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NUMERICAL ANALYSIS OF NON-RELIABLE RETRIAL QUEUEING SYSTEMS WITH COLLISION AND BLOCKING OF CUSTOMERS

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The aim of the investigation is a closed retrial queueing system with a finite source. The server can be reached from the source (primary request) or from the orbit (secondary request). If an incoming (primary or secondary) job finds the server busy, two modes are distinguished: the job is transferred to the orbit (no collision) or the job under service is interrupted and both of them are transferred to the orbit (collision). Requests in the orbit can retry reaching the server after a random waiting time. The nonreliable case when the server is subject to breakdown is also investigated. In case of breakdown, when the server is under repair, also two cases can be investigated. For the first, primary calls from the source can reach the system, and they will be sent to the orbit. For the second, the source is blocked, so primary customers are not able to step into the system. This paper focuses on the unreliable system with collision and blocking of parameters. These types of systems can be solved by numerical, asymptotical, and simulation methods. Our goal is to provide a new approach to the algorithmic solution for calculating the steady-state probabilities of the system. Using these quantities the main performance characteristics (utilization of the server, response time, etc.) can be calculated. Examples illustrate the effect of different parameters on the distribution of requests in the system.

1. Introduction

Retrial queueing systems (RQ-systems) are effective tools for modeling and investigating real systems from different fields of real life situations. The dynamic behavior of a general RQ-system can be described by the following characteristics: when an incoming job from the outside world (from the sources or from the queue of the system) finds the server busy, joins the orbit, and after a random, usually exponentially distributed waiting time it retries to reach the server again. The orbit is an abstract waiting cloud, and it is assumed to be infinitely large and jobs keep retrying until they are served. The commonly known application fields of an RQ-systems are call centers, computer networks, telecommunication systems, telephone switching systems, and recently the different types of networks of a smart city networks, etc. Infinite source models have been investigated and applied by many authors; they have a very large amount of literature. But in several cases the finite source models (finite number of customers in the source) are more adequate to describe the behavior of the considered system. The most characteristic examples are mobile networks, sensor networks, some IoT systems, and cognitive radio systems. The random and multiple access protocols for these type of systems have been investigated, for example, in [3, 9].

In real life situations, unfortunately, the reliable operation of the considered systems cannot be assumed. They are subject to breakdowns. That is why this situation has to be investigated. In the modeling process of the system, some random server failures and the corresponding repairs are included. The system characteristics and performance measures are highly dependent on the nonreliable operation of the systems. Finite-source RQ-systems with server breakdowns and repairs have been investigated in several recent papers, for example in [2, 6, 7, 16, 17].

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In the present paper, an $M/M/1//N$ retrial queueing system with collisions of customers is considered. For nonreliable systems, blocking and nonblocking cases will be distinguished and compared. Collisions of requests can occur with significant numbers in unsynchronized communication systems with limited number of resources, for example communication channels. In this case the transmission is lost and the interrupted customers need to be retransmitted; consequently the performance of the system is sub-optimal. There is great importance in developing methods, procedures, and protocols that are able to shield the system from customers conflicts or at least try to optimize the performance. In this direction some recent results can be found in [1, 4, 8, 10–12, 15].

In this paper a new approach of a recursive numerical solution for calculating the steady-state probabilities of the system is presented. Using these probabilities the most important performance measures can be computed. Several examples illustrate the effect of different parameters of the distribution of requests in the system.

2. Description of the system

A finite source closed retrial queuing system of type $M/M/1//N$ is considered. As the Kendall's notation says, this is a single server system with the number of sources equal to N. Three working modes of the server can be investigated:

- The system is reliable, that is, there is no server breakdown during the operation.
- The nonblocking mode. The system is nonreliable, that is, the server is subject to random breakdowns after an exponentially distributed time. When the server is idle at the time of the breakdown, the breakdown parameter is γ_0. When the server is busy at the time of breakdown, the breakdown parameter is γ_1. This means, for example, that if the server is busy, it will go wrong after an exponentially distributed random time with parameter γ_1. Because of the memorylessness property of the exponential distribution, this is so for every chosen time point from the busy period. Furthermore, it is assumed that the job under service is sent to the orbit. The repair starts immediately after the breakdown. The distribution of the repair time is also exponential with parameter γ_2. During the repair period of the server, the sources are supposed to be able to generate requests. These jobs find an unavailable server and they will be sent into the orbit. From the orbit these requests retry reaching the server again after an exponentially distributed time with parameter σ/N. The customers keep trying for service until they can reach the available idle server.
- The blocking mode. All of the nonblocking conditions hold, except one. During the repair period of the server the sources are not able to generate requests. They are blocked.

This paper investigates mainly the nonreliable case with nonblocking and blocking working modes.

The workflow of the system is as follows. The sources generate a job (request, customer) towards the server. The job generation inter-request times are exponentially distributed with parameter λ/N. After generating a request, the source waits for successful service. Until the end of service of the job, the source cannot generate a new request. The generated customer reaches the server, which can be busy or idle. When the server is empty (idle), the service of the job begins immediately, and the service times are assumed to be exponentially distributed with parameter μ. When the server is busy, two different scenarios can be considered:

- No collision: when an arriving customer finds the server busy, it is transferred into the orbit. From the orbit this request retries reaching the server again after an exponentially distributed time with parameter σ/N. The customers keep trying for service until they can reach an available idle server.
- Collision: when an incoming request finds the server busy, it is involved in collision with the customer under service, and both customers are transferred into the orbit. From the orbit the jobs will have the same behavior as in the "No collision" case. See the model in Fig. 1.

