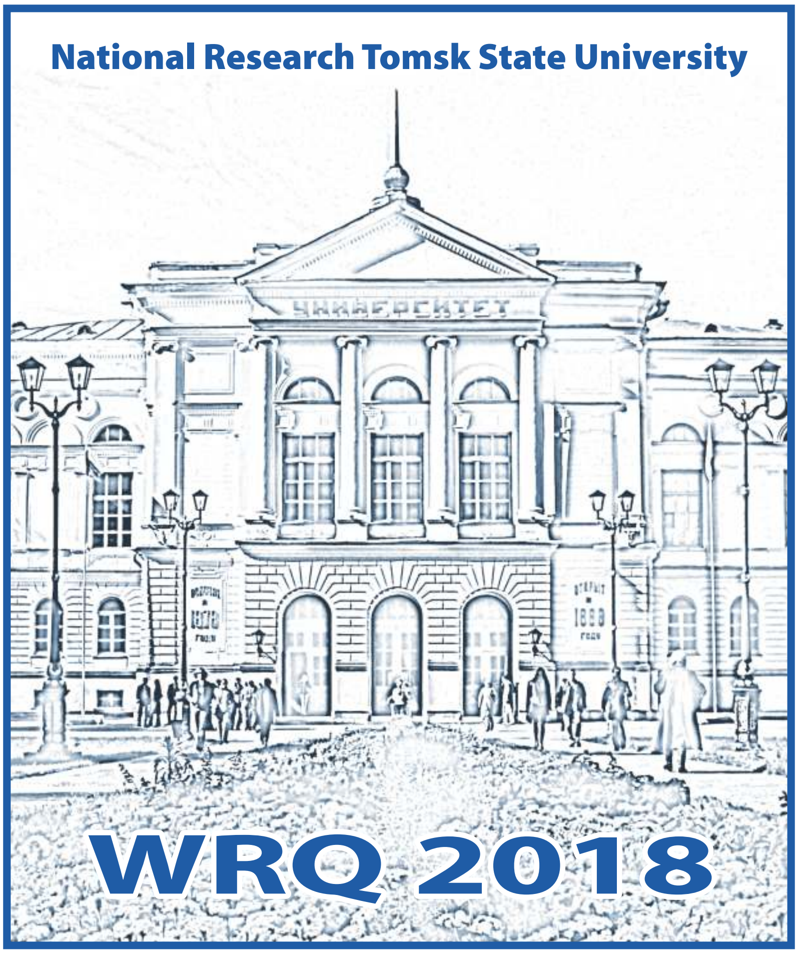
12th International Workshop on Retrial Queues and Related Topics



Tomsk, Russia

September 10–15, 2018

About Tomsk

Tomsk was founded in 1604 and served as a fortress, a merchants' city, a centre of the gold rush, and the centre of a huge province covering several regions of today's Russia and Kazakhstan. The establishment in 1888 of the first university beyond the Urals changed Tomsk dramatically. The city is both old and always young; its humming life is filled with hopes, talents, and youthful energy.

Four universities are found within one and a half kilometers! No other city in the world can boast such close proximity of its higher educational institutions. The two more universities are also quite close by; in Tomsk, everything you want is right outside. More than 72,000 students attend classes in Tomsk institutions of higher education, so every eighth resident of Tomsk is a student.

Tomsk has many local cultural institutions, including several drama theaters, a children's theater, and a puppet theater. Major concert venues in the city include the Conservatory Concert Hall and the Tomsk Palace of Sport. The city also has cultural centres dedicated to the German, Polish, and Tatar languages and culture.

One of the most remarkable features of Tomsk is its picturesque architecture, created by the hands of masters from Europe: Marfeld, Gut, and Orzeszko (Poland), Langer (Austria), Geste (Scotland), Gibert (France), Turskiy (Germany), and Tatarchuh and Homich (Poland). Their masterpieces of different genres, borrowed from Europe, decorated Tomsk. Tomsk architecture has always been its "must see!" There is probably no one who is indifferent to the variety of Tomsk architectural styles. Here one can find wooden architecture, art nouveau, baroque, classicism, renaissance, and eclecticism. Architects from all over the world dreamed of "conquering" Tomsk, which was situated in the most eastern part of Western Siberia and possessed a tremendous cultural potential. They managed to create a "European corner" in a Russian provincial city that attracted scientists and gifted people from around the world. Each of them made a great impact on the history and culture of the city, making it up-to-date and expressive.

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Tomsk State University

KEY DATES

1878 The establishment by decree of Emperor Alexander II of the Siberian Imperial University in Tomsk, the first university in Siberia.

1888 Opening of Imperial Tomsk University, and the first enrollment of students conducted at the single, Medical Faculty.

1898 Opening of the Faculty of Law.

1917 Opening of the Physics and Mathematics Faculty and the History and Philology Faculties.

1928 Creation of the Siberian Physical and Technical Institute, the first major physics research center in Siberia.

1968 Opening of the Research Institute of Applied Mathematics and Mechanics and the Research Institute of Biology and Biophysics.

1998 The inclusion of TSU in the State Code of Particularly Valuable Objects of Cultural Heritage of the Peoples of the Russian Federation.

2005 Receiving from National Quality Assurance a certificate of TSU's compliance with the Quality Management System to international standard ISO 9001: 2000.

2006 Acceptance into the European University Association (EUA).

2010 The designation of Tomsk State University as a National Research University.

2011 Renaming into the Federal State Educational Institution of Higher Professional Education as National Research Tomsk State University.

2013 Start of the implementation of the Programme to Improve the Competitiveness of the University.

Nowdays

CAMPUS

- 10 museums
- Herbarium
- Siberian Botanical Garden
- Accommodation
- Swimming pool
- Centre of Culture
- Research Library (4,000,000 volumes)

FACULTY

- 20 faculties and educational institutes
- 130 Master's programmes (including English programmes)
- 152 areas and specialities
- 152 departments and 38 centres of preuniversity training and vocational guidance in Siberia and Kazakhstan
- Siberian Physical-Technical Institute
- about 15,000 students (and about 2,000 international student)
- Tomsk Regional Teleport
- Supercomputer SKIF Cyberia
- Scientific-educational channel TV-University
- Innovation-based Technological Business-Incubator
- MBA Centre
- Institute of Innovations in Education
- Institute of Distance Education
- Institute of Applied Mathematics and Mechanics
- Research Institute of Biology and Biophysics
- Institute of Strength Physics and Materials Science

RESEARCH

- 42 leading scientific schools
- 12 centres of multiple access to unique equipment
- 47 science-based educational centres
- 47 small-scale innovation enterprises
- 42 academic schools entered the presidential list of Russian leading scientific schools
- 24 dissertation councils, about 20 Doctor of Science and 100 Candidate of Science dissertations are defended annually
- 31 academicians and corresponding members of Russian Academy of Sciences, Russian Academy of Medical Sciences, Russian Academy of Education, Russian Academy of Agricultural Science, 51 Russian State Prize Winners
- About 100 members of the Russian Academy of Sciences, Academy of Medical Sciences and the CIS states, more than 150 State Prize winners, two Nobel Prize winners studied and worked at TSU
- The University has more than 500 Doctors and 1000 Candidates of Sciences
- Young scientists and students were awarded 29 medals of the Russian Academy of Sciences, over 500 students received medals and diplomas of the Ministry of Education and Science
- During the last 10 years 4 TSU teams of scientists were awarded the State Prize of the Russian Federation in the field of science and technology, the RF Government Prize in Science and Technology, and the Russian President Award in the field of education

FACILITIES

- Multilevel educational system: preuniversity training, specialist training, undergraduate studies, Master's course, graduate course, Doctor of Science level programme, retraining and advanced training, second higher education
- Exchange programmes with the leading international institutions of higher education, prestigious scholarships and grants of the largest international funds, educational and research organizations
- Innovative approaches and techniques in the sphere of science and education, their integrated implementation at all levels of the educational and scientific process, innovative educational trajectories, continual upgrading of the academic disciplines by means of introducing scientific research findings to the academic programmes
- Training and advanced training of the teaching staff and specialists, including those from different educational institutions; development of the export of education services
- Large-scale programmes of cooperation with Russian and international universities and research centres, the leading institutes of the Russian Academy of Sciences, Russian Academy of Medical Sciences, Russian Academy of Agricultural Science, the enterprises RusAtom, RusKosmos and others
- In the list of TSU partners are 750 enterprises and organizations; over 130 contracts for cooperation and strategic partnership in education and scientific activities have been signed with the largest Russian and international science-based educational foundations, banks, and enterprises of the real sector of economy.

History of the Workshops on Retrial Queues

The series of international workshops on Retrial Queues started in Madrid, September 22–24, 1998.

The 1st International Workshop on Retrial Queues (WRQ'98) gathered 25 participants from 12 countries. The Chairman was Professor Jesus R. Artalejo. The Proceedings were published in a special issue of Top (Volume 7, Number 2, 1999).

Professor Alexander N. Dudin chaired the 2nd International Workshop on Retrial Queues (WRQ'99), Minsk, June 22–24, 1999. The workshop was conducted jointly with the 15th Belarussian Workshop on Queueing Theory. The Proceedings were published in the monograph' Queues, Flows, Systems and Networks', Belarus State University Publications.

The 3rd International Workshop on Retrial Queues (WRQ'00) was held in Amsterdam, March 13–15, 2000, at the Tinbergen Institute. The Program Chair was Professor Henk C. Tijms from Vrije University of Amsterdam. About 23 participants from both Western and Eastern European countries, USA, Canada, Israel, Sweden and Japan attended the meeting. The European Commission gave support to the above conferences through the INTAS project 96-0828 entitled 'Advances in Retrial Queueing Theory'.

The 4th International Workshop on Retrial Queues (WRQ'02) was held in Cochin, December 17–21, 2002, at the Cochin University of Science and Technology. The conference Chairman was Professor A. Krishnamoorthy. The Proceedings were published in the book 'Advances in Stochastic Modelling', Notable Publications, Inc., New Jersey.

Professor B.D. Choi chaired the 5th International Workshop on Retrial Queues (WRQ'04) at the Telecommunication Mathematics Research Center, Korea University. This meeting was combined with a Workshop on Performance Evaluation of Telecommunication Systems.

The 6th International Workshop on Retrial Queues (WRQ'06) was held at "La Cristalera", Miraflores de la Sierra, Madrid, July 8–10, 2006. The Program Chair was Prof. A Gomez-Corral. This edition gathered 23 participants from 9 countries. A selection of original, high quality contributions will be published at the special issue "Advances in Retrial Queues" of the European Journal of Operational Research.

The 7th International Workshop on Retrial Queues (WRQ'08) was held in Athens, July 17–19, 2008. The Conference Chairman was Prof. A. Economou. 31 participants from 12 countries attended this workshop. A selection of the best papers was published in the special issue "Algorithmic and Computational Methods in Retrial Queues" of the journal Computers & Operations Research (Volume 37, Number 7, 2010).

Professor Q.L. Li was the Chairman of the 8th International Workshop on Retrial Queues (WRQ'10) which was held at the Tshinghua University, Beijing, July 27–29, 2010. This edition gathered together about 40 participants from 12 countries. A selection of papers was published in Operational Research: An International Journal.

Prof. P. Moreno was the Chairwoman of the 9th International Workshop on Retrial Queues (WRQ'12) which was held at the Universidad Pablo de Olavide, Seville, Spain, July 28–30, 2012. This edition gathered together about 20 participants from 13 countries. A selection of papers will be published soon in Asia-Pacific Journal of Operational Research.

