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Modeling Cellular Networks Using MOSEL

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Abstract

This paper investigates a multiserver infinite-source retrial queueing system for the performance modeling of cellular mobile communication networks. The influence of customer redials in case of blocked fresh and dropped handoff calls and the guard channel scheme are included in the model.

The objective is to demonstrate how performance tool MOSEL (Modeling, Specification and Evaluation Language) can be efficiently used in the modeling of cell based networks. The novelty of our analysis is that the blocked and dropped users are treated separately, that is they redial with different probabilities and different rates, with reducing the state space by maximizing the number of redialing customers with appropriately large values (i.e. when the ignored probability mass can be neglected). Furthermore, not only the active but also both types of redialing customers are allowed to depart to other cells, which was not the case in the previous works.

As the benefit of the tool the effects of various system parameters on the fresh call blocking probability, on the handoff call dropping probability and on the grade of service are displayed and analyzed graphically.

Keywords: Retrial queues, Cellular systems, Performance tools, MOSEL tool, Grade of service.

1 Introduction

Queueing network models are widely used in the traffic modeling of cellular mobile systems, such as GSM (Global System for Mobile Communications), GPRS (General Packet Radio Service) and UMTS (Universal Mobile Telecommunication System). Most of the papers consider queueing systems without retrials (see [1], [2] and references therein for some recent results), but after the study of Tran-Gia and Mandjes [3], which demonstrated in the context of cellular

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systems that the retrial phenomenon is not neglectable because of the significant negative influence on the system performance measures, authors more and more take it into consideration in their cellular mobile network model.

The main characteristic of retrial queues (or queueing systems with repeated attempts) is that if an arriving customer finds all servers busy, he leaves the service area, but after some random time repeats his demand. For some fundamental results on retrial queues, see for example: [4], [5].

Cellular systems with customer redials are treated in [6], where an approximate technique is proposed for finite and infinite population Markovian models. The authors reduce the state space of the continuous-time Markov chain model by registering only that if there are retrying blocked and dropped customers in the system or not. In the works [7] and [8], various infinite-source queueing models are studied. In [7], not only customer redials, but also automatic retrials by the cellular system are taken into consideration, but the dropped customer redials handled as generating new fresh call attempts in the new cell and in case of blocking the call is treated as a blocked fresh call. It is probably less realistic, because an interrupted customer may try to reestablish the call with higher probability in shorter time intervals. In [8], the blocked new and dropped handoff calls are not distinguished, but the involved random variables have general phase type distributions.

In this paper, we discuss an infinite-source retrial queueing model of GSM networks, based upon to the ones that were studied by [3], [6], [7] and [8]. We calculate the main system measures quite easily using the efficient software tool MOSEL, developed at the University of Erlangen, Germany (see [9] and [10]). It makes possible to analyze more difficult models, what often not feasible because of the largeness of the state space and the difficulty of the calculations. The blocked and dropped users are treated separately, that is they redial with different probabilities and different rates, like in [6], but we reduce the state space by maximizing the number of redialing customers with appropriately large values (i. e. when the ignored probability mass can be neglected). In [3], [7] and [8], these two types of redialing customers were not distinguished. Furthermore, in our model we allow not only the active but also both types of redialing customers to depart to other cells, what was not allowed in the previous works. The current study can be considered as an initial step towards the analysis of more complex third generation systems focusing on the quality of service issues.

In cellular networks, the most important quality of service measures are the following:

- the fresh call blocking probability \( (P_f) \), i. e. the fraction of new call requests in the cell that cannot be served due to the lack of free channels, and
- the handoff call dropping probability \( (P_h) \), that is the average fraction of incoming handoff calls that are terminated because of the lack of free channels.

The grade of service (GoS) is generally defined as the combination of these two probabilities, for example as

\[
GoS = \frac{P_f + 10P_h}{11}.
\]

Because of the fact, that the handoff call dropping probability has more significant impact on the grade of service, it is important to reduce it even at the expense of increased fresh call blocking probability. In order to prioritize handoff calls, several channel allocation schemes are utilized. One of the most popular policies is the guard channel scheme [2], [3], [6], [8],
where some channels are reserved for the calls that move across the cell boundary, that is if there are $g$ reserved channels in the cell, a new fresh call is only accepted if there are at least $g + 1$ available channels. A handoff call is rejected only if all the channels in the cell are occupied.

