Matrix-Geometric Solutions for the Models of Perishable Inventory Systems with a Constant Retrial Rate

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Abstract. The model of perishable inventory system with orbit is examined under (s, S) and (s, Q) policies. The stability condition of the system is derived and the joint distribution of the number of customers in orbit and the inventory level is obtained by using matrix-geometric method. Formulas for calculation of the performance measures are developed. The behavior of performance measures under given policies are analyzed and comparative numerical results are presented.

Keywords: perishable inventory system · repeated customers · orbit · (s, S) policy · (s, Q) policy · matrix-geometric method · calculation methods · performance measures

1 Introduction

One of the important class of inventory systems (IS) is a perishable inventory systems (PIS) in which an inventory life time is a finite random quantity, for example, blood banks, systems of processing an outdated information, food provision systems, etc. In PIS the inventory level decreasing not only after its release to a customer but also due to the end of inventory life time. Note that a survey works [1–3] and a monograph [4] contain references of numerous literature sources in this direction.

Here consideration is given to models of PIS without service facility. In other words, it is assumed that inventory immediately released to primary customers (p-customers) directly from a system store house, i.e., a service time of p-customers is equal to zero. It means that formation of queue of p-customers is impossible. However, formation of an orbit of repeated p-customers (retrial customers, r-customers) is possible.

An analysis of available literature showed that PIS models without service facility had not been sufficiently studied. The paper [5] has studied a model of PIS without service station of p-customers which applies (s, Q) replenishment policy. It means that when an inventory level decreases or equals a certain level (reorder point) s a delivery order for inventory of volume $Q = S - s$, with S being the maximum volume of the system store house, is sent to a higher store house. The mentioned paper assumes that

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the lead time is equal to zero, and the inventory life time has an exponential distribution function (d.f.). To study the inventory level, the one-dimensional birth and death process is used. Analogous models with a positive lead time have been studied in [6,7].

In [8] the Markovian model of non-perishable IS with instant service and $(S-1, S)$ policy (i.e. one-to-one ordering policy) being investigated. It is assumed the customers that occur during the stock-out periods enter into the orbit of infinite size. The joint probability distribution of the inventory level and the number of customers in the orbit are obtained in the steady state case by applying matrix-analytical method [9]. Various system performance measures in the steady state are derived.

Note that even in an IS with instant service the queue of p-customers can be formed when inventory level is zero. Such kind models of PIS have been studied in [10–13].

This paper is close in spirit to [8]. The main contributions of this paper are as follows: (i) We extend the model investigated in [8] by considering perishable inventory items; (ii) we take into account that arrived p-customers in accordance Bernoulli scheme either join the orbit or leave the system when the inventory level is zero; (iii) we take into account that r-customers might be impatient, i.e. if upon arrivals of the r-customer the inventory level is zero, then they in accordance Bernoulli scheme either leave the system or re-join the orbit; (iv) we consider different replenishment policies, i.e. here we assume that in the system might be applied (s, Q) or (s, S) policies.

Previously, a similar model with the (s, Q) policy was studied in [14] using an approximate method based on the principles of state space merging of two-dimensional Markov chains [15]. This approach allows to find simple formulas for calculating the performance measures of the system. However, despite the simplicity and effectiveness of the specified method, it can be accepted when certain conditions are met. So, in [14], this method is applied when the following condition is met: the total intensity of the arrival of primary customers and deterioration of inventory is much higher than the intensity of the arrival of retrial customers. If this condition is not met, then the accuracy of this method is significantly reduced. Based on this, in this paper, another numerical method is developed that does not impose any conditions on the initial parameters of the system.

The rest of the paper is organized as follows. The models under study are described in Sect. 2. In Sect. 3, we perform the steady-state analysis of the system under various policies. Firstly, the stability condition of the system is derived by using Level Independent Quasi-Birth-Death Process (LIQBD) theory. Then, the joint distribution of the number of customers in orbit and the inventory level is obtained by using MGM. Main performance measures are computed in Sect. 4. Results of numerical experiments are demonstrated in Sect. 5. Conclusions are given in Sect. 6.