The 10th International Workshop on Retrial Queues was held on 24–26 July 2014, in Tokyo Institute of Technology, Tokyo (Japan). The Chairman was Prof. Tuan Phung-Duc. This workshop is dedicated to the memory of Professor J. R. Artalejo. A selection of high quality papers has been be published in a Special Issue on Retrial Queues and Related Models in Annals of Operations Research.

The 11th International Workshop on Retrial Queues was held in Amsterdam from August 31, 2016 until September 2, 2016. The Chairman was Dr. R. D. Nobel of the Vrije University of Amsterdam in The Netherlands. There was about 35 participants. A steering committee was chosen to continue holding the workshops in the further years.

A B S T R A C T S

Optimal information disclosure in strategic queueing systems

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Since the works of Naor [7] and Edelson and Hildebrand [1], the economic analysis of queueing systems with strategic customer behavior has gained a considerable amount of interest in recent years. Information about the queue length is an important factor for customers who make the decision whether to join a queue or not. Queueing systems with strategic customer behavior are usually divided into two groups: observable and unobservable queues. In an observable queue customers are informed about the queue length upon their arrival, whereas in an unobservable queue customers are not informed about the queue length upon their arrival.

It is important to investigate if it is effective for the service provider (server) to provide the information about the queue length to the customers, with the intention to increase the service provider's profit (revenue). There has been a large amount of research on the effects of the information level on the strategic behavior of customers, and the service provider's profit. If the service provider has a fixed income from each customer who joins the queue, the service provider should maximize the throughput of the system in order to maximize its profit. Simhon et al. [8] studied the optimal information disclosure policies in an M/M/1 queue. They proved that the policy of informing customers about the current queue length when the queue length is below the specified threshold and hiding the information when the queue length is above the threshold, is never optimal.

I present a result on the optimal information disclosure policy studied by Kim and Kim [5]. I also present a recent result on equilibrium pricing in strategic queues with competing service providers.

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Bara Kim is a Professor in the Department of Mathematics at Korea University, Seoul, Korea. He received B.S., M.S. and Ph.D. in Mathematics from Korea Advanced Institute of Science and Technology (KAIST). His research interests include probability theory, queueing theory, mathematical finance, insurance models, game theory, mechanism design, applied operations research and their applications.

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Polling systems with and without retrials

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A polling system is a single-server multi-queue system, in which the server attends to the queues in some, often cyclic, order. Many situations in which several types of users compete for access to a common resource can be described by a polling model. Polling systems received in the past much attention in the literature. For example, many different service disciplines at the queues (like exhaustive, gated, one-limited, ...) were studied in detail. Lateron, considerable unification was obtained by the realization that the generating function of the joint queue-length distribution at all queues, at epochs at which the server arrives at a particular queue, can be obtained explicitly if the service discipline at all queues is of so-called branching type (like exhaustive or gated).

More recently, motivated by an application in optical networks, we started to look at polling systems with retrials in which customers, instead of waiting in a physical queue, will go into an orbit when they arrive at a station at the wrong instant (e.g., whenever the server is serving customers in the other stations). We show that, under certain conditions, also for these types of systems the generating function of the joint distribution of the number of customers in the different stations at several embedded time points can be found by using the theory of multi-type branching processes with immigration. This enables us to do a detailed analyis of the system. Amongst others, we will discuss in this talk heavy traffic analysis, workload decomposition, pseudo-conservation laws and mean waiting time approximations. At the end we also pay some attention to optimization issues in these types of systems.

Jacques Resing is an Assistant Professor in the Stochastic Operations Research group at Eindhoven University of Technology (TU/e), where he began working in 1992. His research is in general focused on applied probability and in particular on queueing theory. Amongst others, he studied several polling systems, fluid queues and tandem queues. Another area of Jacques' interest, both in his research and teaching activities, is Insurance Risk. He is active as teacher in courses of the coherent package Finance and Risk and several of his papers study the analysis of insurance risk models. Finally, in recent years he has also become interested in the optimization of condition-based maintenance models.

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A survey of recent results in finite-source retrial queues with collisions

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The aim of the present paper is to give a review of recent results on single server finite-source retrial queuing systems with collision of the customers. There are investigations when the server is reliable and there are models when the server is subject to random breakdowns and repairs depending on whether it is idle or busy. Tool supported, numerical, simulation and asymptotic methods are considered under the condition of unlimited growing number of sources. In general, we could prove that the steady-state distribution of the number of customers in the service facility can be approximated by a normal distribution with given mean and variance. Using asymptotic methods under certain conditions in steady-state the distribution of the sojourn time in the orbit and in the system can be approximated by a generalized exponential one. Furthermore, it is proved that the distribution of the number of retrials until the successful service in the limit is geometrically distributed.

Keywords: *finite-source queuing system*, *retrial queues*, *collisions*, *server breakdowns and repairs*, *analytic results*, *algorithmic approach*, *stochastic simulation*, *asymptotic analysis.*

Introduction

Finite-source retrial queues are very useful and effective stochastic systems to model several problems arising in telephone switching systems, telecommunication networks, computer networks and computer systems, call centers, wireless communication systems, etc. To see their importance the interested reader is referred to the following works and references cited in them, for example [3, 9, 15, 19]. Searching the scientific databases, we have noticed that relatively just a small number of papers have been devoted to systems when the arriving calls (primary or secondary) causes collisions to the request under service and both go to the orbit, see for example [1, 7, 18, 23, 38].

Nazarov and his research group developed a very effective asymptotic method [37] by the help of which various systems have been investigated. Concerning to finite-source retrial systems with collision we should mention the following papers [24–27, 33].

Sztrik and his research group have been dealing with systems with unreliable server/s as can be seen, for example in [2, 42, 43, 48] and that is why it was understandable that the two research groups started cooperation in 2017.

Our investigations have been based on the analytical, numerical, simulation and asymptotic approached as treated in, for example [3, 5, 6, 10, 16, 20, 22, 28, 29, 32, 37, 40, 41, 47, 49].

The primary aim of the present paper is to give a survey on the results obtained in this field in the near past by means of different methods. Doing so we have tried to unify the notation appeared in different publications and to use the standard notation of Western-style papers which is many times differs from the Russian-style ones.

Model description and notations

In the following we introduce the model in the most general form as it was treated by the help of numerical and asymptotic methods.

Let us consider a retrial queuing system of type M/GI/1//N with collision of the customers and an unreliable server (Fig. 1). The number of sources is N and each of them can generate a primary request during an exponentially distributed time with rate λ / N. A source cannot generate a new call until the end of the successful service of this customer.

If a primary request finds the server idle, he enters into service immediately, in which the required service time has a probability distribution function $B(x)$. Let us denote its service rate function by $\mu(y) = B'(y)(1 - B(y))^{-1}$ and its Laplace-Stieltjes transform by $B^*(y)$, respectively. If the server is

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busy, an arriving (primary or repeated) customer involves into collision with customer under service and they both move into the orbit. The inter-retrial times of customers are supposed to be exponentially distributed with rate σ / N. We assume that the server is unreliable, that is its lifetime is supposed to be exponentially distributed with failure rate γ_0 if the server is idle and with rate γ_1 if it is busy. When the server breaks down, it is immediately sent for repair and the repair time is assumed to be exponentially distributed with rate γ_2. We deal with the case when the server is down all sources continue generation of customers and send it to the orbit, similarly customers may retry from the orbit to the server but all arriving customers immediately go into the orbit. Furthermore, in this unreliable model we suppose that the interrupted request goes to the orbit immediately and its next service is independent of the interrupted one. Of cause, in the case of reliable server $\gamma_0 = \gamma_1 = 0$. All random variables involved in the model construction are assumed to be independent of each other.

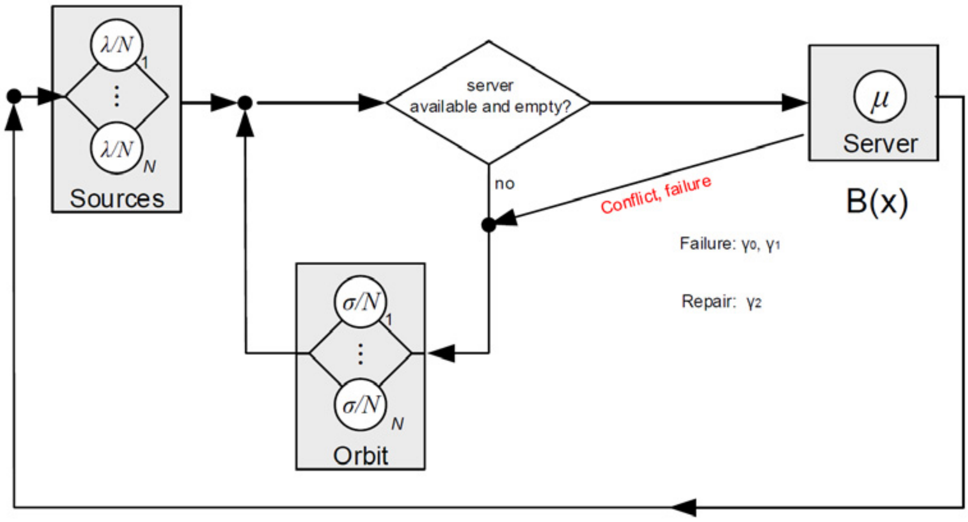

Fig. 1. Retrial queueing system of type M/GI/1//N
with collisions of the customers and an unreliable server

Let $J(t)$ be the number of customers in the system at time t, that is, the total number of customers in the orbit and in service. Similarly, let $K(t)$ be the server state at time t, that is

$$K(t) = \begin{cases} 0, & \text{if the server is idle,} \\ 1, & \text{if the server is busy,} \\ 2, & \text{if the server is down (under repair).} \end{cases}$$

Thus, we will investigate the process $\{K(t),J(t)\}$, which is not a Markov-process unless the service time is exponentially distributed. To be a Markov one we will use the method of supplementary variables, namely, we will consider two variants: the residual service time method and the elapsed service time method depending on what is the aim of the investigation.

Let us denote by $Y(t)$, and $Z(t)$, the supplementary random process equal to the elapsed service time of the customer till the moment t and by $Z(t)$ the residual service time, that is time interval from the moment t until the end of successful service of the customer, respectively.

It is obvious that $\{K(t),J(t),Y(t)\}$ and $\{K(t),J(t),Z(t)\}$ are Markov processes. Let us note, that $Y(t)$ and $Z(t)$ are defined only in those moments when the server is busy, that is, when $K(t)=1$.

Let us define the stationary probabilities as follows:

$$P_0(j) = P\{K = 0, J = j\},$$

$$P_1(j, y) = P\{K = 1, J = j, Y < y\},$$

$$P_1(j, z) = P\{K = 1, J = j, Z < z\},$$

$$P_2(j) = P\{K = 2, J = j\}.$$

Of cause, in the case of exponentially distributed service time the steady-state probabilities are denoted as follows:

$$P_k(j) = P\{K = k, J = j\}, \quad k = 0,1,2, \quad j = 0,...,N.$$

The steady-state distribution of the server's state is denoted by

$$R_k = P(K = k), \quad k = 0,1,2\,,$$

and the distribution of number of customers in the system is designated by $P(j) = P(J = j)$, $j = 0,...,N$.