The paper is organized as follows. In Section 2, the accurate description of the cellular model is given, and in Section 3 it is shown how it can be translated into the description language of MOSEL. Section 4 is devoted to some numerical examples, where the analytical results of the calculations are displayed graphically to demonstrate the effect of the changing of various system parameters on the quality of service measures and on the grade of service. Conclusions and directives for the future work are given in Section 5.

## 2 Model description

In this section we consider the following cell model (illustrated by Figure 1) in a cellular mobile network.

![Figure 1: Retrial queueing model of a cell](image)

In our cellular network model we treat only one cell. The cells are considered identical and to have the same traffic parameters, so it is enough to investigate one cell, and the handoff effect from the adjacent cells to this cell and from this cell to adjacent cells is described by handoff processes. Instead of the frequently used single arrival stream model we distinguish the fresh call and handoff call arrivals, what is gainful if we investigate complex call handling policies.

We assume, that the number of channels in the cell is $C$, and the number of guard channels is $g$, where $g < C$.

The arrival process of the fresh calls is a Poisson process with rate $\lambda_f$. If the number of the active users is smaller than $C - g$, the incoming call starts to be served. Otherwise it is blocked and it starts generation of a Poisson flow of repeated calls (redialing) with probability $\Theta_1$ or leaves the system with probability $1 - \Theta_1$. A blocked customer repeats his call after a random
time which is exponentially distributed with mean $1/\nu_{dr}$, and it can be served or blocked again like the fresh calls. The call duration time is exponentially distributed with mean $1/\mu$.

The arrival process of the handoff calls is a Poisson process with rate $\lambda_h$. If the number of active users is smaller than $C$, the incoming call starts to be served. Otherwise it is dropped (handoff failure) and it starts generation of a Poisson flow of repeated calls with probability $\Theta_2$ or leaves the system with probability $1-\Theta_2$. A dropped customer tries to repeat his call after a random time which is exponentially distributed with mean $1/\nu_{dr}$. If it is blocked it continues redialing with probability $\Theta_2$. The call duration time for handoff calls is also exponentially distributed with mean $1/\mu$.

The active, redialing blocked and dropped customers leave the cell after an exponentially distributed time with mean $1/\mu_a$, $1/\mu_b$ and $1/\mu_{dr}$, respectively.

The number of redialing users because of blocking and dropping is limited to an appropriately large values of $N_{tr}$ and $N_{dr}$ to make the state space finite in order to make the calculations possible by the tools in the steady state.

2.1 The underlying Markov chain

The state of the system can be described with a stochastic process $X(t) = (C(t); N(t); M(t))$, where $C(t)$ is the number of active customers (i.e. the number of busy channels), $N(t)$ is the number of blocked new customers who are sending repeated calls and $M(t)$ is the number of dropped customers at handoff who are trying to redial at time $t$.

Because of the exponentiality of the involved random variables the describing process is a Markov chain with a finite state space $S = \{0, ..., C\} \times \{0, ..., N_{tr}\} \times \{0, ..., N_{dr}\}$. Since its state space is finite, the process is ergodic for all values of the rate of the arrival of new and handoff calls, and we can investigate it in the steady state.

We define the stationary probabilities:

$$P(i; j; k) = \lim_{t \to \infty} P(C(t) = i, N(t) = j, M(t) = k),$$

$i = 0, ..., C$,  $j = 0, ..., N_{tr}$,  $k = 0, ..., N_{dr}$.

Because of the fact that the state space of $(X(t), t \geq 0)$ with sufficiently large $N_{tr}$ and $N_{dr}$ is very large and the functioning of the system is complex, it is very difficult to calculate the steady state probabilities. To simplify these calculations and to make our study more usable in practice, we use the software tool MOSEL to formulate the model and to calculate these probabilities and the system measures. MOSEL has already been used, and it has proved its applicability for the modeling of several computer and communication systems. For some examples about computer systems see [11] (which results are identical to the calculated results of [12]), [13], [14] and in the context of cellular systems [10], [15], [16]. The MOSEL description can be translated automatically into the language of various performance tools and then analyzed by the appropriate tools (at present SPNP – Stochastic Petri Net Package and TimeNET are supported and suitable for this model) to get these measures.