The reason that we deal with rates λ/N and σ/N is that in [13,14] similar systems were treated by an asymptotic method where N tends to infinity and it was proved that the number of customers in the system follows a nearly normal distribution. As we can see in Fig. 2, this happens in our case, too.

Let us denote the state of the system $i(t)$, that is, the number of customers in the service facility that is either in the orbit or under service, and let $k(t)$ denote the status of the server:

$$k(t) = \begin{cases} 0, \text{if the server is up and idle,} \\ 1, \text{if the server is up and busy,} \\ 2, \text{if the server is down and under repair.} \end{cases}$$

Let $\mathsf{P}(k(t) = k, \text{ and } i(t) = i) = \mathsf{P}_k(i,t)$ the probability that at the time t there are i customers in the system and the server is in the state k. Under the assumptions given above, the process $X(t) = \{k(t), i(t)\}$ is a 2-dimensional Markov chain with the state space $\{0,1,2\}x\{0,1,\ldots,N\}$.

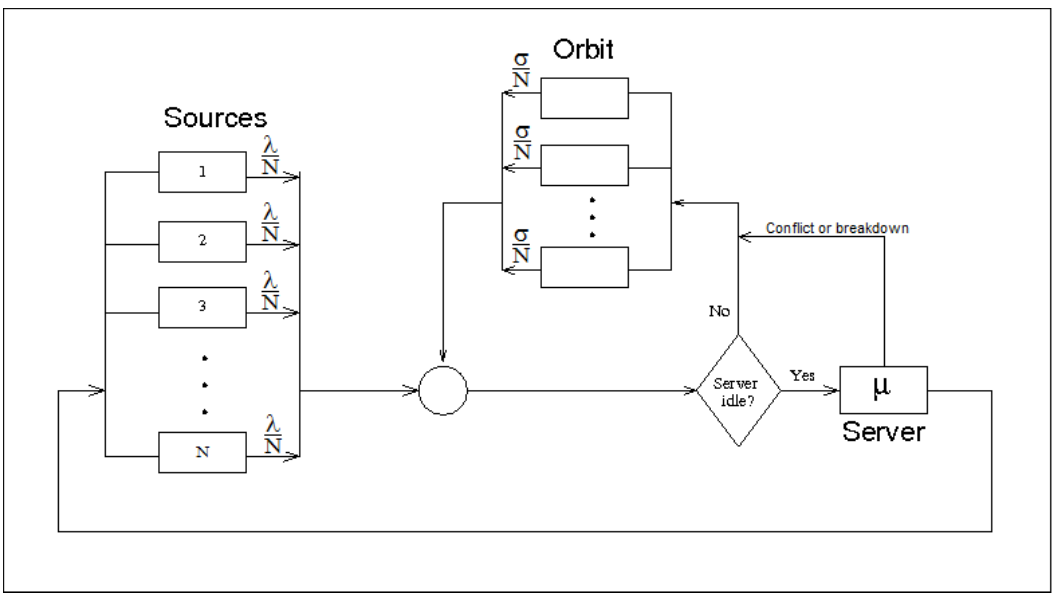

Fig. 1. System model.

When the service of a request is successful, the request goes back to the source. All the random variables involved in the model are assumed to be jointly independent.

The Kolmogorov differential equations for probabilities $\mathsf{P}_k(i,t)$ are as follows (for the nonblocking case see, [10] and [12]):

$$\frac{\partial \mathsf{P}_0(0,t)}{\partial t} = -(\lambda + \gamma_0)\mathsf{P}_0(0,t) + \mu \mathsf{P}_1(1,t) + \gamma_2 \mathsf{P}_2(0,t),$$

$$\frac{\partial \mathsf{P}_1(1,t)}{\partial t} = -\left(\lambda\frac{N-1}{N} + \mu + \gamma_1\right)\mathsf{P}_1(1,t) + \lambda \mathsf{P}_0(0,t) + \frac{\sigma}{N}\mathsf{P}_0(1,t),$$

$$\frac{\partial \mathsf{P}_2(0,t)}{\partial t} = -\gamma_2 \mathsf{P}_2(0,t) + \gamma_0 \mathsf{P}_0(0,t), \tag{1}$$

$$\frac{\partial \mathsf{P}_0(i,t)}{\partial t} = -\left(\lambda\frac{N-1}{N} + \sigma\frac{i}{N} + \gamma_0\right)\mathsf{P}_0(i,t) + \mu \mathsf{P}_1(i+1,t) + \\ + \lambda\frac{N-i+1}{N}\mathsf{P}_1(i-1,t) + \sigma\frac{i-1}{N}\mathsf{P}_1(i,t) + \gamma_2 \mathsf{P}_2(i,t),$$

$$\frac{\partial \mathsf{P}_1(i,t)}{\partial t} = -\left(\lambda\frac{N-1}{N} + \sigma\frac{i-1}{N} + \gamma_1 + \mu\right)\mathsf{P}_1(i,t) + \lambda\frac{N-i+1}{N}\mathsf{P}_0(i-1,t) + \sigma\frac{i}{N}\mathsf{P}_0(i,t),$$

$$\frac{\partial \mathsf{P}_2(i,t)}{\partial t} = -\gamma_2\mathsf{P}_2(i,t) + \gamma_0\mathsf{P}_0(i,t) + \gamma_1\mathsf{P}_1(i,t).$$

The $X(t) = \{k(t), i(t)\}$ process is a finite state Markov chain, so the steady-state operation can be assumed: $\mathsf{P}_k(i,t) = \mathsf{P}_k(i)$.