It is clear that in the case of reliable server all the probabilities where K=2 are 0.

The main aim of the investigations is to get these distributions and other performance measures of the systems, such as the distribution of the sojourn time in the system, distribution of the total service time, distribution of the number of retrials. These are very complicated problems and to the best knowledge of the authors there are no exact analytical formulas to the solutions. That is the reason we have tried to obtain the characteristics of different systems by the help of tool supported, algorithmic, stochastic simulation and asymptotic methods.

Systems with a reliable server

M/M/1 Systems

Algorithmic approach. In papers [25, 33] the steady-state Kolmogorov equations were derived, and the distribution of the system's state were obtained by an algorithmic approach. Then the distribution of the number of customers in the system were calculated and used to validate the asymptotic results.

Asymptotic approach. The main contribution of paper [33] is that the in steady-state the prelimit distribution of the number of customers in the system can be approximated by a normal distribution with given mean and variance. In paper [33] 2nd and 3rd order approximations of the prelimit distribution were compared to the exact distribution obtained by the algorithmic method. In different parameter setup and for different N the applicability of the asymptotic method was validated, and some conclusions were drawn.

A more complicated problem, namely the distribution of the sojourn time in the service facility was investigated in [24] by the help of asymptotic methods as N tends to infinity.

M/GI/1 System

This section deals with the results when the required service times are generally distributed but in the examples the gamma distribution is used due to its useful properties. Namely, it is easy to see that its squared coefficient of variation can be less, equal or greater than 1 depending on the values of the shape and scale parameters.

Algorithmic approach. Paper [26] deals with the algorithmic approach how to get the steady-state distribution of the system. The method of supplementary variable technique with residual service time were applied and several numerical examples were treated with gamma distributed service time. The results helped the validation of asymptotic results for the same model.

Stochastic simulation. Papers [35, 36] are devoted to the asymptotic analysis of the mean total service time, distribution of the sojourn time in the system and the distribution of number of retrials. It must be noted that the results have not been validated by simulation, yet. Meanwhile simulations have been carried out the estimations for the mean and variance of the sojourn time have been obtained, and the distribution of the number of retrials also has been determined. The simulation analysis will be published in the near future.

Asymptotic approach. In this part the asymptotic results published in [35, 36] are summarized. Before doing that, we need some notations, namely

$$B^*(\alpha) = \int_0^\infty e^{-\alpha x} dB(x), \quad \delta(\kappa_1) = \lambda + (\sigma - \lambda)\kappa_1.$$

Then κ_1 can be obtained from

$$\kappa_1 = 1 - \frac{\delta(\kappa_1)}{\lambda} \cdot \frac{B^*(\delta(\kappa_1))}{2 - B^*(\delta(\kappa_1))}, \tag{1}$$

and the distribution of the server's state can be determined by

$$R_0 = \frac{1}{2 - B^*(\delta)}, \quad R_1 = \frac{1 - B^*(\delta)}{2 - B^*(\delta)}.$$

Introducing the notations

$$A_1 = \lambda(1 - \kappa_1), \qquad R_1^*(\alpha) = -\delta R_0 \left[B^*(\alpha) \right],$$

we obtain

$$\kappa_2 = \frac{A_1\big(R_0 \cdot B^*(\delta)[\delta + A_1] - (\delta + A_1 R_0)\big)}{A_1(\sigma - \lambda)\big(R_1^*(\delta) - R_1 - R_0(B^*(\delta) - 1)\big) + \delta\big((\sigma - \lambda)\big(R_1^*(\delta) - R_0 B^*(\delta)\big) - \lambda\big)}.$$

Consequently, the steady-state prelimit distribution of the number of customers in the system can be approximated by a normal distribution with mean $N\kappa_1$ and variance $N\kappa_2$.

For the distribution of the number of retrials/transitions of the tagged customer into the orbit we have the following results.

Let ν be the number of transitions of the tagged customer into the orbit, then

$$\lim_{N \to \infty} \mathsf{E} z^{\nu} = \frac{q}{1 - (1 - q)z},$$

where value of parameter q has a form

$$q = R_0 B^*(\delta).$$

From the proved theorem it is obviously follows that the probability distribution $P\{\nu = n\}$, $n = \overline{0, \infty}$ of the number of transitions of the tagged customer into the orbit is geometric and

$$P\{\nu = n\} = q(1 - q)^n, \ n = \overline{0, \infty}.$$

Consequently, by using the law of total probability for the characteristic function of the sojourn/waiting time W of the tagged customer in the orbit we get

$$\mathsf{E} e^{iuW} \approx q + (1 - q)\frac{\sigma q}{\sigma q - iuN}.$$

In the case of $N \to \infty$ the limiting probability distributions of the sojourn time of the customer in the system T and the sojourn time of the customer in the orbit W coincide, namely

$$\lim_{N \to \infty} \mathsf{E} \exp\left\{ iu \frac{T}{N} \right\} = \lim_{N \to \infty} \mathsf{E} \exp\left\{ iu \frac{W}{N} \right\} = q + (1 - q)\frac{\sigma q}{\sigma q - iu}.$$

Systems with an unreliable server

In many practical situations the server is not reliable and after a random time it can fail and needs repair which also takes a random duration. To deal with these service interruptions several papers have been published, see for example [2, 8, 11, 12, 14, 21, 39, 43, 45, 46, 50]. In the following parts we summarize our results obtained by different methods.

M/M/1 System

Tool supported approach by MOSEL. Because of the fact, that in many practical situations the state space of the describing Markov chain is very large, it is rather difficult to calculate the system measures in the traditional way of writing down and solving the underlying steady-state equations. To simplify this procedure several software packages have been developed and effectively used for performance evaluation of complex systems, see for example [11–14, 17]. In our investigations a similar software tool called MOSEL (Modeling, Specification and Evaluation Language) has been used to formulate the model and to obtain the performance measures. Paper [4] deals with the model formulation, derivation of several performance measures and generation of illustrative examples showing an interesting phenomenon of finite-source retrial queues, that is under specific parameter setup the mean waiting/ sojourn time has a maximum as the arrival intensity is increasing.

Stochastic simulation. To validate the applicability of the asymptotic approach we need either numerical or simulation results. The correct operation of the simulation software was tested by the numerical sample examples. The investigations carried out by the simulation and asymptotic methods have been submitted for publication, see [30, 31].

Asymptotic approach. First we deal with the distribution of the number customers in the system as it has been published in [30]. The first order asymptotic results are the following

$$\lim_{N\to\infty} \mathsf{E}\exp\left\{iw\frac{J}{N}\right\} = \exp\{iw\kappa_1\},$$

where κ_1 is the positive solution of the equation

$$(1-\kappa_1)\lambda - \mu R_1(\kappa_1) = 0,$$

where the stationary distributions of probabilities $R_k(\kappa_1)$ of the server state $k = 0,1,2$ are obtained as follows

$$R_0(\kappa_1) = \left\{\frac{\gamma_0+\gamma_2}{\gamma_2} + \frac{\gamma_1+\gamma_2}{\gamma_2}\cdot\frac{a(\kappa_1)}{a(\kappa_1)+\gamma_1+\mu}\right\}^{-1},$$

$$R_1(\kappa_1) = \frac{a(\kappa_1)}{a(\kappa_1)+\gamma_1+\mu}\cdot R_0(\kappa_1),$$

$$R_2(\kappa_1) = \frac{1}{\gamma_2}[\gamma_0 R_0(\kappa_1) + \gamma_1 R_1(\kappa_1)],$$

here $a(\kappa_1) = (1-\kappa_1)\lambda + \sigma\kappa_1$.

The second order asymptotic results are

$$\lim_{N\to\infty} \mathsf{E}\exp\left\{iw\frac{J-\kappa_1 N}{\sqrt{N}}\right\} = \exp\left\{\frac{(iw)^2}{2}\kappa_2\right\},$$

where κ_2 is

$$\kappa_2 = \frac{\gamma_2\mu(R_1-b_1) + (1-\kappa_1)\lambda\{(\gamma_1+\gamma_2)b_1 + (1-\kappa_1)\lambda R_2\}}{(\lambda+\mu b_2)\gamma_2 - (1-\kappa_1)\lambda(\gamma_1+\gamma_2)b_2}$$

and
$$b_1 = \frac{(1-\kappa_1)\lambda}{a+\gamma_1+\mu}R_0, \qquad b_2 = \frac{(\sigma-\lambda)(R_0-R_1)}{a+\gamma_1+\mu}.$$

Consequently, the prelimit distribution of the number of customers in the system can be approximated by a normal distribution with mean $N\kappa_1$ and variance $N\kappa_2$.

One of the main contributions of paper [31] is that for the limit of the characteristic function of the normalized sojourn time we have

$$\lim_{N\to\infty} \mathsf{E}\exp\left\{iw\frac{T}{N}\right\} = q + (1-q)\frac{\sigma q}{\sigma q - iw},$$

where $q = \dfrac{(1-\kappa_1)\lambda}{(1-\kappa_1)\lambda + \sigma\kappa_1}$.

Consequently, the characteristic function of the sojourn time of the customer in the system in the prelimit situation of finite N can be approximated by

$$\mathsf{E}e^{iuT} \approx q + (1-q)\frac{\sigma q}{\sigma q - iuN}. \tag{2}$$

For the distribution of the number of transitions/retrials of the tagged customer into the orbit we got the following results.

Let ν be the number of transitions of the tagged customer into the orbit, then

$$\lim_{N\to\infty} \mathsf{E}z^{\nu} = \frac{q}{1-(1-q)z},$$

resulting that the probability distribution $P\{\nu = n\}, n = \overline{0,\infty}$ of the number of transitions of the tagged customer into the orbit is geometric and has the form

$$P\{\nu = n\} = q(1-q)^n,\ n = \overline{0,\infty}.$$

Consequently, the prelimit characteristic function of the sojourn/waiting time W of the tagged customer in an orbit can be approximated as

$$\mathsf{E}e^{iuW} \approx q + (1-q)\frac{\sigma q}{\sigma q - iuN}.$$

In the case of $N \to \infty$ the limiting probability distributions of the sojourn time of the customer in the system T and the sojourn time of the customer in an orbit W coincide, namely

$$\lim_{N\to\infty} \mathsf{E}\exp\left\{iu\frac{T}{N}\right\} = \lim_{N\to\infty} \mathsf{E}\exp\left\{iu\frac{W}{N}\right\} = q + (1-q)\frac{\sigma q}{\sigma q - iu}.$$

M/GI/1 System

Stochastic simulation. In paper [44] the required service time is supposed to be gamma distributed and several cases were treated and compared.