Knowing the steady state probabilities the system performance and the quality of service measures can be obtained as follows.

- The mean number of active customers
\[ N_c = \sum_{i=0}^{C} \sum_{j=0}^{N_{bl}} \sum_{k=0}^{N_{dr}} i P(i, j, k). \]

- **The mean number of sources of repeated calls because of the blocking of fresh calls**

\[ N_{bl} = \sum_{i=0}^{C} \sum_{j=0}^{N_{bl}} \sum_{k=0}^{N_{dr}} j P(i, j, k). \]

- **The mean number of sources of repeated calls because of the dropping of handoff calls**

\[ N_{dr} = \sum_{i=0}^{C} \sum_{j=0}^{N_{bl}} \sum_{k=0}^{N_{dr}} k P(i, j, k). \]

- **Fresh call blocking probability**

\[ P_f = \sum_{i=0}^{g} \sum_{j=0}^{N_{bl}} \sum_{k=0}^{N_{dr}} P(C - i, j, k). \]

- **Handoff call dropping probability**

\[ P_h = \sum_{j=0}^{N_{bl}} \sum_{k=0}^{N_{dr}} P(C, j, k). \]

### 3 Model conversion to MOSEL

In this section we discuss the translation of the model into the language of the MOSEL tool. The full MOSEL program can be assembled from the following program parts among the model description in the order of the part numbers.

The number of channels in the cell is \( C \), which is denoted as \( N_{CHS} \) in the program, and the number of guard channels is \( g \), which is denoted as \( N_{G_CHS} \).

In the first part of the MOSEL description, we have to define some other system parameters too, these will be introduced at the appropriate program parts.

(1) \( \text{CONST N_CHS := 15; } \)
\( \text{CONST N_G_CHS := 1; } \)
\( \text{CONST MAX_BLOCKED_USERS := 25; } \)
\( \text{CONST MAX_DROPPED_USERS := 25; } \)
\( \text{CONST call_arrive := 0.5; } \)
\( \text{CONST call_retry_bl := 5; } \)
\( \text{CONST call_retry_dr := 6; } \)
\( \text{CONST call_duration := 0.05; } \)
\( \text{CONST handoff_arrive := 0.4; } \)
\( \text{CONST handoff_departure_ac := 1/3; } \)
\( \text{CONST handoff_departure_bl := 1/3; } \)
\( \text{CONST handoff_departure_dr := 1/3; } \)
The state of the system is described by the number of active users, the number of blocked users who redial after some random time, and the number of users whose calls are dropped at handoff and who are redialing. It can be wrote down in MOSEL as defining the nodes of the system. The number of active users is denoted by $active\_users$. Its maximum value is the number of channels, and it is 0 at the starting time. The number of redialing users because of blocking and dropping is limited to $MAX\_BLOCKED\_USERS$ and $MAX\_DROPPED\_USERS$, which are defined in (1).

(2) NODE $active\_users[N\_CHS] := 0$;
NODE $redialing\_users\_bl[MAX\_BLOCKED\_USERS] := 0$;
NODE $redialing\_users\_dr[MAX\_DROPPED\_USERS] := 0$;

The arrival process of the fresh calls is a Poisson process with rate $\lambda_f$, that is denoted in the program as $call\_arrive$, and defined in (1) like the other parameters. If the number of active users is smaller than $C - g$, the incoming call starts to be served. Otherwise it is blocked and it starts generation of a Poisson flow of repeated calls (redialing) with probability $\Theta_1$ (denoted by $p\_retry\_bl$) or leaves the system with probability $1 - \Theta_1$.

(3) IF $active\_users < N\_CHS-N\_G\_CHS$
FROM EXTERN TO $active\_users$ RATE $call\_arrive$;
IF $active\_users >= N\_CHS-N\_G\_CHS$
FROM EXTERN RATE $call\_arrive$ THEN{
TO $redialing\_users\_bl$ WEIGHT $p\_retry\_bl$;
TO EXTERN WEIGHT $1 - p\_retry\_bl$;
}

The blocked user redials can be handled similar to the fresh call arrivals. If a user is blocked, he repeats his call after a random time which is exponentially distributed with mean $1/\nu_{bl}$. $\nu_{bl}$ is denoted as $call\_retry\_bl$. It can be served or blocked as the fresh calls in the previous part.