Based on the stationarity, the steady-state Kolmogorov equations can be written as

$$-(\lambda + \gamma_0)\mathsf{P}_0(0) + \mu\mathsf{P}_1(1) + \gamma_2\mathsf{P}_2(0) = 0,$$

$$-\left(\lambda\frac{N-1}{N} + \mu + \gamma_1\right)\mathsf{P}_1(1) + \lambda\mathsf{P}_0(0) + \frac{\sigma}{N}\mathsf{P}_0(1) = 0,$$

$$-\gamma_2\mathsf{P}_2(0) + \gamma_0\mathsf{P}_0(0) = 0, \tag{2}$$

$$-\left(\lambda\frac{N-1}{N} + \sigma\frac{i}{N} + \gamma_0\right)\mathsf{P}_0(i) + \mu\mathsf{P}_1(i+1) + \lambda\frac{N-i+1}{N}\mathsf{P}_1(i-1) + \sigma\frac{i-1}{N}\mathsf{P}_1(i) + \gamma_2\mathsf{P}_2(i) = 0,$$

$$-\left(\lambda\frac{N-1}{N} + \sigma\frac{i-1}{N} + \gamma_1 + \mu\right)\mathsf{P}_1(i) + \lambda\frac{N-i+1}{N}\mathsf{P}_0(i-1) + \sigma\frac{i}{N}\mathsf{P}_0(i) = 0,$$

$$-\gamma_2\mathsf{P}_2(i) + \gamma_0\mathsf{P}_0(i) + \gamma_1\mathsf{P}_1(i) = 0.$$

Note that the formulas for the system with conflict and reliable server are obtained if all of the γ_2 parameters and P_2 probabilities are set to zero.

3. Performance measures

To show the effect of the input parameters on the operation of the system, let us define the most important characteristics that can be determined directly from the steady-state probabilities.

- The mean number of customers in the system $\overline{Q}$ and in the orbit $\overline{O}$

$$\overline{Q} = \sum_{i=0}^{N} i\mathsf{P}(i), \qquad \overline{O} = \overline{Q} - \mathsf{P}_1.$$

- The mean arrival rate $\overline{\lambda}$

$$\overline{\lambda} = \sum_{k=0}^{1}\sum_{i=0}^{N}(N-i)\frac{\lambda}{N}\mathsf{P}_k(i).$$

- The mean response time $\overline{T}$ and mean waiting time $\overline{W}$ in the orbit can be obtained by the Little formula

$$\overline{T} = \frac{\overline{Q}}{\overline{\lambda}}, \qquad \overline{W} = \frac{\overline{O}}{\overline{\lambda}},$$

$$\overline{O} = \overline{Q} - \mathsf{P}_1.$$

- The mean total service time $E(T_S)$ and mean total sojourn time in the source $E(\tau)$

$$E(T_S) = \overline{T} - \overline{W}, \qquad E(\tau) = \frac{(N - \overline{Q})\overline{T}}{\overline{Q}}.$$

- The mean number of trials from the source $E(N_{TS})$ and from the orbit $E(N_{TO})$

$$E(N_{TS}) = \frac{\lambda}{N} E(\tau), \qquad E(N_{TO}) = \frac{\sigma}{N}\overline{W}.$$

4. Numerical solution

When we have these system equations, for the steady-state probabilities a recursive solution can be build up. For the nonblocking case it was described in [10] and [12]. For the blocking case the following steps can be used for calculating the steady-state probabilities $\mathsf{P}_k(i)$ recursively:

Step 1. Set the numerical values for parameters $N, \lambda, \mu, \sigma, \gamma_0, \gamma_1, \gamma_2$. Table 1 contains the values of these parameters for the figures presented below.

Step 2. Use the natural assumption $\mathsf{P}_1(0) = 0$.

Step 3. From the third equation of (2), $\frac{\mathsf{P}_2(0)}{\mathsf{P}_0(0)}$ can be calculated as

$$\frac{\mathsf{P}_2(0)}{\mathsf{P}_0(0)} = \frac{\gamma_0}{\gamma_2}.$$

Step 4. From the third equation of (2), $\frac{\mathsf{P}_1(1)}{\mathsf{P}_0(0)}$ can be obtained as

$$\frac{\mathsf{P}_1(1)}{\mathsf{P}_0(0)} = \frac{1}{\mu}\left((\lambda + \gamma_0) - \gamma_2 \frac{\mathsf{P}_2(0)}{\mathsf{P}_0(0)}\right).$$