Asymptotic approach. These results have been published in [34] using supplementary variable technique. The limit of the characteristic function of the scaled number of customers in the systems can be written in the following form

$$\lim_{N\to\infty} \mathsf{E}\exp\left\{iw\frac{J}{N}\right\} = \exp\{iw\kappa_1\},$$

where κ_1 is the positive solution of the equation

$$(1-\kappa_1)\lambda - \delta(\kappa_1)[R_0(\kappa_1) - R_1(\kappa_1)] + \gamma_1 R_1(\kappa_1) = 0,$$

here

$$\delta(\kappa_1) = (1-\kappa_1)\lambda + \sigma\kappa_1,$$

and the stationary distributions of probabilities $R_k(\kappa_1)$ of the server's state $k = 0,1,2$ are determined as follows

$$R_0(\kappa_1) = \left\{\frac{\gamma_0+\gamma_2}{\gamma_2} + \frac{\gamma_1+\gamma_2}{\gamma_2}\cdot\frac{\delta(\kappa_1)}{\delta(\kappa_1)+\gamma_1}\left[1 - B^*(\delta(\kappa_1)+\gamma_1)\right]\right\}^{-1},$$

$$R_1(\kappa_1) = R_0(\kappa_1)\frac{\delta(\kappa_1)}{\delta(\kappa_1)+\gamma_1}\cdot\left[1 - B^*(\delta(\kappa_1)+\gamma_1)\right],$$

$$R_2(\kappa_1) = \frac{1}{\gamma_2}[\gamma_0 R_0(\kappa_1) + \gamma_1 R_1(\kappa_1)].$$

Stochastic simulation of special systems

In paper [44] systems with not only gamma distributed service times but also gamma distributed inter-arrival and gamma distributed retrial times have been investigated.

Conclusions

In this paper tool supported, numerical, simulation and asymptotic methods were considered under the condition of unlimited growing number of sources in a finite-source retrial queue with collisions of customers and an unreliable server. In the near future the two research groups would like to continue their investigations in this direction including systems with impatient customers, systems embedded in a random environment, systems with two-way communications, just to mention some alternative generalizations.

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Modeling two-way communication systems with retrial queues*

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In this paper we consider systems with two-way communication. These systems can be modeled effectively by the help of retrial queueing systems. The research on two-way communications has been becoming more and more popular topic of investigations for the last years [1, 2]. The most important characteristics of two-way communication is that and idle server can look for calls outside and inside of the system (primary and secondary calls).

Authors has been investigated the case, when a secondary outgoing call after servicing is sent back to the source [3]. The novelty of this paper is, that a more realistic case is considered regarding secondary outgoing calls from the orbit. A call being in the orbit implies that the call still has an unserved incoming request. In the model presented here the served secondary outgoing call from the orbit is sent back to the orbit again, where the call is able to retry his request for servicing the original incoming call. In this model an additional operational mode is investigated. When a secondary outgoing call from the orbit arrives to the server, after serving the outgoing call, the pending incoming request will be served immediately, as well. When this two-phase service is finished, the call is sent back to the source. A loss function is also considered.

Mathematical model

The system is modeled by a finite source retrial queueing model with one server.

In the source there are N calls. All of the times, intervals are exponentially distributed and totally independent. λ_1 is the primary generating rate from the source. The call is served at the idle server with parameter μ_1. In case of busy server, the call is sent to the orbit, where it may retry their requests for service after a random waiting time with parameter ν_1

In the other hand, the idle server after some exponentially distributed period can make an outgoing calls:

- The server may call a call from the source to be served (primary outgoing call) with parameter λ_2,
- The server is able to make a call from the orbit, as well (secondary outgoing call). It is performed with parameter ν_1.

The outgoing calls (primary and secondary) are served at the server with parameter μ_2.

For an outgoing call from the orbit, two cases can be considered.

- Case 1 The call came from the orbit, is sent back to the orbit after the outgoing service is finished,
- Case 2 The server is able to serve the incoming request immediately after the outgoing job was finished. That means a two-phase service.

We denote the number of calls in orbit and the server state at time t by $O(t)$ and $S(t)$, respectively. The state space of the process $(S(t), O(t))$ is the set of $\{0, 1, 2, 3\} \times \{0, 1, 2, ..., N-1\}$ in Case 2, and $\{0, 1, 2, 3\} \times \{0, 1, 2, ...,N\}$ in Case 1.

Because of the finite state space these two-dimensional Markov processes are always stable.

The stationary probabilities are computed by the help of MOSEL-2 tool. Using these probabilities and Little formulas the system charactesistics (utilizations, average number of jobs in orbit, response times, waiting times, etc.) can be also calculated.

The expected loss $E(L)$ function also has been considered:

$$E(L) = C_w(1 - U_1 - U_2) + C_1\mu_1U_1 + C_2\mu_2U_2 + C_P(E(O) + U_1 + U_2).$$

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Simulation and numerical examples

Investigating the functionality and the behavior of the system several numerical calculations were performed. Solving the system balance equations described above the MOSEL-2 tool was used. From the probabilities the well known system characteristics are also be calculated. The most interesting performance characteristics obtained by these tools can be graphically presented. As an example, the mean waiting times in Case2 is displayed here for different types of outgoing calls.

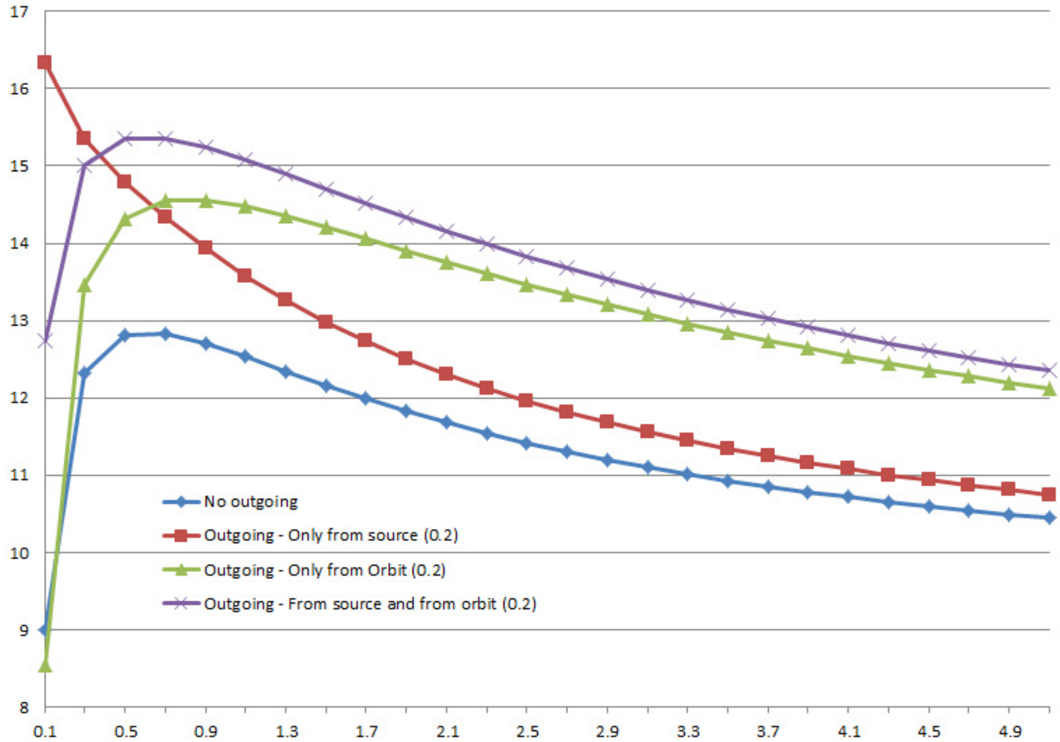

Fig. 1. Mean Waiting Time (Case 2) vs. λ_1

Conclusions

The numerical results proof that in Case 2 the most important performance measures (waiting time, utilization etc.) are better than in Case 1. A loss function keeping balance between utilization and waiting times has been also introduced. In the future it would be interesting to investigate the sensitivity of the loss function to the parameter changing.

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MMAP/M/∞ Retrial Queue with Search for Customers completed Service in the Offer Zone*

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Service providers of telecommunication industry, software vendors and similar business firms adopt various strategies to attract more customers. These strategies include offering free/services at discounted rates for a short period and providing attractive offers on service packages. The mathematical model proposed by Krishnamoorthy et. al in [1] is motivated by such a scenario. In the case of retrial queues, Artalejo et al. [2] introduced the concept of orbital search to ensure the maximum utilization of the server and to minimize the waiting time of a customer. In [3] search helps in minimizing the loss of priority customers. In this paper we propose orbital search as an effective tool to maintain the optimum number of customers in a retrial queueing model working with an offer zone.

Mathematical model

We consider $MMAP/M/\infty$ retrial queueing system with two service stations namely, the main station and the offer zone. The main station is an infinite server station and provides usual paid services. Service packages provided by the offer zone are strategically designed to attract the maximum number of customers to the main station. We assume that the offer zone holds only a finite number N of customers. The offer zone works in n random environments. Each of these environments corresponds to one or more offers and special tariff packages or a combination of these. The duration of the environment i is assumed to follow Phase Type distribution with representation PH(β_i, S_i) of order M_i where $i \in \{1,2,...,n\}$. Let p_i be the probability that the offer zone is in environment i.

Customers arriving to the system are of two types: type A and type B. Type A customers do not wish to have an offer and upon arrival they directly enter the main station. Type B customers are attracted by the offers and they wish to have service at the offer zone. Two types of customers arrive to the service stations according to a Marked Markovian Arrival Process (MMAP) of order m with representation (D_0, D_1, D_2) where $D_1 = pD^*$ and $D_2 = (1-p)D^*$ and $0 \le p \le 1$. If the offer zone is full at the time of arrival of a type B customer, it enters the main station with probability γ or leaves the system with probability $(1-\gamma)$. The service times at the main station and the offer zone in environment i are exponentially distributed with parameters μ and μ_i respectively where $i \in \{1,2,...,n\}$.

Not all the customers who have completed service from the offer zone move to the main station to continue with the usual service. Let η be the probability with which a type B customer move to the main station after completing service in the offer zone. The service providers keep a database with maximum capacity M of type B customers who are not joining the main station after completing service in the offer zone. We consider this database as an orbit. Deletion of customer records from the orbit occur at time intervals exponentially distributed with parameter ζ. Each customer from this orbit makes retrial for service in the main station and the time between successive retrials are exponentially distributed with parameter υ.

When the number of customers in the main station is below a pre-assigned level L, the operators go in search of customers from the orbit. Database formed while the orbital customers were in the offer zone may be used for orbital search. We assume that the orbital customers enter the main station at time intervals exponentially distributed with parameter υ^*.

Denote by $N_1(t)$, $N_2(t)$ and $N_3(t)$ the number of customers in the main station, the offer zone and the orbit at time t respectively. Let $E(t)$, $S(t)$ and $A(t)$ denote the environment, environmental phase and the arrival phase at time t respectively. $X^* = \{N_1(t), N_2(t), N_3(t)), E(t), S(t), A(t)\}$ is a Markov process and it describes the process under consideration. This model can be considered as a Level dependent Quasi-Birth -Death (LDQBD) process.

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Mathematical analysis

Analyzing the model for stability, we can see that the infinite server queueing model considered in this paper is always stable. We use Neuts-Rao truncation Method [4] for the analysis of the model. A steady state solution is obtained by Matrix Geometric Method [5].

Expected number of customers in the main station, expected number of customers entering the offer zone from each environment, expected number of customers in the orbit and expected number of customers entering the main station as a result of orbital search are some of the important measures of performance evaluated using the steady state probability vector. Loss of customers from the system occurs due to capacity restrictions of the offer zone and the orbit. Evaluation of the probabilities of these loses help us to optimize the maximum capacities of the offer zone and the orbit.

An optimization problem that helps us to determine the level L at which the search mechanism is to be switched off is also evaluated. The orbital search proposed in this paper is an efficient tool for maintaining an optimum number of customers in the main station.