(4) IF $active\_users < N\_CHS-N\_G\_CHS$
FROM $redialing\_users\_bl$ TO $active\_users$
RATE $call\_retry\_bl*redialing\_users\_bl$;
IF $active\_users >= N\_CHS-N\_G\_CHS$ FROM $redialing\_users\_bl$
RATE $call\_retry\_bl*redialing\_users\_bl$ THEN{
TO $redialing\_users\_bl$ WEIGHT $p\_retry\_bl$;
TO EXTERN WEIGHT $1 - p\_retry\_bl$;
}

The call duration time is exponentially distributed with mean $1/\mu$. $\mu$ is denoted as $call\_duration$.

(5) FROM $active\_users$ TO EXTERN
RATE $call\_duration*active\_users$;
The arrival process of the handoff calls is a Poisson process with rate $\lambda_h$. $\lambda_h$ is denoted in the program as $\text{handoff\_arrive}$. If the number of active users is smaller than $C$, the incoming call starts to be served. Otherwise it is dropped and it starts generation of a Poisson flow of repeated calls with probability $\Theta_2$ (denoted by $p \_\text{retry\_dr}$) or leaves the system with probability $1 - \Theta_2$.

(6) IF $\text{active\_users} < \text{N\_CHS}$  
FROM EXTERN TO $\text{active\_users}$ RATE $\text{handoff\_arrive}$;  
IF $\text{active\_users} = \text{N\_CHS}$  
FROM EXTERN RATE $\text{handoff\_arrive}$ THEN{
    TO $\text{redialing\_users\_dr}$ WEIGHT $p \_\text{retry\_dr}$;  
    TO EXTERN WEIGHT $1 - p \_\text{retry\_dr}$;
}

The dropped user redials can be handled like the blocked fresh call redials. The customer repeats his call after a random time which is exponentially distributed with mean $1/\nu_{dr}$. $\nu_{dr}$ is denoted as $\text{call\_retry\_dr}$. If it is blocked it continues retrying with probability $\Theta_2$ ($p \_\text{retry\_dr}$).

(7) IF $\text{active\_users} < \text{N\_CHS}-\text{N\_G\_CHS}$  
FROM $\text{redialing\_users\_dr}$ TO $\text{active\_users}$  
RATE $\text{call\_retry\_dr}\_\text{redialing\_users\_dr}$;  
IF $\text{active\_users} = \text{N\_CHS}-\text{N\_G\_CHS}$ FROM $\text{redialing\_users\_dr}$  
RATE $\text{call\_retry\_dr}\_\text{redialing\_users\_dr}$ THEN{
    TO $\text{redialing\_users\_dr}$ WEIGHT $p \_\text{retry\_dr}$;  
    TO EXTERN WEIGHT $1 - p \_\text{retry\_dr}$;
}

The active and redialing customers leave the cell after an exponentially distributed time with parameter $\mu_a$, $\mu_b$ and $\mu_d$, denoted as $\text{handoff\_departure\_ac}$, $\text{handoff\_departure\_bl}$ and $\text{handoff\_departure\_dr}$, respectively.

(8) FROM $\text{active\_users}$ TO EXTERN  
RATE $\text{handoff\_departure\_ac}\_\text{active\_users}$;  
FROM $\text{redialing\_users\_bl}$ TO EXTERN  
RATE $\text{handoff\_departure\_bl}\_\text{redialing\_users\_bl}$;  
FROM $\text{redialing\_users\_dr}$ TO EXTERN  
RATE $\text{handoff\_departure\_dr}\_\text{redialing\_users\_dr}$;

After describing the system functioning, we can define the system measures we would like to calculate, such as the mean number of active and redialing customers because of blocking and handoff failure, the fresh call blocking and the handoff call dropping probabilities.