Step 5. For $1 \leqslant i \leqslant N-1$ the following equations can be derived

$$\frac{\mathsf{P}_0(i)}{\mathsf{P}_0(0)} = \frac{N}{i\sigma}\left\{\left(\lambda\left(\frac{N-i}{N}\right) + \sigma\frac{i-1}{N} + \mu + \gamma_1\right)\frac{\mathsf{P}_1(i)}{\mathsf{P}_0(0)} - \lambda\left(\frac{N-i+1}{N}\right)\frac{\mathsf{P}_0(i-1)}{\mathsf{P}_0(0)}\right\},$$

$$\frac{\mathsf{P}_2(i)}{\mathsf{P}_0(0)} = \frac{1}{\gamma_2}\left(\gamma_0\frac{\mathsf{P}_0(i)}{\mathsf{P}_0(0)} + \gamma_1\frac{\mathsf{P}_1(i)}{\mathsf{P}_0(0)} + \frac{\mathsf{P}_2(i-1)}{\mathsf{P}_0(0)}\right),$$

$$\frac{\mathsf{P}_1(i+1)}{\mathsf{P}_0(0)} = \\ = \frac{1}{\mu}\left\{\left(\lambda\left(\frac{N-i}{N}\right) + \sigma\frac{i}{N} + \gamma_0\right)\frac{\mathsf{P}_0(i)}{\mathsf{P}_0(0)} - \lambda\left(\frac{N-i+1}{N}\right)\frac{\mathsf{P}_1(i-1)}{\mathsf{P}_0(0)} - \sigma\frac{i-1}{N}\frac{\mathsf{P}_1(i)}{\mathsf{P}_0(0)} - \gamma_2\frac{\mathsf{P}_2(i)}{\mathsf{P}_0(0)}\right\}.$$

Step 6. For $i = N$ the following formulas are valid:

$$\frac{\mathsf{P}_0(i)}{\mathsf{P}_0(0)} = \frac{1}{\sigma}\left\{\left(\sigma\frac{N-1}{N} + \mu + \gamma_1\right)\frac{\mathsf{P}_1(N)}{\mathsf{P}_0(0)} - \frac{\lambda}{N}\frac{\mathsf{P}_0(N-1)}{\mathsf{P}_0(0)}\right\},$$

$$\frac{\mathsf{P}_2(N)}{\mathsf{P}_0(0)} = \frac{1}{\gamma_2}\left(\gamma_0\frac{\mathsf{P}_0(N)}{\mathsf{P}_0(0)} + \gamma_1\frac{\mathsf{P}_1(N)}{\mathsf{P}_0(0)}\right).$$

Step 7. Using the normalization condition, $\mathsf{P}_0(0)$ can be determined as

$$\mathsf{P}_0(0) = \frac{1}{\sum_{i=0}^{N}\left(\frac{\mathsf{P}_0(i)}{\mathsf{P}_0(0)} + \frac{\mathsf{P}_1(i)}{\mathsf{P}_0(0)} + \frac{\mathsf{P}_2(i)}{\mathsf{P}_0(0)}\right)}.$$

Step 8. From the values of $\frac{\mathsf{P}_k(i)}{\mathsf{P}_0(0)}$ and $\mathsf{P}_0(0)$, the $\mathsf{P}_k(i)$, $k = 0, 1, 2$, probabilities can be obtained.
Step 9. The one-dimensional marginal distribution can be obtained as

$$\mathsf{P}(i) = \mathsf{P}_0(i) + \mathsf{P}_1(i) + \mathsf{P}_2(i), \quad i = 0, 1, \ldots, N.$$

Several possible tools were taken into consideration to perform the calculations. The choice was a spreadsheet application, namely MS Excel. This software is efficient and comfortable for solving this type of recursive calculations. The parameters are set into specified cells so the effect of changing the different parameters can be computed immediately. During programming the formulas, the absolute and relative cell references are useful methods for handling the recursive elements. The running parameter, the number of sources (i), is set into a column and N can be arbitrary large. Simultaneously with these calculations, the problem was solved by MOSEL-2 tool, as well (MOdeling Specification and Evaluation Language); see [5]. The MOSEL tool builds up the system equations. The steady-state probabilities are the results of these equations. A hard limit can be found for the number of sources. The state space grows extremely fast; consequently the number of sources cannot exceed 200. In Excel we can go far above 200. Other advantages for using this spreadsheet are that the effect of parameter modifications can be seen immediately, and the set of the steady-state probabilities, both two- and one-dimensional, are presented in separate columns and can be used directly for further investigations.

First, the steady-state probabilities are calculated and some effects of the system parameters are presented. A comparison with the MOSEL-calculations is also performed. After these results the behavior of some performance measures are given. These measures are calculated from the steady-state probabilities and are specified in Section 3.

Table 1. Numerical values of model parameters

Figure	Model	λ	μ	σ	N	γ_0	γ_1	γ_2
2	All*	1	1	5	100	0.1	0.1	1
3	Block, Nonblock	1, 2	1	5	100	0.1	0.1	1
4	Block, Nonblock	1	1, 2	5	100	0.1	0.1	1
5	Block, Nonblock	1	1	5	100	0.1, 0.5	0.1	1
6	Block, Nonblock	1	1	5	100	0.1	0.1,0.5	1
7	Block, Nonblock	1	1	5	100	0.1	0.1	1, 5
8	Block	1	1	5	100	0.1	0.1	1
9	Block	x-axes	1	0.1	100	0.1	0.1	1
10	Block	x-axes	1	0.1	100	0.1	0.1	1
11	Block	1	1	0.1	100	x-axes	x-axes	1
12	Block	1	1	0.1	100	x-axes	x-axes	1

* All the four considered models are included in this graph: reliable with no collision, reliable with collision, unreliable with collision and nonblocking, and unreliable with collision and blocking. The

