1/a2$. The times a1, a2 and all delay times of the transitions in the subnets are exponentially distributed.

If one mark is in the place PN1, i.e. a request was generated to occupy the bus, transition tn2 fires with the corresponding multiplicity, as output to P5. The states P5 to PA and their corresponding transitions represent a queue for not yet processed requests. Each processor has a maximum of one request in the system and therefore this queue consists of exactly n states and n-1 transitions.

The amount of tokens in the states of the queue represents the corresponding processor subnet. If, for instance "Processor3" generates a request, then P5 contains exactly three token, and in the case of "Processor2" exactly two.

If the subsequent place in the queue, in our case P6, is empty, transition t5 fires with a multiplicity that corresponds to the amount of tokens in P5. This guarantees, that all token are removed from their prior place and/or arrive at the next place.

If the marks arrive at PA, the request of the corresponding processor is processed immediatelly.

Transition TA, which is modeled by a subnet, existing of P3, P4, T3 and T4, is now activated. If the mark is in P3, TA fires with an exponentially distributed time of 10, and in the other case with 150. These rates model the time, in which the bus is occupied by the following request.

The token arrive now from TA at P8. The transitions t13 to tn3 are responsible for moving the token back to their corresponding subnets, e.g. T12 is activated, if there are exactly two token in P8. The enabling-function of these transitions therefore depends on the amount of tokens, which are in P8. If for example t12 was activated, the token return to the "Processor-Subnet" (P22), transition T12 (i.e. the processor 2) becomes active and is able to generate a new request.

3.1 Analysis with SPNP

SPNP (Stochastic Petri Net Package) is a tool to analyze stochastic Petri nets ([Ciardo91]).

The presented Petri net model was analyzed in three steps:

1. Analysis of the described model:

   The calculation of performance measures for the Petrinet was only possible for up to four processors because of the state space explosion.

   For each new processor, we have to add another subnet to control the rate of the "Processor-Transition", and we have to expand the queue by one place and one transition.

2. Calculation of the net with an uniform environment process:

   To reduce the state space the subnets which form the rates of the processors 2 to x were not modeled.

   The net consists now of P1', P2' and/or T1' and T2' which controls all "Processor-transitions". Of course the multiprocessor system still consists of heterogeneous processors whose exponential distributed service times depend only on one environment process.

   The analysis of the system can be done for models with up to six processors.

3. Analysis of the model with homogeneous processors:

   The following calculation is nearly identical to the last point. The only difference is, that the transitions which symbolize the processors are homogeneous.

The analysis can be done with systems which contain up to six processors.

In close cooperation with Mr. Sczittnick (University of Dortmund, Germany) the existing model was calculated using HIT (The Hierarchical Evaluation Tool [Haring94]). These results, the results of SPNP and the values of Dr. Sztrik's paper ([Sztrik93]) are summarized in the following table.

# Proc.	U_{Palm}	U_{Sztrik}	U_{homo}
3	0,3125	0,2857143	0,3108666
4	0,2461538	0,24	0,2456802
5	0,1993865	0,1983471	0,1992787
6	0,1665815	0,1664355	0,1665614

# Proc.	U_{1_Uproz}	U_{hetero}	U_{HIT}
3	0,3042636	0,3043267	0,3125
4	0,2457828	0,2461382	0,2461537
5	0,1993895		0,1992866
6	0,16657		0,1665815

U_{Palm} : Exact bus utilization (Palm - formula for homogeneous processors).

U_{Sztrik} : Approximated results of the bus utilization (for homogeneous processors).

U_{homo} : Bus utilization of the modeled system with homogeneous processors.

U_{1_Uproz} : Bus utilization of the modeled system with heterogeneous processors which depend only on one environment process.

U_{hetero} : Bus utilization of the modeled system with heterogeneous processors.

U_{HIT} : Bus utilization calculated with HIT.

The final results of the bus utilization, which were calculated with the Palm - formula for homogeneous processors, are nearly identical with the values of the tool HIT. This is because of the high utilization of the processors.

The utilization is very close to 1 and the processors are therefore nearly never in the idle state and therefore the differences between the results are minimal.

Unfortunately, there is no formula to calculate or approximate the performance measures of the heterogeneous multiprocessor system. The only possibility to validate the results of SPNP, is to use tools like HIT and/or MACOM. vspace*0.5cm

References

[Ciardo91] Ciardo, G., Muppala, J.
Manual for the SPNP Package Version 3.1
Duke University, Durham, April 1991

[Haring94] Haring, G.,Wabnig H.
Short Papers and Tool Descriptions of the 7th International Conference on MTT'94
Vienna, May 1994

[Kelling93] Kelling, C.
A New Method to Determine the Initial Checkpoint of the Spectral Variance Analysis
7. ITG/GI-Fachtagung: Messung, Modellierung und Bewertung von Rechen- und Kommunikationssystemen
Aachen, September 1993

[Sztrik93] Sztrik, J., Bolch, G.
Queueing Model for a Markov-Modulated Heterogeneous Multiprocessor System
Internal Report Nr. 2/93, May 1993