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# ANALYSIS OF FINITE-SOURCE RETRIAL QUEUEING SYSTEMS WITH IMPATIENT CUSTOMERS AND CATASTROPHIC BREAKDOWNS USING SIMULATION

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In this paper, a finite-source retrial queueing system is considered with impatient customers and catastrophic breakdowns. The characteristic of the system includes collision which occurs when a new job arrives in the system and the service facility is occupied with a job, they will collide. Both jobs will be forwarded to the virtual waiting room the so-called orbit. Here, the customers initiate other attempts to reach the server after a random time. But they give up retrying after staying in the orbit a while and leave the system which is the impatient attribute of the customers. In case of a negative event, a catastrophic breakdown takes place meaning that all the customers at the server and in the orbit depart from the system. The novelty of this paper is to investigate that feature in a collision environment with impatient customers using different distributions of the service time.

**Keywords:** *Simulation, catastrophic breakdown, retrial queueing system, collision, impatience, sensitivity analysis.*

## Introduction

Designing info-communication systems are essential because of understanding how to optimize a system and also how to handle increasing network traffic. Many tools and mechanisms are available for modeling different systems, and among them, one of the most popular ones is retrial queueing systems. To illustrate real-life problems arising in main telecommunication systems, like telephone switching systems, call centers, computer networks, and computer systems, retrial queues can be effectively applied. In many publications, retrial-queueing systems with repeated calls are utilized to depict their models like in [1]. The specialty of retrial queueing systems relies on the orbit which is assumed to be a virtual waiting room with enough

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capacity to take in every customer. In this way, a job – whose service can not start – is not lost and may launch numerous attempts to get its service requirement. The source is considered to be finite mainly because in many situations a finite number of entities participate in the operation of the system. Impatient behaviour is a natural occurrence of the customers provoking earlier departure without obtaining its service demand. This phenomenon is experienced in many fields of life just to mention some examples: healthcare applications, call centers, telecommunication networks. Not to mention all the papers where the behaviour of impatience is intensively examined, see for example [2]. Real-life systems tend to be subjected to random breakdowns which can be caused by a power outage, human negligence, or other sudden act. Thus, it is important to examine its effect on the operation of the system and the performance measures because it alters significantly the behaviour of a model. Many papers have studied models having service units assumed to be available all the time which is quite unrealistic. These types of systems have been investigated by many authors for example in [3].

The objective of our investigation is to carry out a sensitivity analysis using different distributions of service times on the main performance measures while catastrophic breakdowns eventuate. In the case of these types of events, customers are forced to leave the system due to sudden acts which can be mechanical failures or power outages. Until repair, it is not allowed for any customer to enter the system and detailed studies on catastrophic breakdowns have been examined by several papers. Because we utilize different distributions for the service time of the customers the results are obtained by our simulation program that is based on Simpack [4]. The basic building blocks of the code are used in which we have the opportunity to calculate any desired measure using numerous values of input parameters. Graphical illustrations are provided depicting the effect of different parameters and distributions on the main performance metrics.

## 1. System model

A finite-source retrial queueing system of type  $M/G/1//N$  is considered with an unreliable service unit, impatient customers, the appearance of collisions, and blocking. This model has one service unit and a finite-source where every individual (altogether  $N$ ) may generate request towards the system according to exponential law with parameter  $\lambda/N$  meaning that the inter-arrival times are exponentially distributed with mean  $\lambda/N$ . As there are no queues the service of an arriving job starts immediately following gamma, hypo-exponential, hyper-exponential, Pareto, and lognormal distribution with different parameters but with the same mean and variance

value. In the case of a busy server, an arriving customer brings about a collision with the customer under service, and both are moved into orbit. Jobs residing in the orbit after an exponentially random time with parameter  $\sigma/N$  initiate other tries to be engaged with the server. Since random breakdowns emerge the failure time is also an exponential random variable with parameter  $\gamma_0$  when the server is occupied and with  $\gamma_1$  if idle. Two scenarios are distinguished: general breakdown where the service of a job is interrupted and it is forwarded back to the orbit, other jobs initiated by the individuals of the source can not enter the system until the service unit is functional. The other one is catastrophic breakdown where the service of a job is interrupted but instead of arriving at the orbit it leaves the system as the others from the orbit, no customers are allowed by the system until the server fully recovers.

The repair process starts instantly upon the failure of the service unit which follows an exponential distribution with parameter  $\gamma_2$ . Customers are characterized by impatience implicating that jobs can decide to leave the system after spending an exponentially distributed time with parameter  $\tau$  in the orbit. These requests return to the source being unserved. In paper of [5] similar models are analyzed by an asymptotic method where  $N$  tends to infinity this is why rates  $\lambda/N$  and  $\sigma/N$  are used.

## 2. Simulation results

To obtain the desired results, our self-developed simulation tool was used in which almost all the performance measures can be estimated. Its statistics package utilizes the batch means method where the useful run is divided into a certain number of batches. Batches are long enough in that way sample averages of the batches are approximately independent thus we have a valid estimation. The following article contains more information about that method [6]. The simulations are performed with a confidence level of 99.9%. The relative half-width of the confidence interval required to stop the simulation run is 0.00001. The size of a batch used to detect the initial transient duration is 1000.

Table 1

Numerical values of model parameters

$N$	$\gamma_0$	$\gamma_1$	$\sigma/N$	$\tau$
100	0.05	1	0.05	0.001

Table 1 consists of every parameter that is applied for all the following figures. The parameters of service time of the customers can be found at

Table 2, every chosen parameter is listed resulting in the same mean and variance in every used distribution. The reason for selecting these values is focusing on the interesting situations it must be noted that this model was tested with other values as well, and in most of the cases, the same phenomenon appeared. It is totally intentional that the squared coefficient of variation is more than one, in another scenario we will run the simulations when it is less than one.