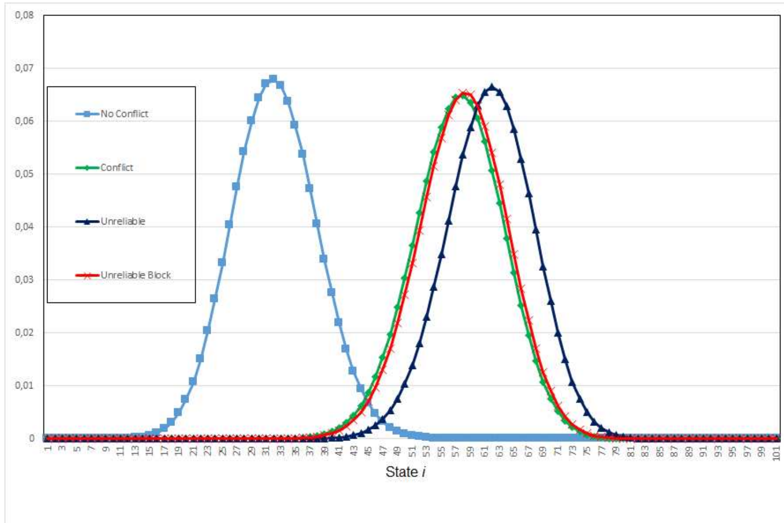

Fig. 2. Reliable no conflict, reliable with conflict, and unreliable with conflict.

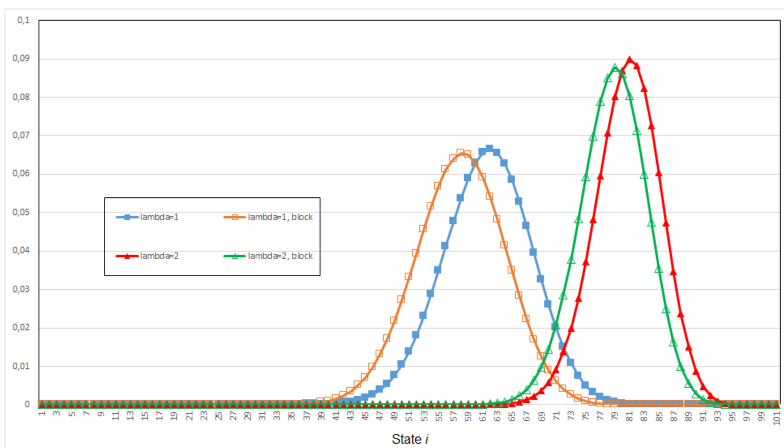

Fig. 3. Effect of generation rate λ.

further figures were generated for comparing the blocking and nonblocking cases for an unreliable server with collision of customers.

In Fig. 2 the steady-state probabilities of the four models are displayed: the basic reliable system with no collision, reliable system with collision, and unreliable system with collision with blocking and nonblocking. In the no collision case the expectation of states are lower than for the other cases. The probabilities of the states have the greatest mean in the unreliable system with nonblocking, as was expected, because the blocking phenomenon prevents from sources entering into system during the phase of the breakdown.

The effects of customer generation rate and service rate can be seen in Figs. 3 and 4 respectively.

The figures reflect the expected behavior: higher generation rates involve higher number of states, thus higher mean; for higher service rates, the mean number of request is lower, as was expected. In the blocking cases the mean number of customers is lower because no customer is generated from the source during the repair period.

In Figs. 5–7 the effect of failure and repair rates is displayed. It can be observed that for significantly higher idle failure rate the mean value of customers in the system is much larger than for the other two cases. Change of the repair rate provides a similar result. The slower the repair, the higher the number

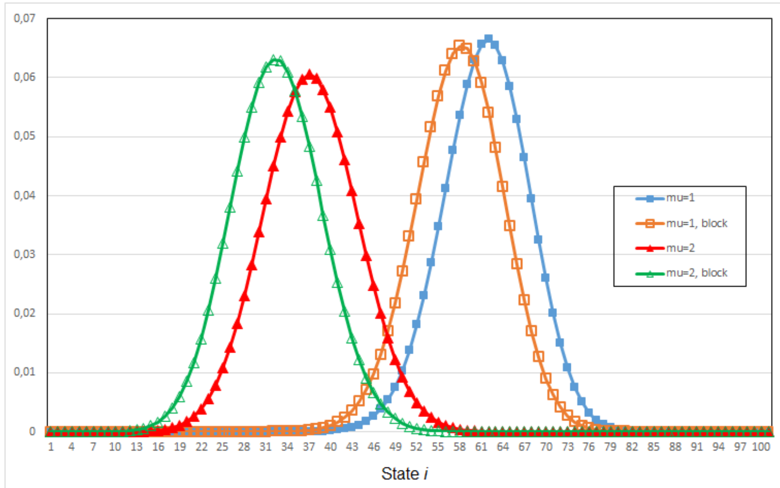

Fig. 4. Effect of service rate μ.