2 Description of the PIS Models

The inventory system has a store house of limited volume S. It is assumed that each item of the inventory, independently of the others, becomes unusable after a random time that has an exponential d.f. with parameter γ, $\gamma > 0$. Input flow of p-customers' forms Poisson stream with rate λ. If at the moment of p-customer arrival the inventory level is positive, then it is instantly serviced and leaves the system; otherwise (i.e. when inventory level is zero) the customer with probability H_p either leaves for infinity orbit to repeat its inquiry or with complementary probability $1 - H_p$ eventually leaves the system. From orbit only r-customer on the head of orbit repeat its inquiry at random time which has exponential d.f. with parameter η, i.e. retrial rate is constant value and it is independent on the number of customers in the orbit. If at the moment of a r-customer arrival inventory level is positive, then such customer is instantly serviced and leaves an orbit; otherwise the r-customer either leaves an orbit with probability H_r or with complementary probability $1 - H_r$ stays there to repeat its inquiry. Here we consider two replenishment policies: (s, S) and (s, Q). In both policies lead time is positive random variables that has exponential d.f. with the meanν^{-1}.

The problem consists in determining a joint distribution of system inventory level and the number of r-customers. This problem solution will allow us to determine performance measures as well.

3 Computation of the Steady-State Probabilities

First consider model with (s, S) policy. Mathematical model of the investigated system is two dimensional Markov chain (2-D MC). States of the indicated 2D MC are defined by 2D vectors (n, m), where n is total number of customers in orbit, $n = 0, 1, ...,$ and m is denote the inventory level, $m = 0, 1, ..., S$. State space of the indicated 2D MC is given by

$$E = \bigcup_{n=0}^{\infty} L(n), \tag{1}$$

where $L(n) = \{(n, 0), (n, 1), ..., (n, S)\}$ called the nth level, $n = 0, 1, 2, ...$

The transition rate from the state $(n_1, m_1) \in E$ to the state $(n_2, m_2) \in E$ is denoted by $q((n_1, m_1), (n_2, m_2))$. The set of all these rates forms the generator of the 2D MC. According to the accepted service scheme and replenishment policy, we obtain the following relations for the determining of the indicated transitions:

$$q((n_1, m_1), (n_2, m_2)) = \begin{cases} \lambda + m_1\gamma & \text{if } m_1 > 0,\ (n_2, m_2) = (n_1, m_1 - 1), \\ \eta & \text{if } n_1 m_1 > 0,\ (n_2, m_2) = (n_1 - 1, m_1 - 1), \\ \eta H_r & \text{if } n_1 > 0,\ m_1 = 0,\ (n_2, m_2) = (n_2 - 1, m_1), \\ \lambda H_p & \text{if } m_1 = 0,\ (n_2, m_2) = (n_1 + 1, m_1), \\ \nu & \text{if } m_1 \le s,\ (n_2, m_2) = (n_1, S), \\ 0 & \text{in other cases.} \end{cases} \tag{2}$$

Hereinafter, the equality of vectors means that their corresponding components are equal to each other.

States from the space E is renumbered in lexicographical order as follows $(0, 0), (0, 1), ..., (0, S), (1, 0), (1, 1), ..., (1, S), ...$ Then indicated 2D MC represents LIQBD for which generator has the following three diagonal form:

$$G = \begin{pmatrix} B & A_0 & . & . & . \\ A_2 & A_1 & A_0 & . & . \\ . & A_2 & A_1 & A_0 & . \\ . & . & . & . & . \\ . & . & . & . & . \end{pmatrix} \tag{3}$$

All block matrices in (3) are square matrices of dimension $S+1$. From relations (2) we conclude that entities of the block matrices $B = \left\| b_{ij} \right\|$ and $A_k = \left\| a_{ij}^{(k)} \right\|$, $i, j = 0, 1, ..., S$, are determined as follows:

$$b_{ij} = \begin{cases} \nu & \text{if } i \le s,\ j = S, \\ \lambda + i\gamma & \text{if } i > 0,\ j = i-1, \\ -\left(\nu + \lambda H_p\right) & \text{if } i = j = 0, \\ -(\nu + i\gamma + \lambda) & \text{if } 0 < i \le s,\ i = j, \\ -(i\gamma + \lambda) & \text{if } s < i \le S,\ i = j, \\ 0 & \text{in other cases;} \end{cases} \tag{4}$$

