Performance Modeling of Finite-Source Cognitive Radio Networks

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Outline

1. Finite-source retrial queueing system
2. Mathematical model
3. Main performance measures
4. Numerical examples
5. Bibliography
Finite-source retrial queueing system

Retrial queueing system
Mathematical model

- $k_1(t)$ is the number of high priority sources at time $t$,
- $k_2(t)$ is the number of low priority (normal) sources at time $t$,
- $q(t)$ denotes the number of high priority requests in the priority queue at time $t$,
- $o(t)$ is the number of requests in the orbit at time $t$.
- $y(t) = 0$ if there is no job in the PCS unit, $y(t) = 1$ if the PCS unit is busy with a job coming from the high priority class, $y(t) = 2$ when the PCS unit is servicing a job coming from the secondary class at time $t$,
- $c(t) = 0$ when the SCS unit is idle and $c(t) = 1$, when the SCS is busy at time $t$. 
The system state at time $t$ can be described by the following continuous time Markov chain with finite state space

$$X(t) = (y(t), q(t); c(t), o(t))$$
Let us denote the steady-state distribution by

\[
P(y, c, q, o) = \lim_{t \to \infty} P(y(t) = y, q(t) = q, c(t) = c, o(t) = o)
\]
Once we have obtained these limiting probabilities the main system performance measures can be derived in the following way:

- **Utilization of the primary server with respect to primary users**
  
  \[ U_{11} = \sum_{q=0}^{N_1-1} \sum_{c=0}^{1} \sum_{o=0}^{N_2-c} P(1, q, c, o) \]

- **Utilization of the primary server with respect to secondary users**
  
  \[ U_{12} = \sum_{q=0}^{N_1-1} \sum_{c=0}^{1} \sum_{o=0}^{N_2-c} P(2, q, c, o) \]
Utilization of the primary server

\[ U_1 = U_{11} + U_{12} \]

Utilization of the secondary server

\[ U_2 = \sum_{y=0}^{2} \sum_{q=0}^{N_1-1} \sum_{o=0}^{N_2-1} P(y, q, 1, o) \]
Average number of jobs in queue

\[ \bar{Q} = E(q(t)) = \sum_{y=0}^{2} \sum_{q=0}^{N_2-1} \sum_{c=0}^{1} \sum_{o=0}^{N_2-c} q P(y, q, c, o) \]

Average number of jobs in the orbit

\[ \bar{O} = E(o(t)) = \sum_{y=0}^{2} \sum_{q=0}^{N_2-1} \sum_{c=0}^{1} \sum_{o=0}^{N_2-c} o P(y, q, c, o) \]
Average number of jobs of primary users in the network

$$\overline{M_1} = \overline{Q} + U_{11}$$

Average number of jobs of secondary users in the network

$$\overline{M_2} = \overline{O} + U_{12} + U_2$$

Average number of jobs in the network

$$\overline{M} = \overline{M_1} + \overline{M_2}$$
Average number of active primary users

\[ \bar{\Lambda}_1 = N_1 - \bar{M}_1 \]

Average number of active secondary users

\[ \bar{\Lambda}_2 = N_2 - \bar{M}_2 \]

Average generation rate of primary users

\[ \bar{\lambda}_1 = \lambda_1 \bar{\Lambda}_1 \]

Average generation rate of secondary users

\[ \bar{\lambda}_2 = \lambda_2 \bar{\Lambda}_2 \]
- **Mean response time of primary user’s jobs**
  \[ T_1 = \frac{M_1}{\lambda_1} \]

- **Mean response time of secondary user’s jobs**
  \[ T_2 = \frac{M_2}{\lambda_2} \]
Numerical examples by the help of MOSEL

MOSEL (MOdeling, Specification and Evaluation Language) developed at the University of Erlangen, Germany, is used to formulate and solved the problem.
**Input parameters**

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Mean Response Time of SU’s jobs vs. $\lambda_1$

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Mean Response Time of SU’s jobs vs. $N_2$
Utilization of PCS vs. $\lambda_1$
Utilization of PCS vs. $N_2$
Utilization of SCS vs. $N_2$
Relative Difference of Mean Orbit Size vs. $N_2$
Difference of utilization of SCS vs. $N_2$
Conclusions

1. Finite-source retrial queueing system with primary and secondary calls
2. Markovian model via MOSEL
3. Effect of some input parameters on performance measures in steady-state
4. Graphical illustrations, case studies
Bibliography


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