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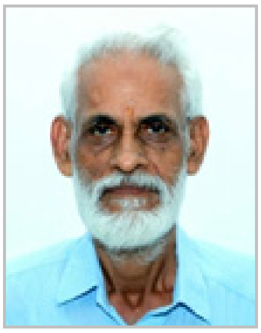

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Stationary probability distribution of the calls number in the orbit for MMPP/M/2 RQ-system with impatient calls

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In this paper we consider the retrial queueing system of MMPP/M/2 type with input MMPP-flow of events and impatient calls.

There are many papers devoted to the RQ-system models where an arriving call joints the orbit with some probability p and leaves the system with the probability $1–p$ when there are not available service devices at the time. Some authors name such customers as non-persistent or p-non-persistent customers [1, 2]. In present research impatient customer is a customer in the orbit that can repeat an attempt to reach the server again or can leave the orbit after a random time without server recalling.

Since real telecommunication systems are usually multiserver retrial queue [3], in the proposed paper RQ-system consisting of two service devices is considered. And we use the Markov Modulated Poisson Process as input flow. Validity of the models with MMPP input flow for multi-server queueing systems description is shown in papers [4, 5]. The problem for MMPP input flow with one server under system heavy load condition is solved in [6]. We consider models with two servers and impatient calls under system heavy load condition.

Asymptotic analysis method is used for research the considered RQ-system.

Mathematical model

We consider a retrial queueing system with an infinite orbit and two servers. The input flow is defined by the Markov Modulated Poisson Process. A customer, who arrives into the system, when at least one of the two servers is free, instantly occupies this server. If all of the devices are busy, the call goes to the orbit, where it stays during a random time. After the delay the customer makes an attempt to reach any server again. If it is free, the call occupies it; otherwise the call immediately joins the orbit. From the orbit calls (impatient calls) can leave the system after a random time. The delay time of calls in the orbit, the calls service time and the impatience time of calls in the orbit have exponential distribution.

The problem is to get stationary probability distribution of the number of calls in the orbit for the system under review.

Asymptotic analysis method results

Asymptotic analysis method is proposed for the solving problem of finding distribution of the number of calls in the orbit under a system heavy load and long time patience of calls in the orbit condition. The theorem about the form of the asymptotic probability distribution is formulated and proved.

Theorem 1. Stationary probability distribution of the calls number in the orbit for RQ-system of MMPP/M/2 type with impatient calls in the orbit under a system heavy load and long time patience of calls in the orbit condition can be approximated by the Gaussian distribution with mean and variance equal to $(\lambda–2\mu)/\alpha$ and λ/α respectively, where $\lambda = \mathbf{r\Lambda e}$, and $\mathbf{\Lambda}$ is the matrix of the input calls flow parameters, $\mathbf{r}$ is the row-vector of the stationary probability distribution of the Markov chain managing the input calls flow, $\mathbf{e}$ is the unit row-vector, μ, σ, α are the exponential distribution parameters, accordingly, of the calls service time, the calls delay time in the orbit, the calls leaving the system from the orbit.

Numerical results

We compared asymptotic and exact distributions for different values of parameters λ and α to demonstrate the applicability area of the asymptotic results depending on parameters of the considered RQ-system. Using the Kolmogorov distance between simulation and approximation results as a mesure and supposing it equals to 0.05 and less as acceptable accuracy of a result, we obtained parameters values in which the approximation can be applied.

Values of the Kolmogorov distance for the system parameters $\mu = 1$, $\sigma = 1$, $\lambda = \{5.297;\ 10.575\}$, $\alpha = \{2;\ 1;\ 0.5;\ 0.1\}$ are presented in Table. In Figures 1, 2 there are examples of comparison of the asymptotic and the exact distribution densities.

Kolmogorov distances between asymptotic and exact distributions under given parameters

	$\alpha = 2$	$\alpha = 1$	$\alpha = 0.5$	$\alpha = 0.1$
$\lambda = 5.297$	0.082	0.025	0.021	0.013
$\lambda = 10.575$	0.040	0.019	0.016	0.017

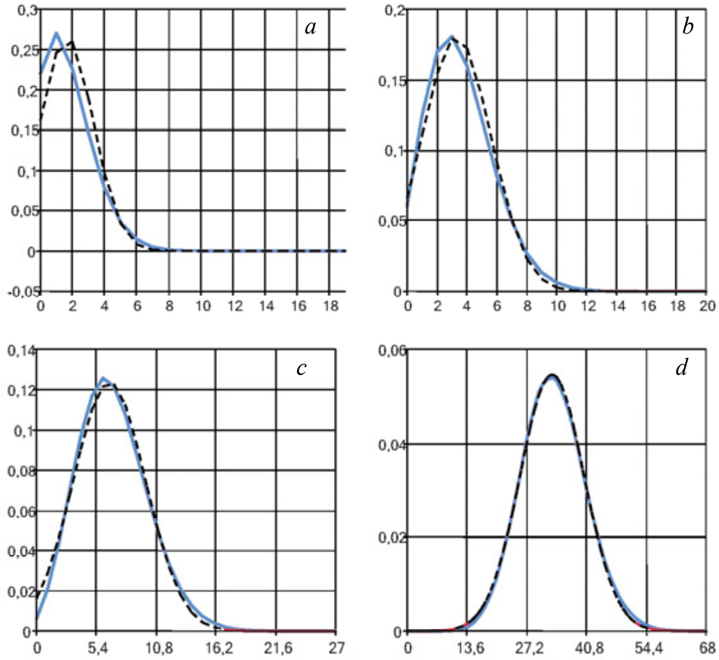

Fig. 1. Comparisons of the asymptotic (dashed line) and the exact (solid line) probability densities when $\lambda = 5.297$ and *a*) $\alpha = 2$, *b*) $\alpha = 1$, *c*) $\alpha = 0.5$, *d*) $\alpha = 0.1$

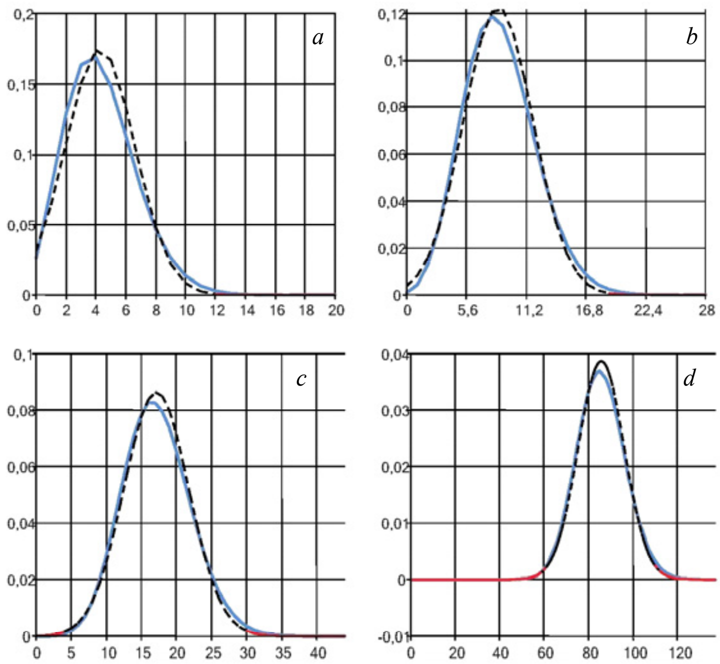

Fig. 2. Comparisons of the asymptotic (dashed line) and the exact (solid line) probability densities when $\lambda = 10.575$ and *a*) $\alpha = 2$, *b*) $\alpha = 1$, *c*) $\alpha = 0.5$, *d*) $\alpha = 0.1$

Conclusions

In the present paper, two servers retrial queueing system of MMPP/M/2 type with impatient customer in the orbit is considered. It is proved that the probability distribution of the calls number in the orbit can be approximated by the Gaussian distribution under the system heavy load and long time patience of calls in the orbit condition.

Numerical results allows drawing a conclusion that increasing of the parameter λ when parameter α is fixed leads to reduction of the Kolmogorov distances between asymptotic and exact distributions, and decreasing of the parameter α when parameter λ is fixed leads to reduction of the Kolmogorov distances between asymptotic and exact distributions.

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MAP/PH/1 retrial queue with server breakdown, repair and search for interrupted customers*

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Interruption to service often happens when using internets, electronic devices or even in our day to day life. In this paper we consider a single server retrial queue with server breakdown, repair and search of interrupted customers with some probability p.

The concept of orbital search in retrial queue is introduced in [1] with a purpose to reduce the waiting time of a customer and also to minimize the idle time of the server. A multi-server retrial queueing model with MAP arrival is considered in [2]. Several researchers have analyzed different types of interruptions since 1958. A natural question arises is that once service is interrupted what should be done on taking back the interrupted customer for service again. By giving priority to the interrupted customers here we bring forth the idea of search mechanism for bringing up the interrupted customer out of the orbit for service again.

Mathematical model

We consider a single server retrial queuing system. Primary customers arrive according to Markovian arrival process with representation (D_0, D_1) of order n. An arriving customer finding a free server enters into service immediately, otherwise the customer enters into orbit I of infinite buffer size. Service time is assumed to follow phase type distribution with representation PH(α, T) of order l. A customer who is in service either successfully complete service or get interrupted due to server breakdown. Interruption occur according to poisson process with parameter γ. Customers whose service get interrupted join the orbit II of finite capacity N. Repair time is assumed to follow phase type distribution with representation PH (β, S) of order m. Both the customers from orbits I and II retries with exponential rates μ_1 and μ_2 respectively. So there arises a competition between primary arrivals, retrials from orbits I and II. At every service completion epoch or after repair the server goes for search of interrupted customers with probability p $(0 < p < 1)$ and remains idle with probability $1-p$. Interruption occur any number of times and the service of an interrupted customer is assumed to repeat identically.

Let $N_1(t)$, $C(t)$, $N_2(t)$, $J_1(t)$, $J_2(t)$, $J_3(t)$ denote the number of customers in orbit I, status of the server, number of customers in orbit II, phases of the service process, repair process and arrival process respectively. $C(t)$ take the values 0, 1 and 2 according to whether the server is idle, in service or under repair. The Markov process of the model is represented by

$X^* = \{(N_1(t), C(t), N_2(t), J_1(t), J_2(t), J_3(t));\ t \geq 0\}$. This process is a continuous time markov chain which turns out to be level independent quasi-birth-death process (LIQBD). This is conveniently and efficiently solved by Neuts Matrix Geometric Method (see [3]).

Mathematical analysis

Stability condition of the model is obtained. Steady-state vector is obtained using Matrix Geometric Method. Several performance measures are obtained of which some important performance measures are expected number of customers in orbit I, expected number of interrupted customers in orbit II, expected number of customers in the system when the server is idle, expected number of customers in the system when the server is in service, expected number of customers in the system when the server is under repair, expected number of customers in the system, probability that the server is idle, probability that the server is in service, probability that the server is under repair, probability that the interrupted customers are blocked from entering into orbit II. Several numerical examples are illustrated.

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A numerical example

In Fig.1 x-axis is taken as search probability p and y-axis is taken as expected number of interrupted customers. Expected number of interrupted customers decreases with search probability increases as expected.

