RETRIAL QUEUES FOR PERFORMANCE MODELLING AND EVALUATION OF HETEROGENEOUS NETWORKS

J. Roszik, J. Sztrik
Institute of Informatics, University of Debrecen
Debrecen, Hungary

e-mail: jsztrik@inf.unideb.hu
www: http://irh.inf.unideb.hu/user/jsztrik/
OUTLOOK

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The queueing model
Applications

• magnetic disk memory systems
• local area networks with CSMA/CD protocols
• collision avoidance local area network modeling
Mathematical model

\[ P(0; 0) = \lim_{t \to \infty} P(C(t) = 0, N(t) = 0) \]

\[ P(j; 0) = \lim_{t \to \infty} P(\alpha_1 = j, N(t) = 0), \quad j = 1, \ldots, K \]

\[ P(0; i_1, \ldots, i_k) = \lim_{t \to \infty} P(C(t) = 0, \beta_1 = i_1, \ldots, \beta_k = i_k), \quad k = 1, \ldots, K-1 \]

\[ P(j; i_1, \ldots, i_k) = \lim_{t \to \infty} P(\alpha_1 = j, \beta_1 = i_1, \ldots, \beta_k = i_k), \quad k = 1, \ldots, K-1. \]
Once we have obtained these limiting probabilities the main system performance measures can be derived in the following way.

1. The server utilization with respect to source $j$

\[ U_j = P(\text{the server is busy with source } j) \]

that is, we have to summarize all the probabilities where the first component is $j$. Formally

\[ U_j = \sum_{k=0}^{K-1} \sum_{i_1, \ldots, i_k \neq j} P(j; i_1, \ldots, i_k) \]
Hence the server utilization

\[ U = E[C(t) = 1] = \sum_{j=1}^{K} U_j. \]

Let us denote by \( P_{W}^{(i)} \) the steady state probability that request \( i \) is waiting (staying in the orbit). It is easy to see that

\[ P_{W}^{(i)} = \sum_{j=0, j \neq i}^{K} \sum_{k=1}^{K-1} \sum_{i \in \{i_1, \ldots, i_k\}} P(j; i_1, \ldots, i_k). \]

Similarly, it can easily be seen, that the steady state probability \( P^{(i)} \) that request \( i \) is in the service facility (it is under service or waiting in the orbit) is given by

\[ P^{(i)} = P_{W}^{(i)} + U_i. \]
2. Mean response time of source $i$

Let us denote by $E[T_i]$ the mean response time of customer $i$, and by $\gamma_i$ the **throughput** of request $i$, that is, the mean number of times that request $i$ is served per unit time. These are related by

$$\gamma_i = \frac{1}{E[T_i] + 1/\lambda_i} = \lambda_i(1 - P^{(i)}) = \mu_i U_i, \quad i = 1, \ldots, K. \quad (1)$$

For $P^{(i)}$ we have

$$P^{(i)} = \frac{E[T_i]}{E[T_i] + 1/\lambda_i} = \frac{\gamma_i E[T_i]}{1 - \gamma_i/\lambda_i} = 1 - \frac{\gamma_i}{\lambda_i} \quad i = 1, \ldots, K. \quad (2)$$

which represents **Little’s theorem** for request $i$ in the service facility. It is easy to see that as a consequence of (??) we have

$$P^{(i)} = 1 - \frac{\mu_i U_i}{\lambda_i}$$
and
\[ P_{W}^{(i)} = P^{(i)} - U_i = 1 - \frac{\mu_i + \lambda_i}{\lambda_i} U_i. \]

Alternatively, by the help of (??) we can express the mean response time \( E[T_i] \) for request \( i \) in terms of \( U_i \) as
\[
E[T_i] = \frac{P^{(i)}}{\lambda_i(1 - P^{(i)})} = \frac{1 - \frac{\mu_i}{\lambda_i} U_i}{\mu_i U_i} = \frac{\lambda_i - \mu_i U_i}{\lambda_i \mu_i U_i}. \tag{3}
\]

3. Mean waiting time of source \( i \)

The mean waiting time of request \( i \) is given by
\[
E[W_i] = E[T_i] - \frac{1}{\mu_i} = \frac{1}{\gamma_i} - \frac{1}{\lambda_i} - \frac{1}{\mu_i} = \frac{\lambda_i - (\mu_i + \lambda_i) U_i}{\lambda_i \mu_i U_i}. \tag{4}
\]
At the same time we have another **Little’s theorem** for request \( i \) waiting for service. Namely

\[
P_W^{(i)} = \frac{E[W_i]}{E[T_i] + 1/\lambda_i} = \gamma_i E[W_i] \quad i = 1, \ldots, K.
\]

4. Mean number of calls staying in the orbit or in service

\[
M = E[C(t) + N(t)] = \sum_{i=1}^{K} P^{(i)} = \sum_{i=1}^{K} (1 - \frac{\mu_i U_i}{\lambda_i}) = K - \sum_{i=1}^{K} \frac{\mu_i U_i}{\lambda_i}.
\]

5. Mean number of sources of repeated calls

\[
N = E[N(t)] = \sum_{i=1}^{K} P_W^{(i)} = \sum_{i=1}^{K} (1 - \frac{\mu_i + \lambda_i U_i}{\lambda_i}) = K - \sum_{i=1}^{K} \frac{\mu_i + \lambda_i U_i}{\lambda_i}.
\]
6. Mean rate of generation of primary calls

\[ \bar{\lambda} = \sum_{i=1}^{K} \gamma_i = \sum_{i=1}^{K} \lambda_i (1 - P(i)) = \sum_{i=1}^{K} \mu_i U_i. \]

7. Blocking probability of primary call \( i \)

\[ B_i = \frac{\lambda_i \sum_{j=1, j \neq i}^{K} \sum_{k=0}^{K-1} \sum_{i_{1}, \ldots, i_k \neq i, \ldots, i_k}^{i_k} P(j; i_1, \ldots, i_k)}{\bar{\lambda}}. \]

Hence blocking probability of primary calls

\[ B = \sum_{i=1}^{K} B_i. \]
In particular, in the case of **homogeneous calls**

\[ U_i = E[C(t)]/K, \quad i = 1, \ldots, K, \quad N = K - \frac{(\lambda + \mu)U}{\lambda}, \]

\[ \bar{\lambda} = \lambda E[K - C(t) - N(t)] = \mu U, \]

\[ E[W] = \frac{N}{\bar{\lambda}} = K - \frac{1}{\mu U} - \frac{1}{\lambda} - \frac{1}{\mu}, \]

\[ B = \frac{\lambda E[K - C(t) - N(t); C(t) = 1]}{\lambda E[K - C(t) - N(t)]}. \]
Evaluation Tool MOSEL

MOSEL (Modeling, Specification and Evaluation Language) developed at the University of Erlangen, Germany, is used to formulate and solve the problem.

Case studies
$E[T]$ versus retrial rate
$E[T]$ versus service rate
$E[T]$ versus primary request generation rate
$E[T]$ versus primary request generation rate
$E[T]$ versus primary request generation rate with homogeneous service and heterogeneous retrial
$E[T]$ versus primary request generation rate with homogeneous retrial and heterogeneous service
$E[T]$ versus retrial rate with homogeneous service and heterogeneous primary request generation.
$E[T]$ versus retrial rate with homogeneous primary request generation and heterogeneous service
$E[T]$ versus primary request generation rate with heterogeneous service and heterogeneous retrial
References