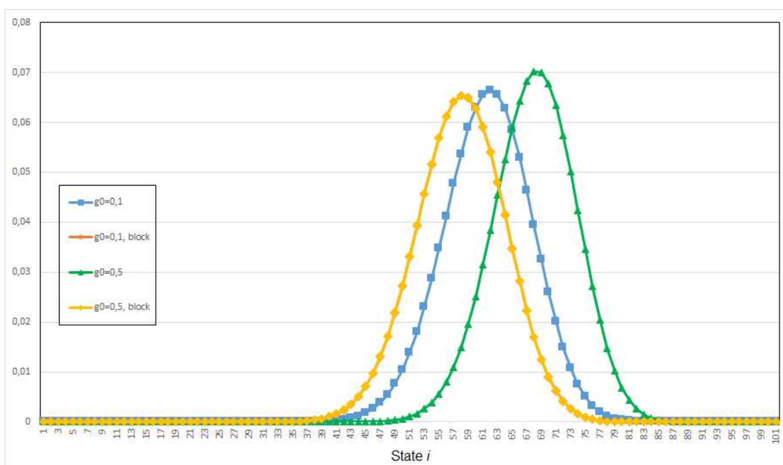

Fig. 5. Effect of failure rate γ_0.

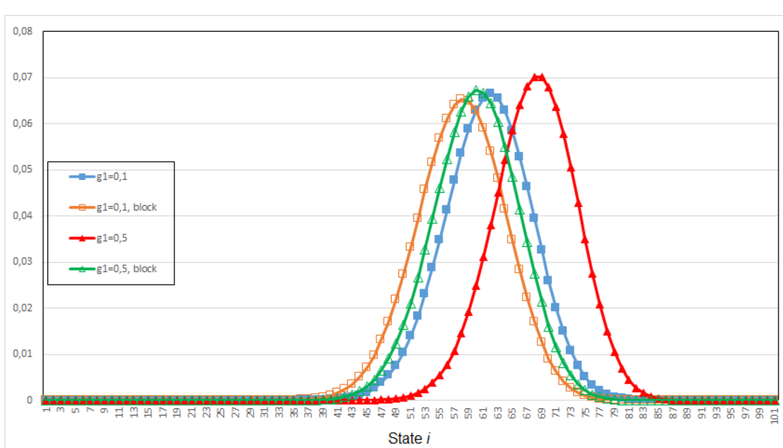

Fig. 6. Effect of failure rate γ_1.

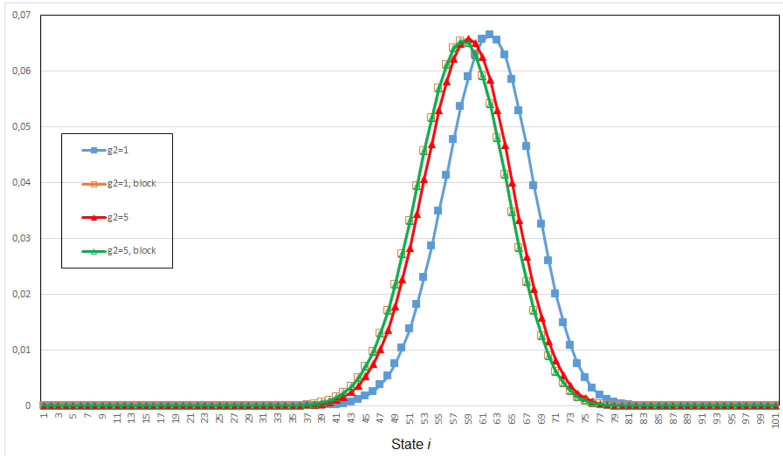

Fig. 7. Effect of repair rate γ_2.

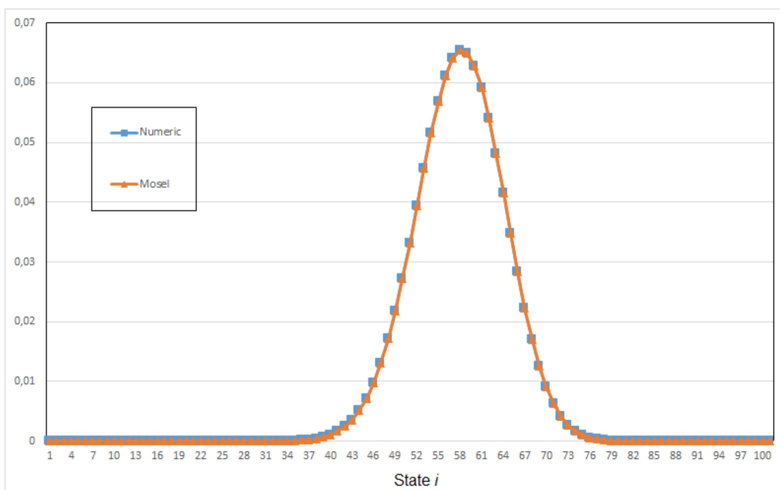

Fig. 8. Numerical calculations vs. MOSEL-2.

of customer. It is interesting that for γ_0 and γ_2 different blocking cases are identical.

In Figs. 8 the result of comparison between the numerical and MOSEL-calculation is displayed. The empirical distribution functions are calculated by cumulating the steady-state probabilities, and the Kolmogorov distance of distribution functions is applied.

The Kolmogorov distance is defined as

$$\Delta = \max_{0 \leqslant k \leqslant N} \left| \sum_{i=0}^{k} \mathsf{P}_{Num}(i) - \sum_{i=0}^{k} \mathsf{P}_{Mos}(i) \right| = 4.37E - 07.$$

It can be stated that the results of the two different calculations (numerical and MOSEL) are almost identical.