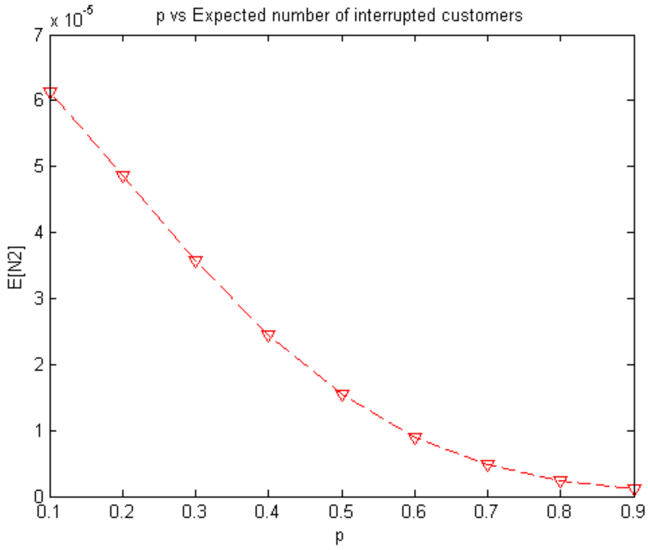

Fig.1. Search probability p vs Expected number of interrupted customers

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A finite-source retrial queue with two types of customers

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This presentation deals with a single server retrial queue where the server serves a finite number of customers (sources of demands). These customers are of two types, called regular customers and subscribed customers, respectively. Queueing models combining a finite population and retrials have many applications in practice: our daily activity, telephone switching systems, telecommunication and computer networks, call centers, cellular and local area networks, etc. Recently, a single server retrial queues with finite number of customers and different additional features of the service have been extensively studied. This includes service with an unreliable server ([4, 6]), service with two phases of the service times ([5]), service with collisions ([3]), service with random access ([2]), service with two-way communication ([1]), etc. To the best of our knowledge there are no investigations about a retrial queue with one server that serves a finite number of customers, some of which have a special status and are called subscribed or special customers. The motivation for studying such model are many real situations like call centers, repair centers, or medical centers, where along with the regular customers there is a special group of subscribed customers, or customers (patients) under special care whose service consists mainly of preventive activities, initiated by the server (operator) when being idle.

The model

We consider a queueing model with one server that serves N customers: K regular customers and (N-K) subscribed customers. Each of these customers produces a Poisson flow of demands with intensity λ_1 and λ_2, respectively. At any time t the server can be in one of three possible states – idle, busy with service of a regular customer (regular service) or busy with a special customer (special service). This is indicated by the variable $C(t)$ equal to 0,1 or 2 respectively. If the server is idle at the time of a regular customer arrival, the customer starts to be served. Otherwise it enters a virtual waiting room, called orbit and after an interval, exponentially distributed with parameter μ repeats its attempt for service. The customers in the orbit are called secondary or repeated customers, while those that are outside it - primary regular customers or regular customers in free state. The service duration of primary and secondary regular customers follows the same arbitrary probability distribution. After the service is over the regular customers of both types (primary or secondary) move to a free state, i.e. can produce a Poisson flow of demands with intensity λ_1.

The behaviour of the subscribed customers is as follows. If the server is idle at the time t of a subscribed customer arrival ($C(t)=0$), the service of this customer starts. If $C(t)=1$, then the subscribed customer waits till the current regular service is over and then is accepted immediately for service. We assume that no more than one special customer is allowed to wait for the next service. i.e. if at the time t a special customer arrives and if $C(t)=1$ and one special customer is waiting, the system state does not change. Finally, if the server is busy with a subscribed service, and a subscribed customer arrives, the system state does not change, i.e. we assume that the rejected subscribed customers do not join the orbit. The service duration of subscribers follows an arbitrary distribution, different from the distribution of the regular customers. After the service any subscribed customer is free to produce his/her usual demands that form a Poisson flow with intensity λ_2.

Results and future work

Introducing a supplementary variable $z(t)$, equal to the elapsed service time, the state of the system at time t can be described by the Markov process

$X(t)=\{C(t),R(t),z(t)\}$ where $R(t)$ is the number of repeated regular customers at time t (orbit size). Because of the finite state space of the Markov process $X(t)$ the stationary regime exists, and we can define the limiting probabilities (densities)

$$p_{i,j}(x)dx = \lim_{t\to\infty} P(C(t)=i, R(t)=j, x \le z(t) < x+dx),\ i=1,2,$$
$$p_{i,j} = \lim_{t\to\infty} P(C(t)=i, R(t)=j),\ i=0,1,2,\ j=0,1,\dots,K.$$

In a general way we obtain the equations of statistical equilibrium and solve them using the discrete transformations technique. In this presentation we derive recurrent formulas for computing the steady state joint distribution of the server state and the orbit size, $p_{i,j}(x), p_{i,j}$, and present numerical examples illustrating the influence of the input system parameters on the main macro characteristics of the system performance: server state distribution, mean orbit size, mean rate of generation of primary regular customers, mean waiting time of a customer in the orbit and blocking probability that an arriving primary regular customer will find the server busy and will be forced to join the orbit.

As a future work we plan to extend the investigation of this model by studying the busy period, the waiting time process as well as the number of lost subscribed customers during the busy period.

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Retrial queueing model with two-way communication, unreliable server and resume of interrupted call for cognitive radio networks*

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We are reviewing retrial queueing system *M/GI/GI/*1/1 with two-way communication [1, 2, 3], unreliable server [4] and afterservice of interrupted calls. For that system we have obtained probability distribution of server states, condition for the existence of a stationary mode and probability distribution of the number of calls in the system.

Our model reflects a real situation in cognitive radio networks where secondary users utilize the licensed channel of primary users when the primary user is not present in the system. Secondary users in cognitive networks correspond to incoming calls in our model. The service of incoming calls may be interrupted by primary calls. This feature is reflected in the breakdown mechanism where the

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breakdown event corresponds to the arrival of a primary user. The service time of primary user corresponds to the time to repair in our model. The unique feature of this paper is we provide a buffer for the interrupted secondary user so that its service is restarted upon the departure of the primary user.

Model description and problem definition

We consider a single server queueing model with two types of calls: incoming calls and outgoing calls. Incoming calls arrive at the system according to a Poisson process with rate λ.

Incoming call enters the system and goes into service if the server is free. The server then starts service for a time duration, distributed with a function $B_1(x)$. If at the moment of entering system the server is busy, the call instantly goes to the orbit and stays there for an exponentially distributed duration of time with a rate σ, after which the call retries to go into the server.

If the server is idle (empty) it starts making outgoing calls to the outside (not to the orbit) with rate α, the service time of an outgoing call follows the distribution function $B_2(x)$.

We will be reviewing a system with unreliable server, which crashes with intensity γ and recovers with intensity μ while servicing incoming calls. In a free state and while servicing outgoing calls the server is reliable and unable to crash.

If while servicing an incoming call the server crashes, the incoming call stays at the server and as soon as server recovers the call goes into afterservice. When the server is serving an incoming call or the server is recovering, incoming calls enter the orbit.

We have obtained:

1. The condition for the existence of a stationary mode in the retrial queue described above.
2. Characteristic function and stationary probability distribution of the number of calls in the system.

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Comparing sampling methods through retrial queues

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Simulation methods including Monte Carlo (MC) methods are considered as approximation methods, they have known and are still experiencing rapid development due to the development of computers. Their applications can be found in various fields, such as queuing systems and other fields. The Random Sampling (RS) method is well known and intensively used to represent the stochastic behavior of random variables. RS, the so-called MC method is defined by several authors such as (Dimov, 2008) and so on, but it is an inaccurate sampling procedure, so, the random behavior of stochastic input variables are not well represented. Several sampling methods representing better than RS the random behavior of stochastic input variables can be found in the literature such as Refined Descriptive Sampling (RDS) (Tari and Dahmani, 2006). RDS is proved to be more accurate than RS method when applied to real problems for instance, (Tari and Dahmani, 2005[a,b], Baghdali-Ourbih et al., 2017). Several research on sampling methods has been conducted in the field of queueing simulation. The main feature of RDS is that, in contrast to RS method, the produced estimates are each time, the most efficient.

This research takes a closer look at the RDS method especially through its varianc**e** reduction and seeks how RDS behaves when applied to M/G/1 retrial queues. This paper is concerned by the evaluation of performance measures of such system under the strong condition by using both RS and RDS for generating input samples to be used by the M/G/1 retrial model.

Mathematical model

An M/G/1 retrial system for which an analytical solution exists and can therefore serve as a basis for comparing both RDS and RS sampling methods is considered for a variety of service time distributions. Under the condition of ergodicity, the mean number of customers in the system and the mean number of customers in the orbit are evatuated through simulation. These two performance measures are selected for the comparison of both sampling methods because they are usually studied in existing literature and have practical interest.

Conclusions

We have shown that the simulated retrial M/G/1 queues for a given time period and for different service time distributions, that proper use of RDS through its getRDS generator reduces the variance of the estimates of all considered output random variables parameters.

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Analysis of discrete time retrial queueing model with changes in vacation times for energy saving in WSNs

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We propose a discrete time retrial queueing model with customer impatience and possibility of changing the vacation period. If an arriving customer finds the server idle, his service is started immediately. On the other hand, if the server is busy, he decides either to join the pool of blocked customers called orbit with probability θ or decides to quit the system with complementary probability $1-\theta$. Upon completion of a service, the server chooses to go on vacation with probability p or continues to serve the next customer with complementary probability $1-p$. During a vacation period, changes in vacation times are permitted based on requirements.

Our model finds its application in modeling a Wireless Sensor Networks (WSNs) to extend the lifetime of the batteries using the possibility of making changes in the vacation period. This model is well suited to discuss the energy saving in the batteries of each sensor node thus extending the lifetime of the WSN. This is achieved by keeping the transmitter in off mode(vacation) when there are no data packets to be sent to the sink node. If the transmitter in the node is busy or in the switched off mode (vacation), the arriving packets may not wait in that sensor node but balk.

The time period the transmitter is in the off mode (vacation period) may be extended if no further messages have arrived at the end of a vacation period or may be shortened and the transmitter changed to on mode if any messages have arrived before the end of the vacation.

A detailed study of the system is performed. Using the probability generating function approach the probability generating functions of the orbit size and system size are derived. The effect of the parameters on the performance measures are analytically derived and numerically validated.

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Analysis of a two-stage multiserver tandem retrial queueing system with servers subject to breakdowns and repairs

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We study a Markovian two-stage multiserver tandem queueing system in which the servers are subject to breakdowns and repairs. We formulate this retrial queueing system as a continuous-time level-dependent quasi-birth-and–death (LDQBD) process. A sufficient condition for the ergodicity of the system is studied. The joint steady-state probability distribution of the number of customers in the system and the state of the server is obtained by employing the matrix analytical method. Besides, some important and interesting performance measures of the system are discussed. A detailed algorithmic analysis is presented in order to determine the distributions of the time needed to reach a certain level of congestion in the orbit and the number of service completions during that time period. Finally, extensive numerical results are carried out to gain various insights into the system performance measures.

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Asymptotic sojourn time analysis of closed M/M/1 retrial queuing system with two-way communication

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The aim of the present paper is to investigate a retrial queuing system M/M/1 with a finite number of sources and two-way communication.

The first results on infinite source retrial queueing systems with two-way communication was published by Falin [5] followed by some recent ones, see for example [1, 2, 3, 8, 9].

Finite-source retrial queueing systems with two-way communication has not been investigated intensively, yet. To the best knowledge of the authors only the paper of Dragieva and Phung-Duc [4] dealt with this problem. They investigated an M/M/1//N retrial model with exponentially distributed retrial times where the primary and retrial outgoing call generation and service times are also exponentially distributed.