$$a_{ij}^{(0)} = \begin{cases} \lambda H_p & \text{if } i = j = 0, \\ 0 & \text{in other cases;} \end{cases} \tag{5}$$

$$a_{ij}^{(1)} = \begin{cases} \nu & \text{if } 0 \le i \le s,\ j = S, \\ \lambda + i\gamma & \text{if } i > 0,\ j = i-1, \\ -\left(\nu + \lambda H_p + \eta H_r\right) & \text{if } i = j = 0, \\ -(\nu + i\gamma + \lambda + \eta) & \text{if } 0 < i \le s,\ i = j, \\ -(i\gamma + \lambda + \eta) & \text{if } i > s,\ i = j, \\ 0 & \text{in other cases;} \end{cases} \tag{6}$$

$$a_{ij}^{(2)} = \begin{cases} \eta H_r & \text{if } i = j = 0, \\ \eta & \text{if } i > 1,\ j = i-1, \\ 0 & \text{in other cases.} \end{cases} \tag{7}$$

Let $A = A_0 + A_1 + A_2$. Stationary distribution that correspond to the generator A is denoted by $\pi = (\pi(0), \pi(1), ..., \pi(S))$, i.e. we have

$$\pi A = \mathbf{0}, \pi \mathrm{e} = 1, \tag{8}$$

where $\mathbf{0}$ is null row vector of dimension $S+1$ and $\boldsymbol{e}$ is column vector of dimension $S+1$ that contains only 1's.

From relations (5)–(7) we obtain that entities of generator $A = \left\| a_{ij} \right\|$, $i, j = 0, 1, ..., S$,are determined as

$$a_{ij} = \begin{cases} -\nu & \text{if } i = j = 0, \\ \nu & \text{if } 0 \le i \le s,\ j = S, \\ \lambda + i\gamma + \eta & \text{if } i > 0,\ j = i-1, \\ -(\lambda + i\gamma + \nu + \eta) & \text{if } 0 < i \le s,\ j = i, \\ -(\lambda + i\gamma + \eta) & \text{if } i > s,\ j = i, \\ 0 & \text{in other cases.} \end{cases} \tag{9}$$

Proposition 1. Under (s, S) policy the investigated system is ergodic if and only if the following relation is fulfilled:

$$\lambda H_p \pi(0) < \eta (1 - (1 - H_r) \pi(0)). \tag{10}$$

Proof. From relations (9) we obtain that system of equations (8) has following explicit form:

$$(\nu + (m\gamma + \lambda + \eta)(1 - \delta_{m,0}))\pi(m) = ((m+1)\gamma + \lambda + \eta)\pi(m+1)\ , 0 \le m \le s; \quad (11)$$

$$\begin{aligned}&(m\gamma + \lambda + \eta)\pi(m) = ((m+1)\gamma + \lambda + \eta)\pi(m+1)\chi(s+1 \le m \le S-1)\\&+\nu\textstyle\sum_{m=0}^{s}\pi(m)\delta_{m,S},\ s+1 \le m \le S.\end{aligned} \quad (12)$$

Hereinafter $\delta_{x,y}$ are denote Kronecker delta and $\chi(A)$ is indicator function of event A.

From (11) and (12) all values $\pi(m)$, $m = 1, ..., S$, are expressed by $\pi(0)$ as follows:

$$\pi(m) = \begin{cases} a_m\pi(0), \text{ if } 1 \le m \le s+1,\\ b_m\pi(0), \text{ if } s+1 < m \le S,\end{cases} \quad (13)$$

where $a_m = \prod_{i=1}^{m}\frac{\Lambda_{i-1}+\nu}{\Lambda_i}$; $b_m = \frac{\Lambda_{s+1}}{\Lambda_m}\prod_{i=1}^{s+1}\frac{\Lambda_{i-1}+\nu}{\Lambda_i}$; $\Lambda_i = \lambda + \eta + i\gamma$, $i = 1, 2, ..., S$.

The probability $\pi(0)$ is determined from normalizing condition, i.e.

$$\pi(0) = \left(1 + \sum_{m=1}^{s+1} a_m + \sum_{m=s+2}^{S} b_m\right)^{-1}.$$

In accordance to [9] (chapter 3, pages 81–83) investigated LIQBD is ergodic if and only if the following condition is fulfilled:

$$\pi A_0 e < \pi A_2 e. \quad (14)$$

By taking into account (5), (7) and (13) after some algebras from (14) we obtain that relation (10) is true.