(9) PRINT $\text{mean\_active\_users} = \text{MEAN}(\text{active\_users})$;  
PRINT $\text{mn\_redialing\_users\_bl} = \text{MEAN}(\text{redialing\_users\_bl})$;  
PRINT $\text{mn\_redialing\_users\_dr} = \text{MEAN}(\text{redialing\_users\_dr})$;  
PRINT $\text{call\_blocking\_prob} =$  
    $\text{PROB}(\text{active\_users} = \text{N\_CHS}-\text{N\_G\_CHS})$;  
PRINT $\text{handoff\_call\_dropping\_prob} =$  
    $\text{PROB}(\text{active\_users} = \text{N\_CHS})$;
Finally, we define two pictures that show the changing of the blocking and dropping probabilities depending on the number of channels. If we use \( N_{CHS} \) as parameter, we have to define it in (1) as follows: \( \text{PARAMETER } N_{CHS} := 6, 7, 8, 9, 10; \)

\[ (10) \text{ PICTURE "Blocking probability versus number of channels"} \]
\[ \text{PARAMETER } N_{CHS} \text{ CURVE call\_blocking\_prob;} \]
\[ \text{PICTURE "Dropping probability versus number of channels"} \]
\[ \text{PARAMETER } N_{CHS} \text{ CURVE handoff\_call\_dropping\_prob;} \]

4 Numerical examples

In this section we consider some sample numerical results to illustrate graphically how the system measures depend on variable system parameters.

In Figures 2 and 3 the fresh call blocking and handoff call dropping probabilities are displayed versus the number of channels with and without user redials. The system parameters belonging to the curves without redials are the same as in the paper of Dharmaraja, Trivedi and Logothetis [2], where a similar model is studied without customer redials (\( \lambda_f = 0.5, \mu = 0.05, \mu_a = \mu_b = \mu_d = 1/3, \lambda_h = 0.4, \nu_{lt} = \nu_{dr} = 10^6, \Theta_1 = \Theta_2 = 10^{-6} \) and for the other curve \( \nu_a = \nu_{dr} = 6, \Theta_1 = 0.8, \Theta_2 = 0.9, \) furthermore the maximum number of redialing customers is 25, respectively). These results are in agreement with theirs in the exponential case.

![Figure 2: Fresh call blocking probability versus number of channels](image)

In Figures 4 and 5 the fresh call blocking and handoff call dropping probabilities are displayed versus the mean handoff call arrival rate. The system parameters are the same as in Figures 2 and 3, except of that \( C = 8, \) and \( \lambda_h \) is on the x axis, like in [2].

The negative influence of the retrial phenomenon is shown in each figures, and we can see that it increases as the handoff call arrival rate increases.
In Figure 6 we can see the fresh call blocking probability, the handoff call dropping probability and the grade of service as the mean fresh call arrival rate increases. The following system parameters were used: $C = 7$, $g = 1$, $\mu = 0.05$, $\mu_a = \mu_b = \mu_d = 1/3$, $\lambda_f = 0.4$, $\nu_{of} = 6$, $\nu_{dr} = 7$, $\Theta_1 = 0.8$ and $\Theta_2 = 0.9$.

In Figure 7 the fresh call blocking and handoff dropping probabilities and the GoS are displayed versus the number of guard channels. We can see that a very few number of guard channels can improve the grade of service significantly, but then only very small handoff dropping advance can be achieved on the great expense of fresh call blocking probability, and the GoS declines. The system parameters are the following: $C = 15$, $\lambda_f = 3$, $\mu = 0.05$, $\mu_a = \mu_b = \mu_d = 1/3$, $\lambda_h = 0.4$, $\nu_{of} = 6$, $\nu_{dr} = 7$, $\Theta_1 = 0.8$ and $\Theta_2 = 0.9$. 
Figure 5: Handoff call dropping probability versus mean handoff call arrival rate

Figure 6: System measures versus mean fresh call arrival rate

5 Conclusion and future work

In this paper a multiserver infinite-source retrial queueing system is studied for the performance modeling of GSM networks. It is shown how easily and efficiently the tool MOSEL can be used, and some numerical examples are presented to show the impact of the retrial phenomenon and some system parameters on the quality of service measures and on the grade of service.

The current study is an initial step towards the analysis of more complex third generation cellular systems. These hierarchical systems may consist two or more layers, and various dynamic channel allocation schemes can be utilized and analyzed. Furthermore, other than exponential distributions can be treated that are supported by both MOSEL and the applied tools.
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