In their Conclusion the authors mentioned studying waiting time process among others. Hence it was our main motivation to investigate the distribution of the waiting and response time distribution of primary incoming calls by using asymptotic methods similar to [6, 7, 10]. Assuming that the number of sources N tends to infinity, it is proofed that the response/waiting time distribution of primary incoming customers in the system/orbit can be approximated by a generalized exponential distribution with given parameters. In addition, the asymptotic average number of customers in the system and in the orbit are obtained. The results are validated by the Little-formula.

Model description and notations

Let us consider a retrial queuing system of type M/M/1//N with two-way communication. The number of sources is N and each of them can generate a primary request with rate λ/N. A source cannot generate a new call until the end of the successful service of this customer. If incoming (primary or retrial) customer finds the server idle, it enters into service immediately, in which the required service time is exponentially distributed random variable with parameter μ_1. Otherwise, if the server is busy, an arriving (primary or repeated) customer moves into the orbit. The retrial times of requests are assumed to be exponentially distributed with rate σ/N. We suppose that if the server is idle, it generates an outgoing call in an exponentially distributed time with rate α/N for outgoing call to the orbit and with rate β/N for primary outgoing calls. The service times of outgoing calls are assumed to be exponentially distributed random variable with parameter μ_2. All random variables involved in the model construction are assumed to be independent of each other.

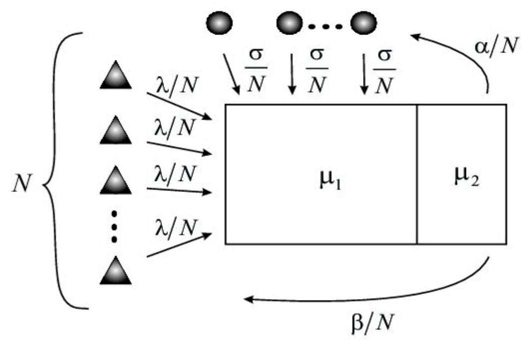

Fig. 1. Retrial queuing system of type M/M/1//N with two-way communication

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Our main aim is to find the sojourn time distribution of the customers in the system and in the orbit, respectively. The method of asymptotic analysis [10] is used in the condition of an unlimited growing number of sources.

First, we will find the first order asymptotic mean normed number of customers in the orbit, the results of which we will apply later on to study the sojourn time distribution of the customer in the system.

First order asymptotic for the number of customers in the orbit

Let $Q(t)$ be the number of customers on the orbit at time t, $C(t)$ be the server state at time t, that is

$$C(t)=\begin{cases}0, & \text{if the server is idle,}\\ 1, & \text{if the server is busy by an incoming call,}\\ 2, & \text{if the server is busy by an outgoing call.}\end{cases}$$

Thus, we will investigate the Markov process $\{C(t),Q(t)\}$.

Let us define the stationary probabilities as follows:

$$P_k(n)=\lim_{t\to\infty}P\{C(t)=k,Q(t)=n\}\,,$$

then we have

Theorem 1. *Let Q be the number of customers in the orbit then*

$$\lim_{N\to\infty}E\exp\left\{iw\frac{Q}{N}\right\}=\exp\{iw\kappa\},\tag{1}$$

where value of parameter κ *is the positive solution of the equation*

$$\lambda(1-\kappa)\left[R_1(\kappa)+R_2(\kappa)\right]-(\alpha+\sigma)R_0(\kappa)\kappa=0.\tag{2}$$

Here the stationary distributions of probabilities $R_k(\kappa)$ *of the service state k depends on* κ *and can be obtained as follows*

$$R_0(\kappa)=\left\{1+\frac{1}{\mu_1}[\lambda(1-\kappa)+\sigma\kappa]+\frac{1}{\mu_2}[\beta(1-\kappa)+\alpha\kappa]\right\}^{-1},\tag{3}$$

$$R_1(\kappa)=\frac{1}{\mu_1}[\lambda(1-\kappa)+\sigma\kappa]R_0(\kappa),\quad R_2(\kappa)=\frac{1}{\mu_2}[\beta(1-\kappa)+\alpha\kappa]R_0(\kappa)\,.$$

Sojourn time distribution of the customer in the system

Let T be the total sojourn time of the tagged customer in the system and $T(t)$ is the time length from moment t until the end of the service of the tagged customer. The total sojourn time T is simply expressed through the residual sojourn time $T(t)$.

Let $S(t)$ describe the server state at time t as follows

$$S(t)=\begin{cases}0, & \text{server is free,}\\ 1, & \text{server is busy by incoming (not tagged) customer,}\\ 2, & \text{server is busy by outgoing (not tagged) customer,}\\ 3, & \text{server is busy by incoming tagged customer,}\\ 4, & \text{server is busy by outgoing tagged customer.}\end{cases}$$

Thus, we can state the following theorem

Theorem 2. *Let T be the total sojourn time of the customer in the system then*

$$\lim_{N\to\infty}\mathsf{E}\exp\left\{iw\frac{T}{N}\right\}=R_0+(1-R_0)\frac{(\alpha+\sigma)R_0}{(\alpha+\sigma)R_0-iw}.\tag{4}$$

Consequently, we have

$$\mathsf{E}\exp\{iuT\} \approx R_0 + (1-R_0)\frac{(\sigma+\alpha)R_0 / N}{(\sigma+\alpha)R_0 / N - iu} = 1 - q + q\frac{\gamma}{\gamma - iu},$$

which is the prelimit value, that is for fixed N.

Thus, the mean response time can be approximated by

$$\frac{(1-R_0)}{(\sigma+\alpha)R_0 / N}.$$

Since the service time of a primary incoming customer is bounded then the limiting distribution of the normalized response time and the waiting time coincide. Similarly, the limiting distribution of the normalized number of customers in the system and in the orbit are the same.

Hence the mean arrival rate to the system is $\lambda(1-\kappa)$.

We will use the Little-formula to check our results, namely we have

$$\lambda(1-\kappa)\frac{(1-R_0)}{(\sigma+\alpha)R_0} = \kappa,$$

which is equivalent to equation (2) from which κ was determined.

Conclusions and future work

In this paper a finite-source retrial queuing system of type M/M/1//N with two-way communication was considered. The research has been performed by the method of asymptotic analysis under the condition of unlimited growing number of sources. As the result of the analysis it was shown that the limiting sojourn/waiting time of the customer in the system has a generalized exponential distribution with given parameters. The authors plan to continue their research, among others modeling finite-source retrial queuing systems with two-way communication for the case of generally distributed service times.

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Performance analysis of an M/G/1 retrial queueing systems under LCFS-PR discipline with general retrial and setup times

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This paper deals with the analysis of an M/G/1 retrial queueing systems with general retrial and setup times. The customers are served under the preemptive resume priority last-come, first-service (LCFS-PR) discipline and only the customer at the head of the orbit queue is allowed to access the server. For such a system, the necessary and sufficient condition for the system to be stable is investigated. Using the generating functions technique, the joint steady-stat distribution of the server state and the number of customers in the orbit are obtained along with some interesting and important performance measures. Besides, the general stochastic decomposition property is discussed. Finally, some numerical examples to illustrate the effect of system parameters on several performance characteristics are carried out.

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Optimal control of an M/G/1 queue with two phase of service

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Consider the problem of a dynamic routing control retrial queue with a single server providing two phases of service. Every customer must receive service in two phases before leaving the system. Upon completion of the first phase, the server can either continue with the second phase for the same customer or stop the current service sequence in the first stage. In the latter case, the customer is placed in the retrial box, from where he is recalled for the second phase after a random length of time before leaving the system. Using Markov decision theory, we prove that there exists an optimal policy that minimizes the expected total discounted cost for the system. In the case of socially optimal routing policies, we show that such a policy can be described by a switching curve based on the number of customers in the system. We present a conjecture regarding the structure of this policy. Numerical results for the optimal threshold for different parameter values are obtained.

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Approximation of an unreliable M/G/1 retrial queue with impatience

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This paper studies M/G/1 retrial queue with persistent and impatient customers having different general service distribution. The server is subject to active and passive breakdowns. The considered model takes into account two types of arbitrarily distributes maintenances: preventive for improving system performances and random breakdowns, and corrective for restoring the service when a failure occurs. The explicit expressions of the probability generating functions of distribution of server state and orbit size are well known from early study. We obtain asymptotic behavior of the random variable representing the number of customers in the retrial group under some extreme conditions: heavy traffic, low retrials, instantaneous connection of impatient customers. Some numerical illustrations are also given.

The mathematical model

We consider an M/G/1 retrial queue with unreliable server and two types of primary calls: persistent and impatient. These calls arrive according to independent Poisson processes with rate $\lambda > 0$ and $\gamma > 0$ respectively. There is no queue in the classical sense. If an arriving primary call (persistent or impatient) finds the server available and free, it immediately occupies the server and leaves the system after service completion. If an arriving persistent call finds the server blocked it becomes a source of secondary call and returns later to try again until it finds the server free and available; the collection of all secondary calls is called orbit. If an arriving impatient call finds the service blocked, it leaves the system forever. Any customer accepted for service upon arrival or on retrial leaves the system forever after service completion. We assume that each customer in orbit comes back, independently of others, to the server after an exponential amount time with parameter $v > 0$. The service times of the persistent customers are independent with common probability distribution function $H(x)$, Laplace-Stieltjes transform $h(s)$ and first order moment h_1 and h_2. The service times of the impatient customers are independent with common probability distribution function $F(x)$, Laplace-Stieljes transform $f(s)$ and first order moments f_1 and f_2. The server is subject to active and passive breakdowns. The failures occur according to Poisson processes with rates $\theta_1 > 0$ and $\theta_2 > 0$ when the server is busy and idle respectively. A persistent customer whose service is interrupted joins the retrial group while an impatient one leaves the system. Two types of maintenance are performed: preventive and corrective. The preventive maintenance is initiated from time to time in order to improve system performance according to a Poisson process with rate $\delta > 0$. Its duration is a random variable with probability distribution function $G(x)$, Laplace-Stieltjes transform $g(s)$ and first two moment g_1 , $g_2 >0$. If a preventive action occurs when a service is in course, then it is postponed to an ulterior date. The corrective maintenance (also called repair) is launched when the server fails. Its duration is a random variable with probability distribution function $R_1(x)$, Laplace-Stieltjes transform $r_1(s)$ and first order moment r_{11} and r_{12}, given that the breakdown occurs in a busy period and with probability distribution function $R_0(x)$, Laplace-Stieltjes transform $r_0(s)$ and first two moment r_0 and r_{02}, given that the breakdown occurs in an idle period. All the considered variables are assumed to be mutually independent and all moments are assumed to be finite.