Note 1. The ergodicity condition (10) has probabilistic meaning. Indeed, left side of (10) is equal to the rate of the primary customers in the orbit subject to inventory level is zero while right side of (10) represent weighted average total rate of the retrial customers leaving with the purchase of inventory (when the inventory level is positive) and without purchase of inventory (when inventory level is zero). Therefore, relation (10) means the following: the conditional rate of primary customers to orbit should be less than the weighted average total rate of retrial customers leaving the system.

Special Cases: 1) Fully impatient retrial customers, i.e. when $H_r = 1$ we have following ergodicity condition: $\lambda H_p\pi(0) < \eta$. 2) Patient retrial customers, when $H_r = 0$ we have following ergodicity condition: $\lambda H_p\pi(0) < \eta\ (1 - \pi(0))$. In both cases, if in addition we set $H_p = 1$, then ergodicity condition requires that the rate of primary customers to orbit should be less than the rate of retrial customers leaving the system. Note that in all cases, ergodicity condition depends on size of warehouse, as well as on perish rate of inventory and lead time.

Steady-state probabilities that corresponds to the generator matrix G we denote by $p = (p_0, p_1, p_2, ...)$, where $p_n = (p(n,0), p(n,1), ..., p(n,S))$, $n = 0, 1, ...$ Under the ergodicity condition (10) the steady-state probabilities are calculated from the following equations:

$$p_n = p_0 R^n,\ n \ge 1, \tag{15}$$

where R is nonnegative minimal solution of the following quadratic matrix equation:

$$R^2 A_2 + RA_1 + A_0 = 0.$$

Bound probabilities p_0 are determined from following system of equations with normalizing condition:

$$\begin{aligned} p_0\,(B + RA_2) &= \mathbf{0}. \\ p_0\,(I - R)^{-1}\,e &= 1, \end{aligned} \tag{16}$$

where I is indicated identity matrix of dimension $S+1$.

Now consider model with (s, Q)policy. State space for the model under (s, Q) policy is same with previous one, i.e. it is defined by set E and generator of appropriate 2D MC is determined by following relations:

$$q\left((n_1, m_1), (n_2, m_2)\right) = \begin{cases} \lambda + m_1\gamma & \text{if } m_1 > 0,\ (n_2, m_2) = (n_1, m_1 - 1), \\ \eta & \text{if } n_1 m_1 > 0,\ (n_2, m_2) = (n_1 - 1, m_1 - 1), \\ \eta H_r & \text{if } n_1 > 0,\ m_1 = 0,\ (n_2, m_2) = (n_2 - 1, m_1), \\ \lambda H_p & \text{if } m_1 = 0,\ (n_2, m_2) = (n_1 + 1, m_1), \\ \nu & \text{if } m_1 \le s,\ (n_2, m_2) = (n_1, m_1 + S - s), \\ 0 & \text{in other cases.} \end{cases} \tag{17}$$

By using the indicated above lexicographical order of renumbering of states we conclude that for this model generator matrix has the following form:

$$\tilde{G} = \begin{pmatrix} \tilde{B} & A_0 & . & . & . \\ A_2 & \tilde{A}_1 & A_0 & . & . \\ . & A_2 & \tilde{A}_1 & A_0 & . \\ . & . & . & . & . \\ . & . & . & . & . \end{pmatrix}$$

Entities of matrices $\tilde{B}$ and $\tilde{A}_1$ in $\tilde{G}$ are calculated as follows:

$$\tilde{b}_{ij} = \begin{cases} \nu & \text{if } i \le s,\ j = i + S - s, \\ \lambda + i\gamma & \text{if } i > 0,\ j = i - 1, \\ -\left(\nu + \lambda H_p\right) & \text{if } i = j = 0, \\ -(\nu + i\gamma + \lambda) & \text{if } 0 < i \le s,\ i = j, \\ -(i\gamma + \lambda) & \text{if } s < i \le S,\ i = j, \\ 0 & \text{in other cases;} \end{cases} \tag{18}$$

$$\tilde{a}_{ij}^{(1)} = \begin{cases} \nu & \text{if } 0 \le i \le s,\ j = i + S - s, \\ \lambda + i\gamma & \text{if } i > 0,\ j = i - 1, \\ -\left(\nu + \lambda H_p + \eta H_r\right) & \text{if } i = j = 0, \\ -(\nu + i\gamma + \lambda + \eta) & \text{if } 0 < i \le s,\ i = j, \\ -(i\gamma + \lambda + \eta) & \text{if } i > s,\ i = j, \\ 0 & \text{in other cases;} \end{cases} \tag{19}$$