Table 2

Parameters of service time of primary customers

Distribution	Gamma	Hyper-exponential	Pareto	Lognormal
Parameters	$\alpha = 0.054$ $\beta = 0.077$	$p = 0.473$ $\lambda_1 = 1.353$ $\lambda_2 = 1.5$	$\alpha = 2.027$ $k = 0.355$	$m = -1.839$ $\sigma = 1.722$
Mean	0.7			
Variance	9			
Squared coefficient of variation	18.367			

On Figure 1 and 2 on the X-axes  $i$  represents the number of customers located in the system, and on the Y-axes  $P(i)$  denotes the probability that exactly  $i$  customer are situated at the server and in the orbit altogether. In both Figure 1 and 2 the distribution of the number of customers in the system is displayed when  $\lambda/N$  is 0.1 using various distributions of service time. Catastrophic breakdown feature is applied and interestingly the mean number of customers in the system differs from each other. In the case of the gamma distribution, customers tend to spend less time in the system compared to Pareto distribution. It is also noticeable that for both types of breakdowns the distribution of the number of customers tends to follow Gaussian distribution.

Figure 2 depicts the comparison of different failure modes besides gamma and hyper-exponential distributions. Naturally more customers are in the system using the general breakdown method but the shape of the curves curiously are slightly disparate. In case of catastrophic breakdown, the peak is not that high and the mean number is fewer but other than that curves follow the same tendency.

### 3. Conclusion

We simulated a retrial queueing system of type  $M/G/1//N$  with impatient customers in the orbit and with an unreliable server using two different failure mechanisms when blocking is applied. Results are obtained by our program to carry out a sensitivity analysis on different performance measures like the distribution of the number of customers in the system. Un-



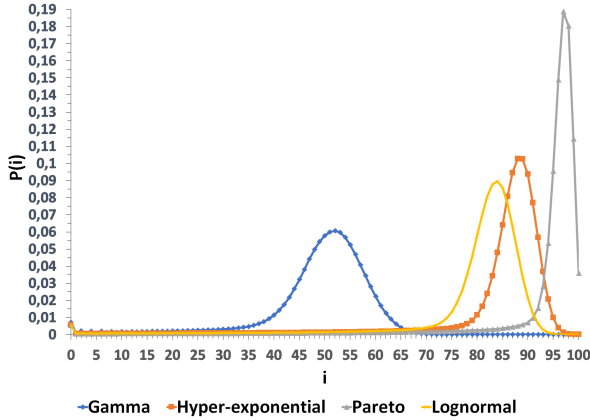


Figure 1. Distribution of the number of customers in the system

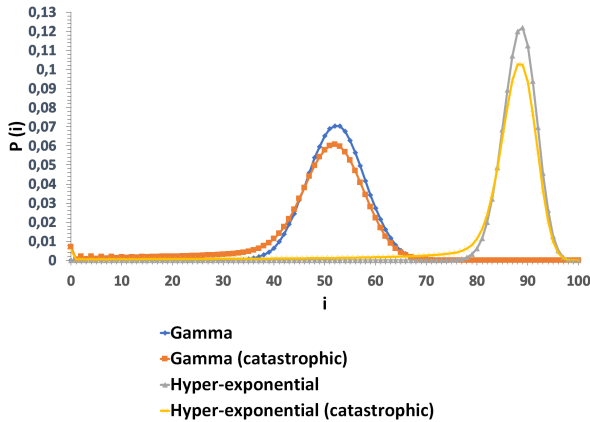


Figure 2. Comparison of distribution of the number of customers in the system using different failure modes

der various parameter settings, the most interesting measures were chosen which were graphically illustrated. When the squared coefficient of variation is more than one significant deviation is experienced between the distributions in almost every aspect of the investigated measures. Consistently, it was also revealed that besides catastrophic breakdown less customer is in the system than in the case of a normal breakdown which is an expected

phenomenon but the shape of the curves follows the same tendencies. In future works, the authors aim to carry on investigating the effect of catastrophic breakdown in other models and performing sensitivity analysis for other variables like the failure rate or the impatience of the customers.

## REFERENCES

1. *Fiems D., Phung-Duc T.* Light-traffic analysis of random access systems without collisions // *Annals of Operations Research*. 2019. Vol. 277. P. 311–327.
2. *Kumar R., Jain N., Som B.* Optimization of an  $M/M/1/N$  Feedback Queue with Retention of Reneged Customers // *Operations Research and Decisions*. 2014. Vol. 24. P. 45–58.
3. *Tóth Á., Sztrik J., Pintér Á., Bács Z.* Reliability Analysis of Finite-Source Retrial Queueing System with Collisions and Impatient Customers in the Orbit Using Simulation // 2021 International Conference on Information and Digital Technologies (IDT), 2021, Zilina: IEEE, 2021. P. 230–234.
4. *Fishwick P.A.* SimPack: Getting Started With Simulation Programming In C And C++ // In 1992 Winter Simulation Conference, New York: Association for Computing Machinery, 1992. P. 154–162.
5. *Nazarov A., Sztrik J., Kvach A., Bérczes T.* Asymptotic analysis of finite-source  $M/M/1$  retrial queueing system with collisions and server subject to breakdowns and repairs // *Annals of Operations Research*. 2019. Vol. 277. № 2. P. 213–229.
6. *Chen J.E., Kelton D.W.* A Procedure for Generating Batch-Means Confidence Intervals for Simulation: Checking Independence and Normality // *SIMULATION*. 2007. Vol. 83. № 10. P. 683–694.

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