In Fig. 9 the mean response time calculated by the formulas presented in Chapter 3 is displayed as a function of the overall generation rate. The expected maximum characteristic can be observed in this figure, as well. Under some parameter settings the finite-source retrial queueing systems have this maximum feature for several performance measures, e.g., response time. The reason is the special coincidence of the high generation rate and the low number of active tokens in the source (the number of jobs in the system is usually high in this situation).

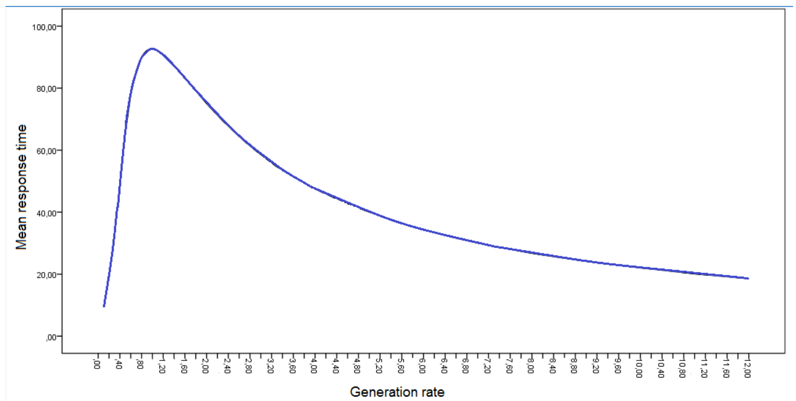

Fig. 9. Mean response time vs. λ.

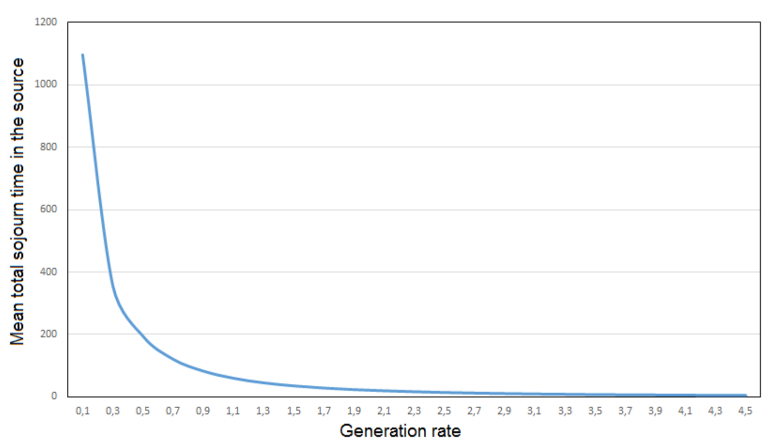

Fig. 10. Mean total sojourn time in source vs. λ.

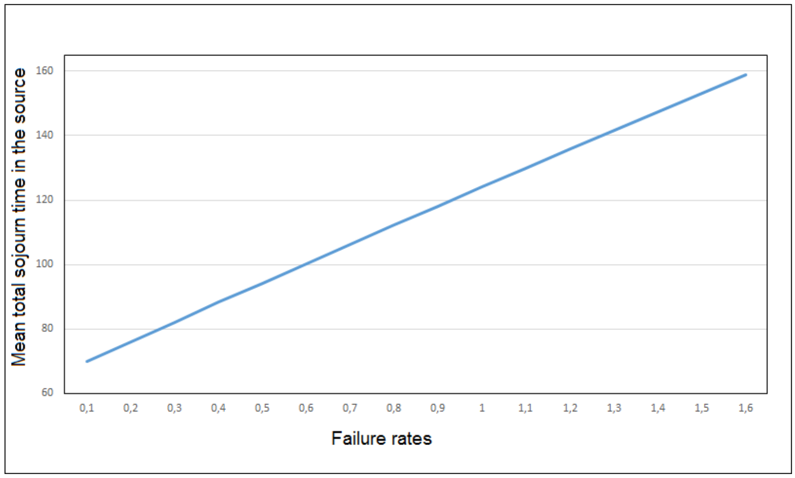

Fig. 11. Mean total sojourn time in the source vs. γ_0 and γ_1.

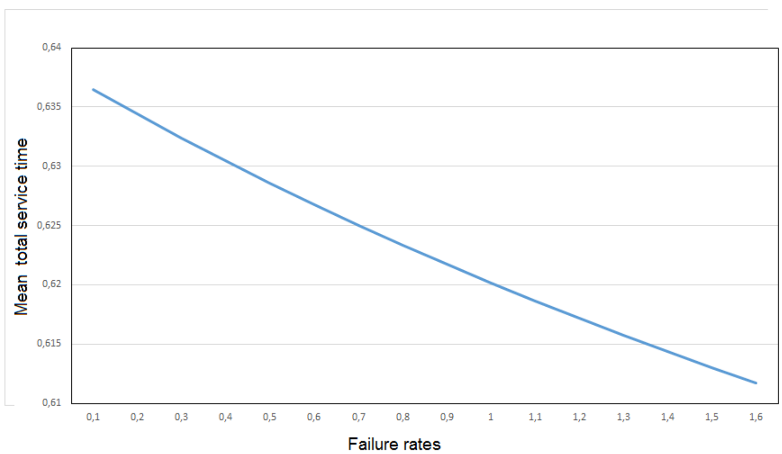

Fig. 12. Mean total service time vs. γ_0 and γ_1.