Therefore, from relations (5), (7) and (19) we obtain that in this model entities of generator $\tilde{A} = A_0 + \tilde{A}_1 + A_2$ are determined as

$$\tilde{a}_{ij} = \begin{cases} -\nu & \text{if } i = j = 0, \\ \nu & \text{if } 0 \le i \le s,\ j = i + S - s, \\ \lambda + \eta + i\gamma & \text{if } i > 0,\ j = i - 1, \\ -(\lambda + \eta + i\gamma + \nu) & \text{if } 0 < i \le s,\ j = i, \\ -(\lambda + i\gamma + \eta) & \text{if } i > s,\ j = i, \\ 0 & \text{in other cases.} \end{cases} \quad (20)$$

Proposition 2. Under (s, Q) policy the investigated system is ergodic if and only if the condition (10) is fulfilled where $\pi(0)$ is defined as $\pi(0) = c_0\pi(s+1)$, where

$$\pi(s+1) = \left(\sum_{m=0}^{s} c_m + \sum_{m=s+1}^{S-s} d_m + \sum_{m=S-s+1}^{S} f_m \right)^{-1};$$

$$c_m = \prod_{i=m+1}^{s+1} \frac{\Lambda_i}{\nu + \Lambda_{i-1}};\ d_m = \frac{\Lambda_{s+1}}{\Lambda_m};\ f_m = \frac{\nu}{\Lambda_m} \sum_{i=m-S+s}^{s} c_i .$$

Proof. From relations (20) we obtain that balance equations for state probabilities $\pi(m)$, $0 \le m \le s$, coincide with equations (11) and balance equations for state probabilities $\pi(m)$, $s+1 \le m \le S$, has following explicit form:

$$\begin{aligned} &(m\gamma + \lambda + \eta)\pi(m) = ((m+1)\gamma + \lambda + \eta)\pi(m+1)\chi(s+1 \le m \le S - s) \\ &+\nu\pi(m - S + s)\chi(S - s + 1 \le m \le S),\ s+1 \le m \le S. \end{aligned} \quad (21)$$

From (11) and (21) we obtain

$$\pi(m) = \begin{cases} c_m\pi(s+1), \text{ if } 0 \le m \le s, \\ d_m\pi(s+1), \text{ if } s+1 \le m \le S - s, \\ f_m\pi(s+1), \text{ if } S - s + 1 \le m \le S. \end{cases} \quad (22)$$

By taking into account (22) after some algebras from (14) we conclude that the fact stated above is true. Further by using system of equations (15) and (16) the steady-state probabilities for this model are calculated.

4 Performance Measures

In both models performance measures are calculated via steady-state probabilities. Main performance measures are following ones.

Average inventory level (S_{av}):

$$S_{av} = \sum_{m=1}^{S} m \sum_{n=0}^{\infty} p(n, m);$$

Average order size under (s, S) policy (V_{av}):

$$V_{av} = \sum_{m=S-s}^{S} m \sum_{n=0}^{\infty} p(n, S - m);$$

Note 2. Average order size under (s, Q) policy is constant and equal to $S - s$.

Average number of customers in orbit (L_o):

$$L_o = \sum_{n=1}^{\infty} n \sum_{m=0}^{S} p(n, m);$$

Average reorder rate (RR):

$$RR = (\lambda + (s+1)\gamma) \sum_{n=0}^{\infty} p(n, s+1) + \eta \sum_{n=1}^{\infty} p(n, s+1);$$

Loss probability of p-customers $\left(P_p\right)$:

$$P_p = \left(1 - H_p\right) \sum_{n=0}^{\infty} p(n, 0);$$

Loss probability of r-customers (P_r):

$$P_r = H_r \sum_{n=1}^{\infty} p(n, 0).$$

5 Numerical Results

In this section results of numerical experiments will be discussed and presented. The behavior of performance measures vs s under (s, S) and (s, Q) policies are depicted in Fig. 1 and Fig. 2.