Result 1. Approximation under heavy traffic (p→1-)

As $p \to 1\text{-}0$, the scaling random variable (1–p) follows a Gamma distribution, i.e.

$$(1-p)R \sim \Gamma(1+\frac{2(1+\gamma\omega_1+\theta_1 r_{01}+\delta g_1)v_1}{u_2 v},\frac{2(v_1)^2}{v_2})$$

where $\Gamma(a, b)$ is the Gamma distribution with parameters (a, b) and

$$v_1 = \frac{1-h_1(\theta_1)}{h(\theta_1)}(\frac{1}{\theta_1}+r_{11})$$

Result 2. Approximation under low rate of retrials (*v*→0)

As *v*→0, the random variable *R* follows a Gaussian distribution with mean

$$a_1 = \frac{\lambda(p+\gamma\omega_1+\theta_0 r_{01}+\delta g_1)}{v(1-p)}$$

and variance

$$b_1 = \frac{\lambda_3(1+\omega_1+\theta_0 r_{01}+\delta g_1)v_2}{v(1-p)} +$$

$$+\frac{[\lambda^2(\gamma\omega_2+\theta_0 r_{02}+\delta g_2)+2\lambda(p+\gamma\omega_1+\theta_0 r_{01}+\delta g_1)](1-p)}{v(1-p)}.$$

Result 3. Instantaneous connection to impatient customers (γ→∞)

As γ→∞, the random variable R follows a Gaussian distribution with mean

$$a_2 = \frac{\lambda\gamma\omega_1}{v(1-p)}$$

and variance

$$b_2 = \frac{\lambda^3\omega_1 v_2+[\lambda^2\omega_2+2\lambda\omega_1](1-p)}{2(1-p)^2}.$$

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Cost optimization and performance analysis of double orbit retrial queueing model with unreliable server and balking

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The present study is concerned with the performance analysis of the unreliable single server retrial queueing system. The concept of customers' balking behavior and provision of double orbits are incorporated to depict the realistic scenarios of many not wish to join the system for the service and leave the system without getting served. On joining the system, if the customers see the server is busy then they are directed to accommodate on one of the two retrial orbits, i.e., ordinary orbit or executive orbit; the high paying customer would like to join the executive orbit. Chapman-Kolmogorov equations are framed and solved by analytical method by using probability generating functions of the queue size distributions. Several performance indices are established explicitly. Furthermore, various analytical results are established which are further compared by using soft computing artificial neuro fuzzy interface system (ANFIS) approach by taking Gaussian membership function for fuzzy input parameters. A numerical illustration is provided to figure out the effects of the system parameters on the several performance indices. The cost function is also constructed and minimized by using quasi-Newton method to determine the optimal service rate.

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Dynamic control of a retrial queueing system with abandonments: manpower planning in a call center

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In call centers dynamic manpower planning is an important issue, because there is a trade-off between the cost of keeping many idle servers active, i.e. available to start the service of an incoming call, and the cost of long waiting times for the callers before being serves due to a lack of available [free] servers. On the one hand it is a waste of resources when too many servers stay idle and active for a long time, and on the other hand it is a nuisance for the customers, and so indirectly also for the call center when too many callers have to wait very long before they find a free server. In call centers it is common that callers who find all servers busy upon arrival do not wait in a queue, but instead temporarily leave the system and try to approach the center anew after some random time. But if they have to wait too long before finding a free server they will give up, and abandon the system forever, which is also a loss for the call center. So, this setup of a call center asks for the analysis of a queuing model where customers who find no free server upon their first arrival *retry* to enter the system repeatedly until they find a free server, and will abandon the system, when after a series of unsuccessful retrials their patience time has expired. It will be clear that/ due to the stochastic character of the arrival stream of the callers and the service time, having a constant number of servers active [idle or busy] might not be optimal from a cost perspective: some form of dynamic manpower planning is required to reduce the total operational cost for the call center.

To get an insight into an optimal manpower planning policy in this environment we consider a multi-server retrial queueing model where new customers [callers] arrive according to a Poisson process and customers will be non-persistent, i.e. they will abandon the system after their patience time has expired. The number of active servers is controllable at arrival epochs and at service completion epochs. Servers who are not active are sleeping [turned off], and they do not incur any cost. An active server is always idle or busy. Keeping an idle server active for incoming calls, i.e. newly arriving customers, requires a standby cost per unit time. When upon arrival of a customer at least one of the active servers is idle, the newly arrived customer goes into service immediately. Otherwise the decision must be made to send the customer into orbit, a virtual waiting area from which the customers try to enter the system anew after some random time, or to activate a sleeping server for immediately service of the arrived customer into orbit, a virtual waiting area from which a set-up cost. A customer in orbit tries to renter the system some random time later, but he will abandon the orbit forever once his patience time has expired. Retrial times and patience times follow an exponential distribution. For customers in orbit linear holding costs are incurred per unit time and every customer who abandons the orbit incurs a penalty cost for the system. To reduce standby costs, the system has the option to deactivate an idle server, i.e. send him into sleep, at service completion epochs. The service time of the customers are independent and follow a Coxian-2 distribution. This choice enables to study the sensitivity of the variance of the service times for the activating/deactivating policies. The problem is to find the optimal policy for activating new servers at arrival epochs and deactivating idle servers at service completion epochs which guarantees a minimal long-run average cost per unit time.

Using Markov decision theory this policy can be calculated in principle, but due to the large state space showing up in the mathematical description of the system a straightforward application of well-known methods like policy-iteration algorithm is hampered. By introducing so-called fictitious decision epochs it is shown how the problem of a large state space can be circumvented. Numerical results will illustrate that the optimal policies are characterized by so called threshold policies. This structural phenomenon can greatly facilitate any practical implementation.

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A retrial queueing system with a batch Markovian arrival process and non-exponential inter-retrial times

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Retrial queueing systems describe the operation of many switching telephone systems, modern telecommunication networks, contact centers, etc. Such queues have been extensively studied under a variety of scenarios for single and multiple server cases, for references see, e.g., surveys [1, 2] and books [3, 4]. In the most of research the systems with a stationary Poisson input and exponential distributionof inter-retrial times are analyzed.

A small number of publications deals with *M/G/*1 and *M/M/*1 retrial queue with non-exponential inter-retrial time distribution. But all these publications consider so-called constant retrial policy. However, in most real-life systems where the effect of retrials is observed, systems operate under the classical retrial policy, where each orbital customer generates a flow of repeated attempts independently of the rest of the customers in the orbit. At the same time, as far as we know, retrial queues with classical retrial policy and non-exponential inter-retrial time distribution were considered only in articles [5, 6, 7].

In [5], the author developed an approximate method for calculating the steady state distribution of *M/G/*1 retrial queue with inter-retrial time that are mixtures of Erlangs. The authors of [6, 7] assume that the elapsed retrial time for any orbital customer is a random variable independent of other orbital customer's elapsed retrial times. Such an assumption greatly simplifies the mathematical analysis of the system.

In the present paper, we consider *BMAP/PH/N* retrial queue with alternating distribution of inter-retrial times. We assume that inter-retrial times have *PH* distribution if the number of customers in the orbit does not exceed some large threshold *K* and have exponential distribution otherwise. We suppose that, under a large value of *K* our model can be considered as a good approximation of the *BMAP/PHN*1 retrial queue with *PH* distribution of inter-retrial times. This supposition is based on our internal convictions which, in turn, are based on the theorems by A. Ya. Khinchin, G.A. Ososkov, B.I. Grigelionis about superpositions of the large number of small flows. Our model allows to some extent take into account the realistic nature of retrial process and, at the same time, to avoid a large increase in the dimensionality of the state space of this process.

Customers arrive at the system according a Batch Markovian Arrival Process (*BMAP*) which models well the correlated bursty traffic in modern telecommunication networks. Customers which find the servers busy enter the orbit of infinite size and try their luck after some random time. We suppose that inter-retrial times have phase-type distribution if the number of customers in the orbit does not exceed some threshold and have exponential distribution otherwise. We derive the condition for stable operation of the system, calculate the stationary distribution and the main performance measures of the system.

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An M/G/1 queueing system with differentiated vacation

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Mathematical formulation

We consider a model of *M/G/*1 type queue with differentiating (or adaptive) vacation. Customers arrive at the system according a Poisson process with rate λ and request for service which takes a random duration S with general probability distribution $H(x)=P(S<x)$, Laplace-Stieltjes transform $h(s)$ and first two moments h_1 and h_2.

The server starts a type I vacation when the server becomes idle. The length of this vacation follows an exponential distribution with mean $1/\gamma_1$. On returning from this vacation, if the server is still idle, it takes a type II vacation whose duration follows an exponential distribution with mean $1/\gamma_2$. Type II vacations are repeated as long the system is empty upon the completion of a vacation. On returning from either type I or II vacation, if there are some customers in the system, the server immediately start servicing customers until the system is empty again. All the considered random variables are assumed to be mutually independent and all moments are assumed to be finite. Such a model has been related recently with power-saving mode where the server is turned off in order to save energy in communication and computer systems [1–3]. Such a model has been analyzed using difference equations or in discrete time and revisited in [4] using generating function approach which conduct to more simple formulas, but in the Markovian context. In this paper we consider one of these models in a non-Markovian framework when the service time distribution is arbitrary distributed by using generating and Laplace transforms.

Joint probability distribution of server state and number of customers in the queue

Let $S(t)$ denote the server state

$$S(t)=\begin{cases}0, \text{ the server is busy by the sevice of a customer,}\\ 1, \text{ the server is on vacation of type I,}\\ 2, \text{ the server is on vacation of type II.}\end{cases}$$

We introduce also a continuous random variable $\xi(t)$ on $\mathbb{R}^+$ and which represents the residual service time at time t, if $S(t)=0$.

Let $N(t)$ be the number of customers in the system which is not Markovian. However, the stochastic process $\{S(t),\xi(t)\}$ is now a Markov process defined on the sate space

$$\{(0,j):j\in N\}\cup\left\{(1,j):j\in\mathbb{Z}^+\right\}\cup\left\{(2,j):j\in\mathbb{Z}^+\right\}\otimes\mathbb{R}^+.$$

Let

$$P_i(m)=\lim_{t\to\infty}P\{S(t)=i,N(t)=m\},i=1,2,m\in\mathbb{Z}^+$$

and

$$P_i(m,x)=\lim_{t\to\infty}P\{S(t)=i,N(t)=m,\xi(t)<x\},i=1,2,m\in\mathbb{Z}^+,x\in\mathbb{R}^+$$

the steady-state probabilities of system state.

These probabilities are solution of a system of partial differential equations in a discrete variable $m \in \mathbb{Z}^+$ and a continuous one $x \in \mathbb{R}^+$. This system is solved using Fourier transforms: Generating function for the discrete variable and Laplace function for the continuous one.

Using some properties of these transforms and after some algebraic calculations we obtain explicit expressions for the generating functions of the server state and number of customers in the queue and the generating function of the unconditional distribution of the number of customers in the queue. This yields all the required moments and some interesting optimal problems for practical purposes.

Remark 1. As noted in reference [3], if $\gamma_2 \to \infty$, the system converges to the M/G/1 queue with single vacation.

Remark 2. Following the work [3] we conjecture that we can exhibit a stochastic decomposition property for the number of customers in our M/G/1 queue

$$E(N) = E(N)^{FIFO} + E(N)_{1+2}^{VACATION}.$$

The first term is the number of customers in an M/G/1 FIFO-queue while the second term corresponds to the number of customers that arrive during the remaining time of vacations of type I or II. According to this remark, the computation of all performance measures will be simpler.

Remark 3. According to the distributional Little's law we can compute the Laplace – Stieltjes transform of the sojourn time distribution using formula

$$w(s) = Q\left(1 - \frac{s}{\lambda}\right)$$

and all the desired moments.

Conclusions

In this paper, we have considered a queueing model with differentiated vacation in the case of arbitrary distributed service time. It will be interesting to extend this model to general vacation time distributions for which the system equation is more complex. Some attention can be paid also to the other variants considered in [4].

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