Figure 10 shows the mean total sojourn time in the source as a function of the generation rate. The other parameters are the same as in Fig. 9. With increasing generation rate, a decreasing sojourn time was expected. The only question was the shape of the function. Here it is exponentially decreasing. (When this measure is calculated as a function of failure rates, the correspondence is linear. See Fig. 11.)

In Figs. 11 and 12 the mean total sojourn time in the source and the mean total service time are presented. Sojourn times have been investigated in many papers (see, e.g., [10]), but for this blocking case the sojourn time in the source has not been described yet. The running parameters are the failure rates γ_0 and γ_1. For simplicity, $\gamma_0 = \gamma_1$ is considered. An increasing sojourn time was expected for higher failure rates. In Figs. 11 a positive linear correspondence can be observed between the sojourn time and the failure rate. For increasing failure rate the response time and the waiting time (in orbit) will be increasing, as well. The mean total service time is the difference of $\overline{T}$ and $\overline{W}$, and in Fig. 12 it can be seen that this difference is almost constant and is slightly decreasing.

5. Conclusions

In this paper a finite-source retrial queueing model was introduced. Mainly the cases of an unreliable server and collision of customers were investigated and the blocking and nonblocking behaviors were compared.

The goal of the paper was to provide an alternative solution for the tool supported (MOSEL) numeric calculations of the steady-state probabilities in the nonblocking case, as well. A robust software package, the MS Excel, proved to be a useful and efficient solution method.

The main advantage of the algorithmic approach is that there is no memory limitations, and the values of the system probabilities are immediately ready for further use and investigations. The results of the blocking and nonblocking scenarios were investigated. In addition, the results of the calculations were compared with the results of the MOSEL-output. With the help of the Kolmogorov distance the two sets of probabilities were found to be almost identical.

Acknowledgments

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REFERENCES

1. A.A. Ali and S. Wei, "Modeling of coupled collision and congestion in finite source wireless access systems," in: *Wireless Communications and Networking Conference (WCNC)*, (2015), pp. 1113–1118.
2. B. Almási, J. Roszik, and J. Sztrik, "Homogeneous finite-source retrial queues with server subject to breakdowns and repairs," *Math. Comput. Model.*, **42**, No. 5–6, 673–682 (2005).
3. J.R. Artalejo and A. Gómez-Corral, *Retrial Queueing Systems: A Computational Approach*, Springer, Berlin (2008).
4. S. Balsamo, G.L. Dei Rossi, and A. Marin, "Modelling retrial-upon-conflict systems with product-form stochastic petri nets," in: *International Conference on Analytical and Stochastic Modeling Techniques and Applications*, Springer (2013), pp. 52–66.
5. T. Bérczes, A. Tóth, A. Nazarov, and J. Sztrik, "Performance modeling of finite-source retrial queueing systems with collisions and nonreliable server using MOSEL," *Commun. Comput. Inform. Sci.*, **700**, 248–258 (2017).
6. V.I. Dragieva, "Number of retrials in a finite source retrial queue with unreliable server," *Asia-Pac. J. Oper. Res.*, **31**, No. 2 (2014).
7. N. Gharbi and C. Dutheillet, "An algorithmic approach for analysis of finite-source retrial systems with unreliable servers," *Comput. Math. Appl.*, **62**, No. 6, 2535–2546 (2011).
8. J.S. Kim, "Retrial queueing system with collision and impatience," *Commun. Kor. Math. Soc.*, **25**, No. 4, 647–653 (2010).
9. J. Kim and B. Kim, "A survey of retrial queueing systems," *Ann. Op. Res.*, **247**, No. 1, 3–36 (2016).
10. A. Kvach and A. Nazarov, "Sojourn time analysis of finite source markov retrial queuing system with collision," *Commun. Comput. Inform. Sci.*, **564**, 64–72 (2015).
11. T. V. Lyubina and A. A. Nazarov, "Research of the non-Markov dynamic retrial queue system with collision," *Her. Kemerovo State Univ.*, **1**, No. 49, 38–44 (2012).
12. A. Nazarov, A. Kvach, and V. Yampolsky, "Asymptotic analysis of closed Markov retrial queuing system with collision," *Commun. Comput. Inform. Sci.*, **487**, 334–341 (2014).
13. A. Nazarov, J. Sztrik, A. Kvach, and T. Bérczes, "Asymptotic analysis of finite-source M/M/1 retrial queueing system with collisions and server subject to breakdowns and repairs," *Ann. Op. Res.*, `https://doi.org/10.1007/s10479-018-2894-z` (2018)

14. A. Nazarov, J. Sztrik, and A. Kvach, "A survey of recent results in finite-source retrial queues with collisions " *Commun. Comput. Inform. Sci.*, **912**, 1–15 (2018).

15. Y. Peng, Z. Liu, and J. Wu, "An M/G/1 retrial G-queue with preemptive resume priority and collisions subject to the server breakdowns and delayed repairs," *J. Appl. Math. Comput.*, **44**, No. 1–2, 187–213 (2014).

16. J. Wang, L. Zhao, and F. Zhang, "Analysis of the finite source retrial queues with server breakdowns and repairs," *J. Ind. Manag. Opt.*, **7**, No. 3, 655–676 (2011).

17. F. Zhang and J. Wang, "Performance analysis of the retrial queues with finite number of sources and service interruptions," *J. Kor. Stat. Soc.*, **42**, No. 1, 117–131 (2013).