We used the following parameters for numerical experiments:

$$\lambda = 40, \eta = 25, H_p = 0.7, H_r = 0.3, \nu = 10, \gamma = 25, S = 20$$

S_{av} under (s, S) policy is increasing with the increase of s as opposed to (s, Q). This behavior is expected as with higher s the inventory is replenished more frequently up to S which results in higher average inventory level. But under (s, Q) the replenishment amount is fixed $(S - s)$ and becomes lower with higher s which in turn results in lower average inventory level. Average order size V_{av} is also proportional to s which is reflected in graph. We excluded (s, Q) series from V_{av} as it is fixed for given s. RR is also lower under (s, S) policy due to higher average inventory level.

The average number of customers L_o is lower for (s, S) policy because of higher inventory level S_{av}. The higher inventory level results in more number of served customers that in turn keeps the average orbit size lower. Customer loss probabilities decrease for higher values of s due to higher S_{av} under (s, S) policy. In contrary, under (s, Q) policy for the higher values of s the slight increase in P_p and P_r is observed due to lower S_{av} which results in lesser number of served customers.

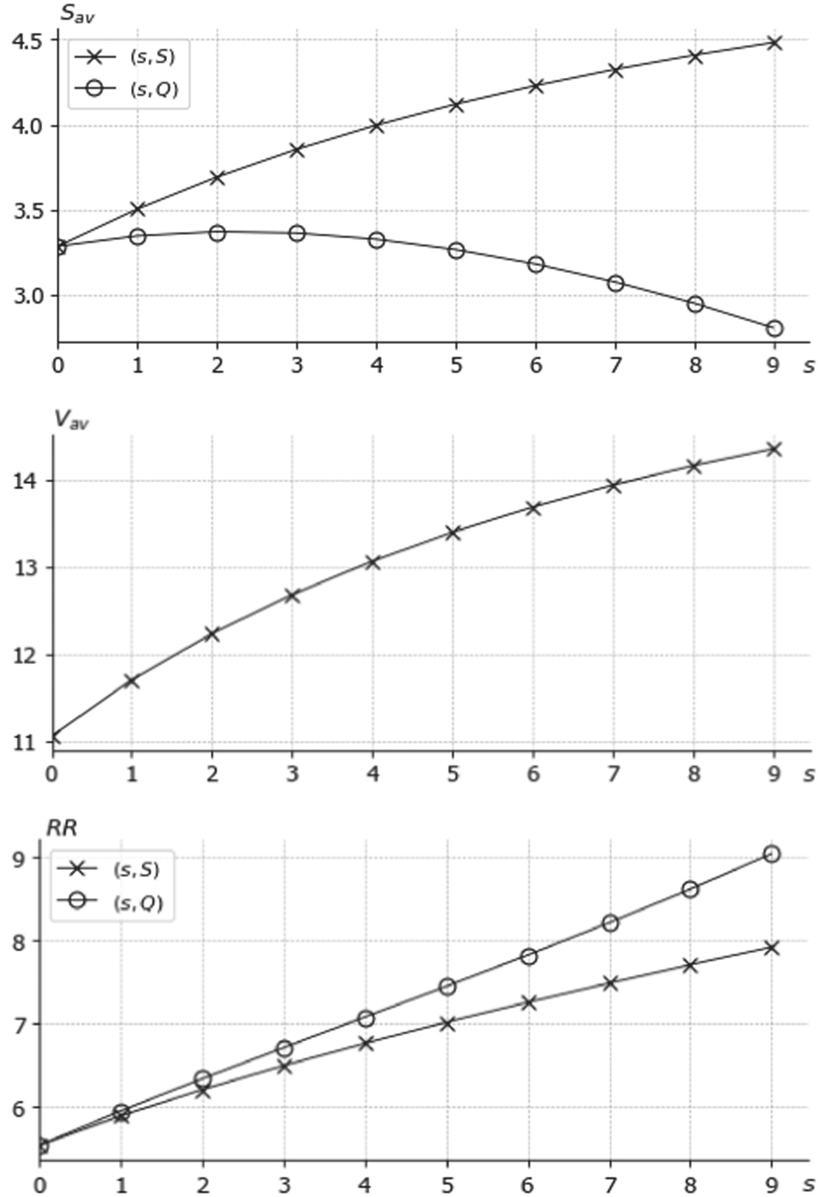

Fig. 1. Dependence of inventory related performance measures on the reorder level s under (s, S), (s, Q) policies

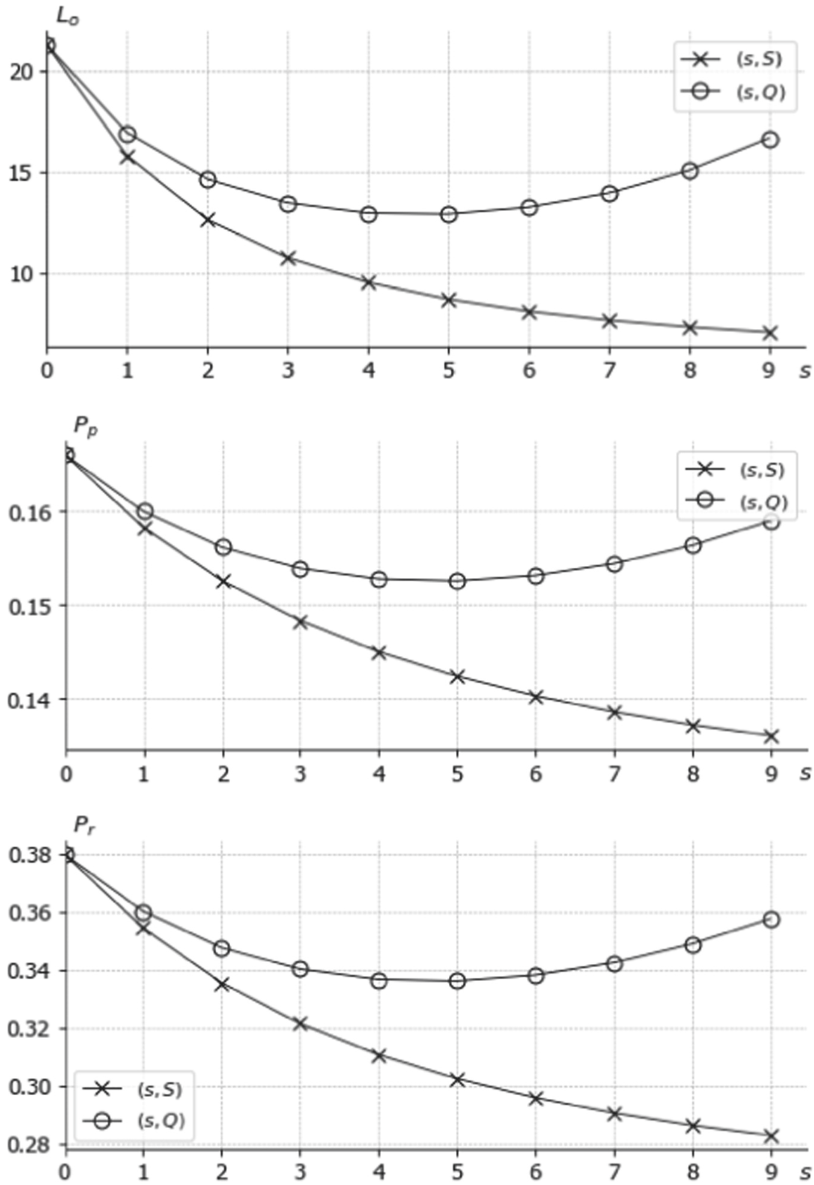

Fig. 2. Dependence of customer related performance measures on the reorder level s under (s, S), (s, Q) policies

6 Conclusion

The queuing-inventory model with perishable inventory and infinite orbit size was presented under (s, S) and (s, Q) replenishment policies. Joint distribution of the inventory level and the number of customers in the orbit was found using matrix-geometric method. Formulas for performance measures were developed. Numerical experiments were performed using developed formulas. The behavior of performance measures were analyzed under both replenishment policies and results were analyzed and presented in graphical forms.

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